value to those inputs. That process produces an output that is intended to meet or exceed customer requirements. The SIPOC is typically used at the early stages of a project to help characterize a process and to identify appropriate team members. This model is applicable to both product and service processes.

Steps to Create

1. Define the process, name it, and define the start and stop points.
2. Identify suppliers and the critical inputs the process receives from them.
3. Identify the customers of the process (those who receive the outputs) and the outputs of the process that respond to customer needs.
4. Identify the five to eight major process steps that produce the output.
5. Validate the process map by working with the key functions that perform the major steps.

Example An improvement team was created to address the order-receiving process. To help identify the high-level steps and the scope of their project, the team created the SIPOC shown in Figure 18.22, beginning the process with receiving the order and following the process through to the time the product is scheduled for production.

Supplier	Input	Process	Output	Customer
Store location	Electronic order	Receive order	Queue-created file	Order sorter system
Order sorter system	Queue-created file	Hold order in queue	Purchase order TIF file	Order entry
Order entry	Purchase order TIF file	Enter order in system	Order ready for scheduling	Order checker
Order checker	Electronic order	Check order	Scheduling form	Scheduling
Scheduling	Scheduling form	Schedule production	Paper work	Production plant

Figure 18.22 SIPOC, a Six Sigma tool. (*Juran Institute, Inc. Used with permission.*)

Statistical Process Control

Purpose

The daily life of many employees involves operating a process within intended boundaries, that is, to maintain it according to specifications established through quality planning and improvement. Historically, this has relied heavily on inspection, with detection and elimination of nonconforming product after the fact. In contrast, the concept of control over a process entails predicting its performance, within certain limits. Rather than merely detecting nonconforming output ("inspecting quality into a product"), control is forward looking and seeks incremental but continuous improvement by identifying and eliminating special causes that create unpredictable variation (and potentially, but not necessarily, nonconformity to specification).

Statistical process control is the application of statistical methods to the measurement and analysis of variation in a process. A process is a collection of activities that converts inputs into outputs or results. Through use of control charts, statistical process control assists in detecting special (or assignable) causes of variation in both in-process parameters and end-of-process (product) parameters. The objective of a control chart is not to achieve a state of statistical control as an end in itself but to reduce variation.

Before proceeding with the steps to create a control chart, further discussion is warranted regarding common and special cause variation in the context of process control. A statistical control chart compares process performance data to computed “statistical control limits,” drawn as limit lines on the chart. The process performance data usually consist of groups of measurements (called rational subgroups) from the regular sequence of production while preserving the order of the data. A prime objective of a control chart is detecting special (or assignable) causes of variation in a process. Knowing the meaning of “special causes” and distinguishing them from common (random or chance) causes is essential to understanding the control chart concept.

There are two kinds of process variations: (1) common (random or chance), which are inherent in the process, and (2) special (or assignable), which cause excessive variation (see Table 18.1). Ideally, only common causes are present in a process because they represent a stable and predictable process that leads to minimum variation. A process that is operating without special causes of variation is said to be in a state of statistical control. The control chart for such a process has all of the data points within the statistical control limits and exhibits no discernible patterns.

Random (Common) Causes	Assignable (Special) Causes
Description	
Consists of many individual causes	Consists of one or just a few individual causes
Any one random cause results in a minute amount of variation (but many random causes act together to yield a substantial total).	Any one assignable cause can result in a large amount of variation.
Examples are human variation in setting control dials, slight vibration in machines, NS slight variation in raw material.	Examples are operator blunder, a faulty setup, or a batch of defective raw materials.
Interpretation	
Random variation cannot be eliminated from a process economically.	Assignable variation can be detected; action to eliminate the causes is usually economically justified.
An observation within the control limits of random variation means that the process should not be adjusted.	An observation beyond control limits means that the process should be investigated and corrected.
With only random variation, the process is sufficiently stable to use sampling procedures to predict the quality of total production or do process optimization studies.	With assignable variation present, the process is not sufficiently stable to use sampling procedures for prediction.

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Table 18.1 Distinctions between Random and Assignable Causes of Variation

The control chart distinguishes between common and special causes of variation through the choice of control limits. These are calculated by using the laws of probability so that highly improbable causes of variation are presumed to be due to special causes not to random causes. When the variation exceeds the statistical control limits, it is a signal that special causes have entered the process and the process should be investigated to identify these causes of excessive variation. Random variation within the control limits means that only common (random) causes are present; the amount of variation has stabilized, and minor process adjustments (tampering) should be avoided. Note that a control chart detects the presence of a special cause but does not find the cause—that task must be handled by subsequent investigation of the process.

Steps to Create

Setting up a control chart requires taking the following steps:

1. Choosing the characteristic to be charted.
2. Giving high priority to characteristics that are currently running with a high defective rate. A Pareto analysis can establish priorities.
3. Identifying process variables and conditions that contribute to the end-product characteristics to define potential charting applications from raw materials through processing steps to final characteristics. For example, the pH, salt concentration, and temperature of a plating solution are process variables contributing to plating smoothness.
4. Verifying that the measurement process has sufficient accuracy and precision to provide data that does not obscure variation in the manufacturing or service process. The observed variation in a process reflects the variation in the manufacturing process and also the combined variation in the manufacturing and measurement processes. Anthis (1991) described how the measurement process was a roadblock to improvement by hiding important clues to the sources of variation in a manufacturing process. Dechert (2000) explained how large measurement variation can be controlled and result in effective statistical process control methods.
5. Determining the earliest point in the production process at which testing can be done to get information on assignable causes so that the chart serves as an effective early-warning device to prevent defectives.
6. Choosing the type of control chart. Table 18.2 compares three basic control charts. Schilling (1990) provides additional guidance in choosing the type of control chart to use.
7. Deciding on a central line to be used as the basis of calculating the limits. The central line may be the average of past data, or it may be a desired average (i.e., a standard value). The limits are usually set at threes, but other multiples may be chosen for different statistical risks.
8. Choosing the “rational subgroup.” Each point on a control chart represents a subgroup (or sample) consisting of several units of product. For process control, rational subgroups should be chosen so that the units within a subgroup have the greatest chance of being alike and the units between subgroups have the greatest chance of being different.
9. Providing a system for collecting the data. If the control chart is to serve as a day-to-day shop tool, it must be simple and convenient to use. Measurement must be

Statistical Measure Plotted	**Average $\bar{X}$ and Range R**	**Percentage Nonconforming (p)**	**Number of Nonconformities (c)**
Type of data required	Variable data (measured values of a characteristic)	Attribute data (number of defective units of product)	Attribute data (number of defects per unit of product)
General field of application	Control of individual characteristics	Control of overall fraction defective of a process	Control of overall number of defects per unit
Significant advantages	Provides maximum use of information available from data Provides detailed information on process average and variation for control of individual dimensions	Data required are often already available from inspection records Easily understood by personnel Provides an overall picture of quality	Same advantages as p chart but also provides a measure of defectiveness
Significant disadvantages	Not understood unless training is provided; can cause confusion between control limits and tolerance limits. Cannot be use with go/no go type of data	Does not provide detailed information for control of individual characteristics Does not recognize different degrees of defectiveness in units of product	Does not provide detailed information for control of individual characteristics
Sample size	Usually four or five	Use given inspection results or samples of 25, 50, or 100	Any convenient unit of product such as 100 feet of wire or one television set

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Table 18.2 Comparison of Some Control Charts

simplified and kept error free. Indicating instruments must be designed to give prompt, reliable readings. Better yet, instruments should be designed that can record as well as indicate. Recording of data can be simplified by skillful design of data or tally sheets. Working conditions are also a factor.

10. Calculating the control limits and providing specific instructions for the interpretation of the results and the actions that various production personnel are to take (see below). Control limit formulas for the three basic types of control charts are given in Table 18.3. These formulas are based on $\pm 3\sigma$ and use a central line equal to the average of the data used in calculating the control limits. Values of the A2, D3, and D4 factors used in the formulas are given in Table 18.4. Each year, *Quality Progress* magazine publishes a directory that includes software for calculating sample parameters and control limits and for plotting the data. The general rule of thumb is to collect 20 to 30 samples (rational subgroups) before attempting to establish control limits.
11. Plotting the data and interpreting the results.

Chart for	Central Line	Lower Limit	Upper Limit
Averages $\overline{X}$	$\overline{\overline{X}}$	$\overline{\overline{X}} - A_2\overline{R}$	$\overline{\overline{X}} + A_2\overline{R}$
Ranges R	$\overline{R}$	$D_3\overline{R}$	$D_4\overline{R}$
Proportion nonconforming p	$\overline{p}$	$\overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$	$\overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$
Number of nonconformities c	$\overline{c}$	$\overline{c} - 3\sqrt{\overline{c}}$	$\overline{c} + 3\sqrt{\overline{c}}$

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Table 18.3 Control Chart Limits—Attaining a State of Control

Factors for $\overline{X}$ and R Control Charts;* Factors for Estimating s from R[†]				
Number of Observations in Sample	A_2	D_3	D_4	**Factor for s Estimate from $\overline{R}$: $d_2 = \overline{R}/s$**
2	1.880	0	3.268	1.128
3	1.023	0	2.574	1.693
4	0.729	0	2.282	2.059
5	0.577	0	2.114	2.326
6	0.483	0	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
11	0.285	0.256	1.744	3.173
12	0.266	0.284	1.717	3.258
13	0.249	0.308	1.692	3.336
14	0.235	0.329	1.671	3.407
15	0.223	0.348	1.652	3.472

$$\begin{cases} \text{Upper control limit for } \overline{X} = \text{UCL}_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} \\ \text{Lower control limit for } \overline{X} = \text{LCL}_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} \end{cases}$$

$$\begin{cases} \text{Upper control limit for } R = \text{UCL}_R = D_4\overline{R} \\ \text{Lower control limit for } R = \text{LCR}_R = D_3\overline{R} \end{cases}$$

Table 18.4 Factors for $\overline{X}$ and R Control Charts

Stage	Step	Method
Preparatory	State purpose of investigation. Determine state of control. Determine critical variables. Determine candidates for control. Choose appropriate type of chart. Decide how to sample. Choose subgroup size and frequency.	Relate to quality system Attributes chart Fishbone Pareto Depends on data and purpose Rational subgroups Sensitivity desired
Initiation	Ensure cooperation. Train user. Analyze results.	Team approach Log actions Look for patterns
Operational	Assess effectiveness. Keep up interest. Modify chart.	Periodically check usage and relevance Change chart, involve users Keep frequency and nature of chart current with results
Phaseout	Eliminate chart after purpose is accomplished.	Go to spot checks, periodic sample inspection, overall *p*, *c* charts

(*Source:* Schilling 1990.)

Table 18.5 Life Cycle of Control Chart Applications

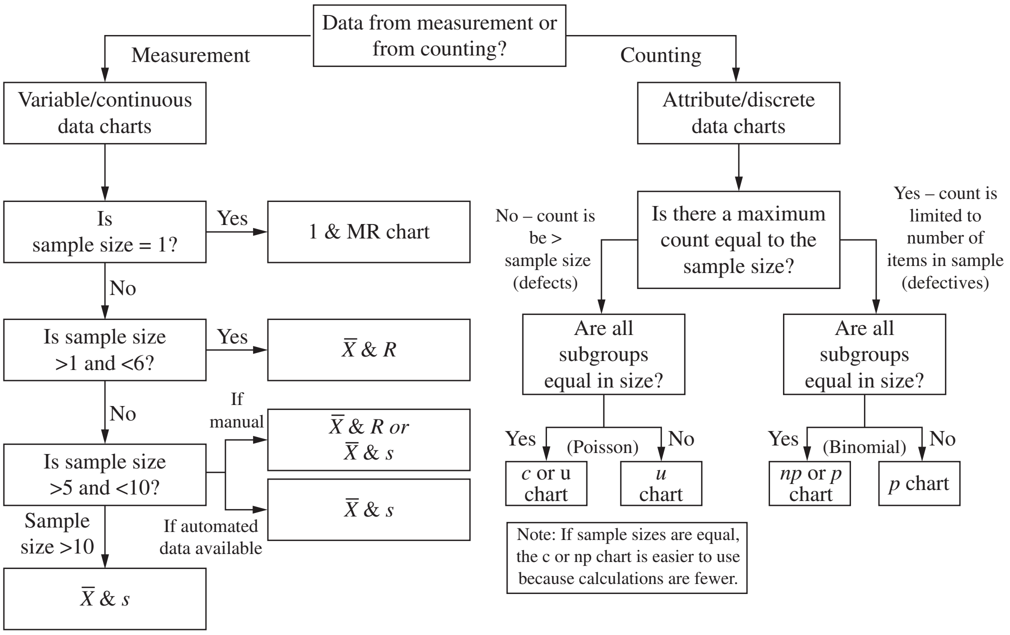
The control chart is a powerful statistical concept, but its use should be kept in perspective. The ultimate purpose of an operations process is to make product that is fit for use—not to make product that simply meets statistical control limits. Once the charts have served their purpose, many should be taken down and the effort shifted to other characteristics needing improvement. Schilling (1990) traces the life cycle of control chart applications (Table 18.5). A given application might employ several types of control charts. Note that, in the phaseout stage, statistical control has been achieved, and some of the charts are replaced with spot checks.

Types of Control Charts

Traditional Shewhart control charts (named for Dr. Walter A. Shewhart; see the *Juran's Quality Handbook* (1999), Section 45 (Juran and Godfrey 1999), for a historical account of their development) are divided into two categories: variable charts (those using continuous, measurement data), and attribute charts (those using count data). Selecting the proper type of control chart is shown in Figure 18.23; the different types are described further below.

Regardless of the specific chart type or statistic (e.g., average, range, standard deviation, proportion), control limits are established such that it would be very unlikely that the values would fall outside if the process were stable; usually this is set at plus or minus three standard deviations.

Examples of Control Charts for Variables Data In these charts, the mean and either range or standard deviation are the typical statistics that are monitored. These statistics are monitored in a pair of charts. The averages chart plots the sample averages, specifically, the average of each rational subgroup (if the rational subgroup size is one, then an individual and moving range chart (X-mR) (also known as an I-mR chart) of individuals is used instead). The range chart or standard deviation chart plots the range or standard deviation of rational subgroups. The specific subtypes are as follows:



Note that "measurement" implies a continuous scale is used to assess a characteristic. "Counting" occurs when outcomes are tabulated as pass/fail, accept/reject, or some other categorization, even if the original measurement was continuous. Also, control limits depend on number of observations in each sample. When subgroup sizes vary, control limits will not be straight lines on any of the charts.

Figure 18.23 Flow chart of control chart selection. (*Juran Institute, Inc. Used with permission.*)

$\bar{X}$ and R chart. Also called the "average and range" chart. $\bar{X}$ refers to the average of a rational subgroup and measures the central tendency of the response variable over time. *R* is the range (difference between the highest and lowest values in each subgroup), and the *R* chart measures the gain or loss in uniformity within a subgroup which represents the variability in the response variable over time. Note that, because specification limits apply to individual values rather than averages (averages inherently vary less than the component individual values), control limits cannot be compared to specification limits which should not be placed on a control chart for averages.

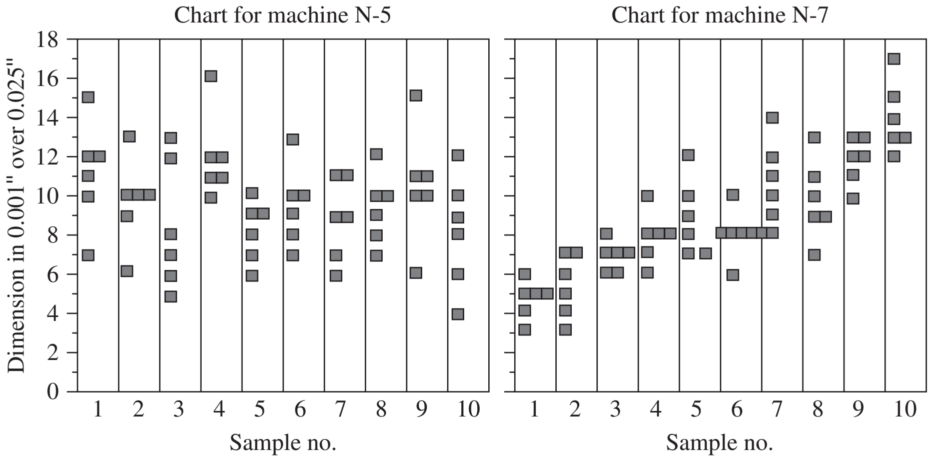
$\bar{X}$ and s *chart.* The average and standard deviation chart is similar to the $\bar{X}$ and *R* chart, but the standard deviation (instead of the range) is used in the *s* chart. Although an *s* chart is statistically more efficient than the range for subgroup sizes greater than 2, a range chart is easier to compute and understand and is traditionally used for subgroup sizes smaller than about 10.

X-mR chart. Also known as an I-mR chart, this charts individual measures and a moving range. It is used when the rational subgroup size = 1 (such that there are no multiple measures from which to obtain an average).

Z-mR chart. This is similar to the X-mR chart, except that the individual values are standardized through a Z transformation. This is useful for short runs in which there are fewer than the recommended 20 to 30 needed to establish one of the preceding charts (see short-run control charts in Chapter 19, Accurate and Reliable Measurement Systems and Advanced Tools).

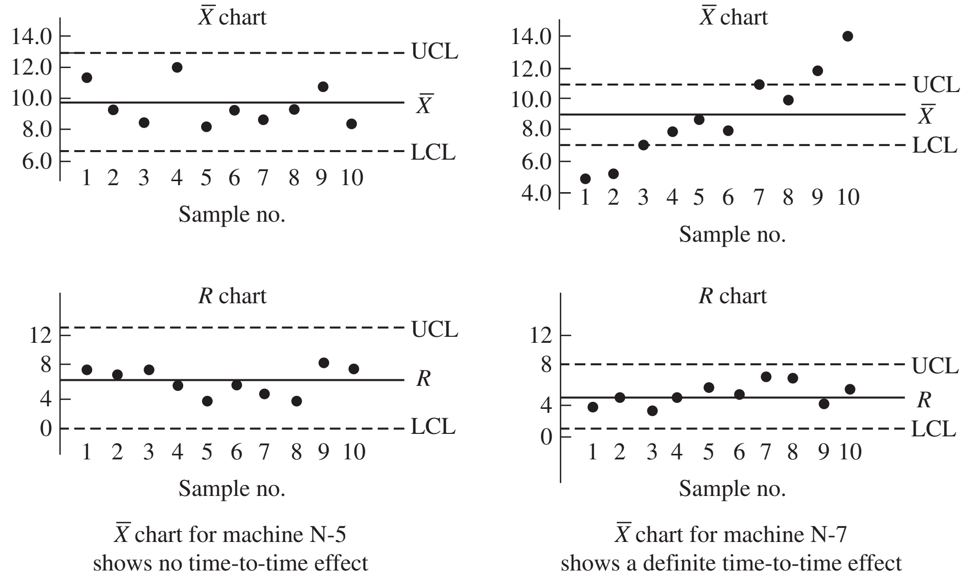
Individuals chart. Also called a run chart, this is an alternative to the $\bar{X}$ and *I* chart, and is simply a plot of individual values against time. In the simplest case, specification limits are added to the chart; in other cases, ±3σ limits of individual values are added. A chart of individual values is not as sensitive as the $\overline{X}$ chart, however.

By way of example of variable control charting, refer to the $\overline{X}$ and R charting in Figure 18.24. The upper part of the figure displays the individual observations for two machines, N-5 and N-7. For each machine, the data consist of 10 samples (with six units



For machine N-5:

Sample	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	11.2	9.7	8.5	12.0	8.2	9.5	8.8	9.3	10.5	8.2
R	8.0	7.0	8.0	6.0	4.0	6.0	5.0	5.0	9.0	8.0



$\overline{X}$ chart for machine N-5 shows no time-to-time effect

$\overline{X}$ chart for machine N-7 shows a definite time-to-time effect

Figure 18.24 $\overline{X}$ and R charts confirm suggested machine differences. (*Quality Planning & Analysis. Juran Institute, Inc., ©2007. Used with permission.*)

in each rational subgroup) plotted in time order of production. The lower portion shows the $\bar{X}$ and R charts for each machine. For machine N-5, all points fall within the control limits, so that (based on this rule), the process appears to be free of assignable causes of variation, and is "in control." However, machine N-7 has both within-sample variation (seen in the range chart) and between-sample variation (seen in the chart of sample averages). The $\bar{X}$ chart indicates some factor (special cause) such as tool wear is present that results in larger values of the characteristic with the passing of time (note the importance of preserving the order of measurements when collecting data).

Interpreting variables charts. Place the charts for $\bar{X}$ and R (or s) one above the other so that the average and range for any one subgroup are on the same vertical line. Observe whether either or both indicate lack of control for that subgroup. Usually, the R (or s) chart is interpreted first because the range or standard deviation is used in calculating limits for the $\bar{X}$ chart.

Rs outside control limits are evidence that the uniformity of the process has changed. Typical causes are a change in personnel, increased variability of material, or excessive wear in the process machinery. If the R or s chart exhibits a special cause variation, then the within-subgroup variation will contain both common and special cause variation, and its use in calculating control limits for the $\bar{X}$ chart will result in excessively large control limits (reducing its ability to detect out-of-control conditions). A single out-of-control R can be caused by a shift in the process that occurred while the subgroup was being taken.

$\bar{X}$s outside the control limits are evidence of a general change affecting all pieces after the first out-of-limits subgroup. The log kept during data collection, the operation of the process, and the worker's experience should be studied to discover a variable that could have caused the out-of-control subgroups. Typical causes are a change in material, personnel, machine setting, tool wear, temperature, or vibration.

Look for unusual patterns and nonrandomness. Nelson (1984, 1985) provides eight tests to detect such patterns on control charts using 3σ control limits (see Figure 18.25). Each of the zones shown is 1σ wide. (Note that test 2 in Figure 18.25 requires nine points in a row, Other authors suggest seven or eight points in a row (see Nelson 1985 for elaboration).

Ott and Schilling (1990) provide a definitive text on analysis after the initial control charts by presenting an extensive collection of cases with innovative statistical analysis clearly described.

Examples of Control Charts for Attribute Data Whereas control charts for variables data require numerical measurements (e.g., line width from a photoresist process), control charts for attribute data require only a count of observations of a characteristic (e.g., the number of nonconforming items in a sample). These also are called categorical data because units are classified into groups such as pass and fail.

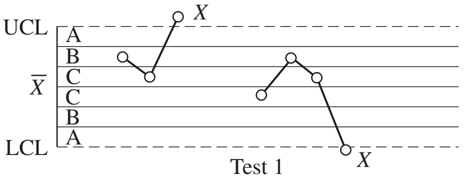
p *chart.* Also called a proportions chart, this tracks the proportion or percentage of nonconforming units (percentage defective) in each sample over time.

np *chart.* This chart is used to track the number of nonconforming (defective) units in each sample over time. An *np* chart should only be used when the number of units sampled is constant (or nearly so).

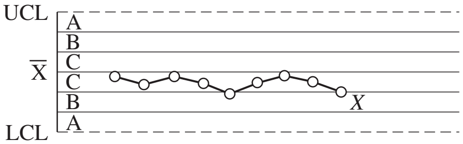
c *chart.* Used to track the number of nonconformities (i.e., defects, rather than defective units as in the *p* chart).

u *chart.* A variation of the *c* chart, and analogous to the *np* chart, this chart tracks the number of nonconformities (defects) per unit in a sample of *n* units. As with the *np* chart, the number of units should be approximately constant.

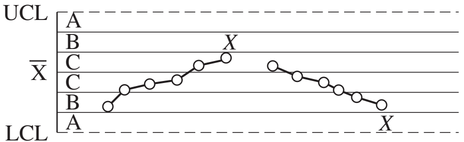
As an example of attribute control charting, the fraction nonconforming (p) chart can be illustrated with data on magnets used in electrical relays. For each of 19 weeks, the number of magnets inspected and the number of nonconforming magnets were recorded. There was a total of 14,091 magnets tested. The total number nonconforming was 1030, or 7.3 percent The resulting control chart (calculating control limits based on average sample size of 741.6)



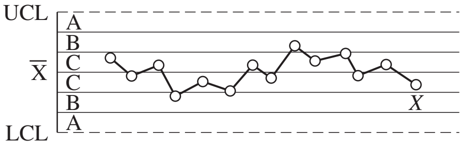
Test 1
One point beyond zone A



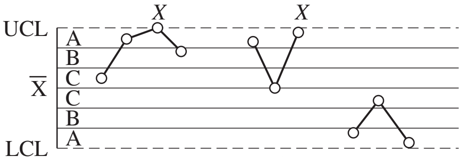
Test 2
Nine points in a row in
zone C or beyond



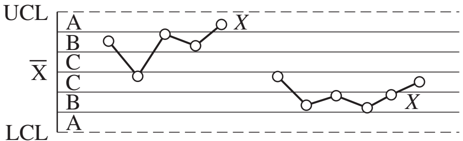
Test 3
Six points in a row steadily
increasing or decreasing



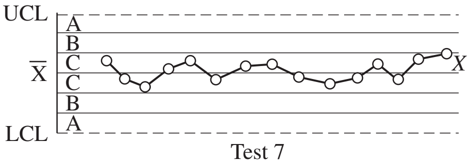
Test 4
Fourteen points in a row
alternating up and down



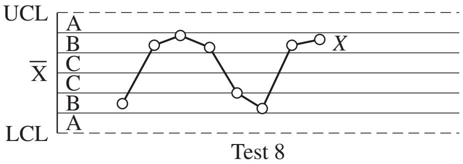
Test 5
Two out of three points in
zone A or beyond



Test 6
Four out of five points in
a row in zone B and beyond



Test 7
Fifteen points in a row in zone C
(above and below centerline)



Test 8
Eight points in a row on
both sides of centerline

Figure 18.25 Tests for special causes applied $\bar{X}$ to control charts. (*Nelson 1984.*)

is shown in Figure 18.26. Note that several points fall beyond the control limits, suggesting that there are special cause(s) at work. In the case of the unusually low point for the last sample, it may be useful to identify and reinforce any special cause of the exceptionally good quality. The same rules as described above in Figure 18.25 also apply to attribute charts.

Stratification

Purpose

Stratification is the separation of data into categories. The most frequent use is during problem analysis to identify which categories contribute to the problem being solved. However, stratification can be applied when identifying projects, analyzing symptoms, testing hypotheses, and developing solutions. Stratification helps answer questions as to the frequency of defects, factors that may be contributing to a quality problem, and the degree to which results may differ across groups (strata).

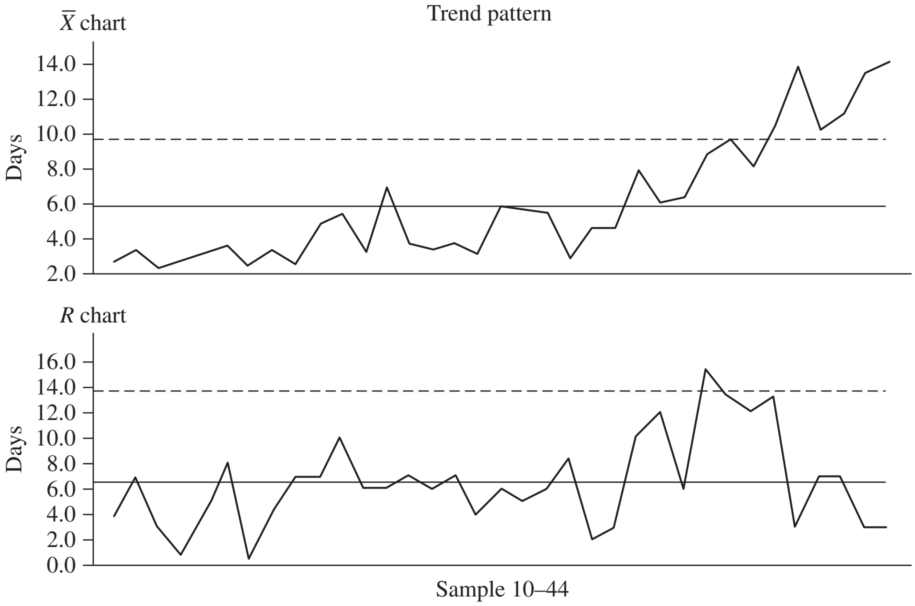


Figure 18.26 Average and range control charts. (*Quality Planning & Analysis. Juran Institute, Inc., ©2007. Used with permission.*)

Steps to Create

1. Select the stratification variables. If new data are to be collected, be certain that all potential stratification variables are collected as identifiers.
2. Establish categories that are to be used for each stratification variable. The categories may be either discrete values or ranges of values.
3. Sort observations into the categories of one of the stratification variables. Each category will have a list of the observations that belong to it.
4. Calculate the phenomenon being measured for each category. These calculations can be a count of the number of observations in the category, an average value for those observations or a display (like a histogram) for each category.
5. Display the results. Bar graphs are usually the most effective.
6. Prepare and display the results for other stratification variables. Repeat steps 2 through 5. Do second-stage stratification as appropriate.
7. Plan for addition confirmation.

Example A manufacturer of mechanical equipment had recently received a rash of complaints about pins (stock number 128B) coming loose from press-fit sockets. The sockets were produced internally by the manufacturer under good statistical process control. The steel pins that fit into the sockets were purchased from three different suppliers. (see Figure 18.27)

The quality improvement team looking into the complaints measured the diameter of 120 pins from inventory, 40 from each of the three suppliers. The nominal value for the pin diameter was 10 mm. The upper specification limit was 10.2 mm, and the lower limit was 9.8 mm.

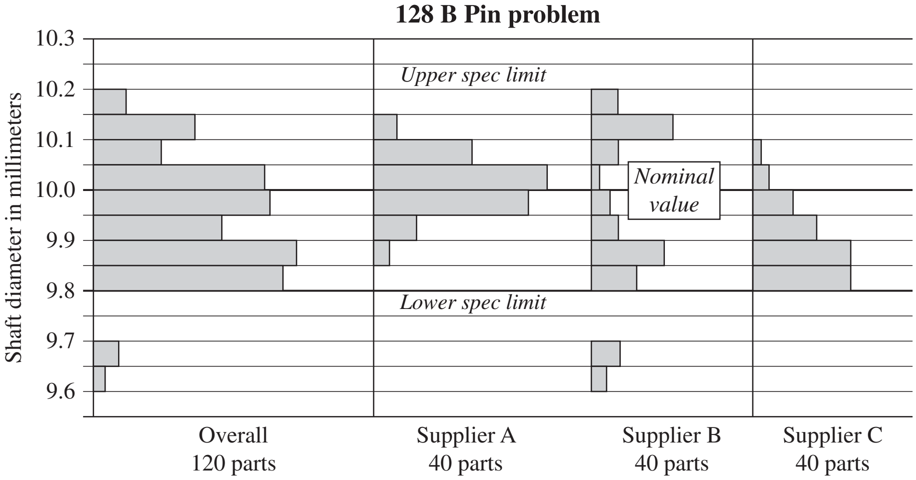


Figure 18.27 Stratification of the 128B pin problem. (*Lean Six Sigma Pocket Guide, Juran Institute, Inc., ©2009. Used with permission.*)

To get a better understanding of the data, the team produced a histogram of all 120 parts. The histogram showed that the pin diameter measurements had a broad, multipeaked distribution, with most of the data between the lower specification limit and the nominal value. Because most of the pins were smaller than nominal, there was indeed a good chance of a loose fit.

This summary histogram, however, did not tell the team much about what the cause of the problem was. So the team decided to stratify the data by supplier and to plot new histograms.

On the basis of the histograms on the previous page, the team drew the following conclusions:

- Supplier A has good controls on its process. Most of the product is close to the nominal value, and because the inherent variability in the process is smaller than the width of the specification limits, there is little chance of producing a part outside the limits.
- Supplier B appears to be running two distinct processes, neither of which has been set up to produce pins with diameters close to the nominal. The shape of the distribution for supplier B looks like the sum of two distributions similar to that of supplier A, one of which has been shifted up a bit, the other shifted down.
- Supplier C has a process that is highly variable and not set up to produce pins at the nominal value. The abruptly ended (or truncated) nature of the distribution suggests that the supplier is using inspection to screen out off-spec pins.

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