Effects of Parameter Estimation on Control Chart Properties: A Literature Review

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Control charts are powerful tools used to monitor the quality of processes. In practice, control chart limits are often calculated using parameter estimates from an in-control Phase I reference sample. In Phase II of the monitoring scheme, statistics based on new samples are compared with the estimated control limits to monitor for departures from the in-control state. Many studies that evaluate control chart performance in Phase II rely on the assumption that the in-control parameters are known. Although the additional variability introduced into the monitoring scheme through parameter estimation is known to affect the chart performance, many studies do not consider the effect of estimation on the performance of the chart. This paper contains a review of the literature that explicitly considers the effect of parameter estimation on control chart properties. Some recommendations are made and future research ideas in this area are provided.

Key Words: ARL; Conditional Distribution; Marginal Distribution; Phase I; Phase II; Run-Length Performance; Sample Size; Shewhart Chart; Statistical Process Control.

CONTROL charts are known to be effective tools for monitoring the quality of processes and are applied in many industries. When the parameters representing some quality characteristic of the process are unknown, control charts can be applied in a two-phase procedure. In Phase I, control charts are used retrospectively to study a historical reference sample. This use of charts includes defining the in-control state of the process and assessing process stability to ensure that the reference sample is representative of the process. Once an in-control reference sample is established, the parameters of the process are estimated from this Phase I sample and control limits

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are estimated for use in Phase II. In Phase II, samples from the process are prospectively monitored for departures from the in-control state. If the successively observed chart statistics are plotted within the control limits, the process is deemed *stable* or *in control*. Chart statistics that are plotted outside of the control limits are signals that the process may be *out of control* and corrective action on the process may be needed.

Most of the research that involves the development and evaluation of Phase II control charts is based of the assumption of some stochastic model that serves as an approximation. In addition, the in-control process parameters are assumed to be known. For example, when univariate process data are assumed to follow a normal distribution, the process parameters of interest are the in-control mean, μ_0, and in-control standard deviation, σ_0. For serially correlated data, parameters include time series model coefficients. The assumption that the in-control values of parameters are known simplifies the development and evaluation of control charts. In practice, the parameters are rarely known, and control charts are usually based on estimated parameters. When estimates are used in place of known parameters, the variability of the estimators can result in chart performance that differs from that of charts designed with known parameters.

The potential weakness of using limited amounts of data or using sample information that is not representative of the process to determine control limits has long been recognized. Shewhart (1939, p. 76) wrote, "In the majority of practical instances, the most difficult job of all is to choose the sample that is to be used as the basis for establishing the tolerance range (control limits)." Collecting a representative sample of sufficient size will ensure accurate control limits. However, much of the literature contains underestimates of the amount of data needed to obtain sufficiently accurate estimates of the control limits. For example, a typical recommendation for $\overline{X}$ charts is to take 20–30 small samples in Phase I (Montgomery, 2005, p. 168).

Woodall and Montgomery (1999) identified the effects of parameter estimation on control chart properties as an important research topic. Prior to 1999, there was relatively little research done in this area and only for a few types of charts. Since that time, there has been much more research done on the issue. In the recent articles, three major questions often arise: "Just how poorly (or well) might a chart perform if designed with estimates in place of known parameters?", "What sample size is needed in Phase I to ensure adequate performance in Phase II?", and "How should the Phase II limits be adjusted to compensate for the size of the Phase I sample?" Research related to these three questions is considered in this review. It should be noted that some procedures, such as self-starting control charts and changepoint methods, avoid the distinction of Phase I and Phase II altogether and provide alternative approaches when process parameters are not known.

An important issue related to the effect of parameter estimation on control chart performance is the selection of the best parameter estimator(s). There has been considerable work published on what makes for appropriate estimator(s) for various types of control charts. In the case of Shewhart charts, see, for example, Cryer and Ryan (1990), Cruthis and Rigdon (1991), and Derman and Ross (1995). In the case of estimation in multivariate control charts, see Sullivan and Woodall (1996, 1998) and Vargas (2003). These authors generally considered properties of estimators such as the mean squared error or the ability to detect various kinds of process changes in Phase I, rather than the Phase II performance of the control chart. While an important issue, this work is not explicitly reviewed in this paper.

Control Chart Performance Measures

Performance measures are needed to study and compare the performance of control charts. Aroian and Levene (1950) considered several performance measures in the case of known parameters and recommended aspects of the run length distribution to evaluate control chart performance. The run length (RL) of a control chart is a random variable that represents the number of plotted statistics until a signal occurs. If the plotted statistics are independent and identically distributed (i.i.d.) random variables and the control limits are known constants, such as can be the case for Shewhart charts, the RL is a geometric random variable with parameter Pr(*signal*), representing the probability that a single chart statistic falls outside of the control limits. Furthermore, the RL follows a geometric distribution no matter the assumed distribution of the data, as long as the plotted statistics are i.i.d. random variables. If the process is in control, the probability of a signal is related to the frequency of false alarms. However, when parameters are estimated, the RL distribution is not geometric and thus the probability of a signal does not have a meaningful interpretation.

An intuitively appealing and more widely applicable measure of control chart performance is the average run length (ARL). The ARL is the expected number of plotted chart statistics before a signal is observed. A typical $\overline{X}$ chart with 3-sigma limits based on known parameters has an in-control ARL of 370 under the assumption of normally distributed data. This indicates that a practitioner can expect to obtain a signal, on average, once in every 370 plotted statistics. The out-of-control ARL is a measure of how quickly an out-of-control situation will be detected. If the RL of a chart is a geometric random variable, the chart's in-control and out-of-control ARLs are computed as $1/(\Pr(\text{signal}))$. The ARL is the most common measure of control chart performance, and much of its popularity is due to its intuitively appealing interpretation. Thus, it is often used to design and to compare the performance of control charts.

Fortunately, it is usually true that the in-control ARL is larger than the out-of-control ARL for any size shift in the process. A control chart for which this property holds is defined as an *ARL-unbiased* chart. Krumbholz (1992) developed a method for obtaining an R chart with this property. Champ and Lowry (1994) gave a method for designing an ARL-unbiased S chart, which was used by Champ (2001) to design an R chart with this property. ARL-unbiased charts were also discussed by Acosta-Mejía and Pignatiello (2000) and Zhang and Chen (2002). In some cases, the maximum ARL occurs when there has been some small shift rather than when there is no shift. A control chart with this property is called an *ARL-biased* chart. The ARL-biased chart is usually undesirable, especially if its difference from the ARL-unbiased chart is extreme.

When the process parameters are unknown and estimates from Phase I are used to construct the control chart, the properties of the run length, including the ARL, must be interpreted carefully. Before the data are gathered in Phase I, the run length of a control chart is dependent on the random parameter estimators. Let the run length be the random variable denoted by T and assume that the unknown in-control process mean and standard deviation, μ_0 and σ_0, are to be estimated using the sample estimators V and W, the observed values of which will be denoted by v and w. The capital letters V and W are used to distinguish the random variables from their observed values, v and w, respectively. Because of the dependence of T on the random variables V and W, it is often useful to consider the conditional probability mass function, $f_{T|V,W}(t \mid v, w) = P(T = t \mid v, w)$, which implies that the run length has a different distribution for each possible value of $V = v$ and $W = w$. Similarly, moments and percentiles of this conditional distribution take different values for each value of v and w; for example, $E_T(T \mid v, w)$ is a random variable which depends on v and w. Once the Phase I data are gathered and the values v and w have been observed, the conditional RL distribution (mass function) of the chart will be $f_{T|V,W}(t \mid v, w)$ and the ARL of the chart will be $E(T \mid v, w)$.

Unfortunately, it is often impossible to tell a practitioner how a specific control chart constructed with estimated parameters will perform using the conditional distribution. In order to evaluate specific chart performance in this case, one needs to know μ_0 and σ_0 in addition to v and w. Of course, if one had the parameter values, then the parameter estimates would not need to be computed, and evaluation of the chart performance would be done via traditional methods for known parameters. However, one can consider hypothetical cases of this conditional distribution in order to gain insight about the best and worst case performance scenarios for charts with estimated parameters. For example, Jones et al. (2001) rewrote the chart statistic and the RL distribution of an exponentially weighted moving average (EWMA) chart with estimated parameters in terms of the random variables

$$Z_0 = \sqrt{m}\frac{V - \mu_0}{\sigma_0/\sqrt{n}}, \tag{1}$$

and

$$Z_1 = \frac{W}{\sigma_0}, \tag{2}$$

where m is the number of subgroups and n is the number of observations per subgroup. Here, Z_0 is a random variable that represents the standardized difference between the in-control mean and the estimated mean, and Z_1 represents the ratio of the estimated standard deviation to the in-control standard deviation. Hypothetical values of Z_0 and Z_1 can be assumed and substituted into the conditional RL distribution, which can be rewritten as $f_{T|Z_0,Z_1}(t \mid z_0, z_1) = P(T = t \mid Z_0 = z_0, Z_1 = z_1)$. For example, the RL performance of a chart constructed with a sample mean that under- or overestimates μ_0 by one, two, or three standard errors can be studied. Similarly, the performance of a chart constructed with a sample standard deviation that under- or overestimates σ_0 by a factor of one half or two can be evaluated. This approach was used by Hawkins and Olwell

(1998, pp. 159–160), Jones et al. (2001, 2004), and Shu et al. (2004).

In contrast with the conditional RL distribution, the marginal RL distribution, $f_T(t)$, takes into account the random variability introduced into the charting procedure through parameter estimation and does not require knowledge of the observed estimates. The marginal RL distribution is the conditional RL distribution averaged over all possible values of the parameter estimators and can be used before the Phase I data are gathered to make decisions regarding the sample size requirements necessary to achieve desired performance. The marginal RL distribution is computed by integrating the conditional distribution over the range of the parameter estimators and if V and W are independent, is given by

$$f_T(t) = \int_0^\infty \int_{-\infty}^\infty f_{T|V,W}(t \mid v, w) f_V(v) f_W(w)\, dv\, dw. \tag{3}$$

Many authors, including Burroughs et al. (1993, 1995), Chen (1997), Chakraborti (2000), and Jones et al. (2001), have used the marginal RL distribution and its moments to evaluate the performance of control charts with estimated parameters. When $f_T(t)$, its moments, or its percentiles are used to measure chart performance, it is important to keep in mind that the marginal distribution does not describe the RL performance of any specific chart, but rather is averaged over all possible charts constructed using sample estimates from an in-control Phase I sample of the same size. Thus, the marginal distribution will not inform a practitioner how a specific chart will perform, but it does measure how charts computed using a given design procedure perform on average.

The marginal and conditional RL distributions serve different purposes in evaluating the performance of control charts with estimated parameters. While some authors have advocated the use of either the marginal or the conditional RL distributions to evaluate the performance of control charts with estimated parameters, both should be considered to gain a complete understanding of chart performance. The marginal RL distribution can be used for general performance evaluations, and, because it is meaningful before the data are gathered in Phase I, it can be used for design recommendations, including the necessary sample size to achieve desired performance. The conditional RL distribution allows practitioners to gain specific information about best- and worst-case estimation scenarios and lets one know how badly a chart with poorly estimated parameters could perform. Note that, after observing the preliminary sample, no more is known about the conditional RL distribution than before these data are observed.

To simplify comparisons and give a more complete picture of the RL distribution, summary values from both the marginal and conditional RL distributions can be used to effectively evaluate the performance of charts with estimated parameters. For example, the ARL, the standard deviation of the run length (SDRL), and the percentiles of the RL distribution are often used. If the RL distribution is geometric, as is the case for the conditional run length distribution of Shewhart charts, then the ARL (and consequently the probability of a signal) completely characterizes the distribution and is the accepted measure of performance. However, conditional and marginal distributions are often more strongly right skewed than the geometric distribution, as is usually the case when parameters are estimated. For this reason, it is recommended to supplement the ARL with the SDRL and various upper and lower percentiles to effectively evaluate these charts. For examples of authors who consider both the marginal and conditional RL distributions and supplement the ARL with other measures to evaluate chart performance when estimates are used, see Jones et al. (2004) or Shu et al. (2004).

Some authors have suggested other performance measures for control charts with estimated parameters. Albers and Kallenberg (2004a, c, 2005) suggested the use of exceedance probabilities to study Shewhart Individuals (X) and $\overline{X}$ charts with estimated parameters. The exceedance probability measures how much larger the probability of a signal would be when compared to the desired probability of signal when the parameters are assumed known. This exceedance probability is then used to study the performance of the charts and to recommend adjustments to the control limits. A limitation is that this measure is only useful for Shewhart charts based on independent observations.

Shewhart Charts

In this section, it is assumed that the process to be monitored yields some quality characteristic values, X_{ij}, $i = 1, \ldots, m$, $j = 1, \ldots, n$, that are normally distributed with in-control values of the mean, μ_0, and standard deviation, σ_0. In Phase I, a sample of size n at each of m time intervals is taken and μ_0 is estimated by the overall sample mean, which is denoted by $\widehat{\mu}_0$. The in-control value of the standard deviation, σ_0, can be estimated in a variety of ways, such as by the average sample range ($\widehat{\sigma}_{\bar{R}}$), the average standard deviation ($\widehat{\sigma}_{\bar{S}}$), the pooled standard deviation ($\widehat{\sigma}_{S_p}$), or an overall standard deviation ($\widehat{\sigma}_{S_O}$). When $n = 1$, as in the case of Individuals charts, $\widehat{\mu}_0$ is still the sample mean and σ_0 is estimated using an average moving range ($\widehat{\sigma}_{MR}$) or the sample standard deviation ($\widehat{\sigma}_S$).

For discussion of the relative merits and properties of the above estimators for Shewhart charts, see Cryer and Ryan (1990), Cruthis and Rigdon (1991), Derman and Ross (1995), Del Castillo (1996a), Trietsch (2001), Champ and Jones (2004), and Montgomery (2005). In general for Phase I applications, $\widehat{\sigma}_{MR}$ is preferred for Individuals charts and $\widehat{\sigma}_{S_p}$ for $\overline{X}$ charts. Robust estimators will be preferable for situations where outliers are present, but their benefit is primarily for Phase I applications. The choice of estimator has an impact on the control chart properties in Phase II, as will be seen in later discussion. It should be noted that the estimator used in the Phase I analysis does not necessarily have to be the same one used to construct control limits for use in Phase II.

Early Investigation

In several early studies, it was recognized that, when using estimated parameters, the probabilities of a signal are different than in the known-parameters case. The usual strategy was to adjust the control limits so that the desired unconditional probability of a false alarm was maintained and the appropriate sample size could then be determined. This approach was taken by King (1954), Proschan and Savage (1960), Hillier (1964, 1967, 1969), and Yang and Hillier (1970) for various Shewhart charts. However, as noted by Ghosh et al. (1981) and Quesenberry (1993), the approach of these earlier papers has limited practical value because the unconditional probability of a signal is treated as a constant (rather than a random variable) to determine control chart performance. In addition, the earlier research ignores the dependence between successive Phase II in- or out-of-control decisions that arises from the use of Phase I estimates to compute the control limits. Tables 1 of Quesenberry (1993) and Maravelakis (2002) show that this dependence is positive and decreases as m and n increase.

$\overline{X}$ Charts

Ghosh et al. (1981) considered the marginal distribution of the RL for one-sided and two-sided $\overline{X}$ charts when $\widehat{\sigma}_{S_p}$ is used. They showed that the lower bound on the in-control ARL is the value of the ARL for the known-parameter case, and that the RL distribution converges to a geometric distribution as $m \to \infty$. Using numerical integration to evaluate the marginal RL distribution, they found for a limited number of sample sizes and shifts, that both the in-control and out-of-control ARLs for the estimated parameter case are higher than for the known-parameter case. This result was confirmed in an empirical study by Ng and Case (1992) of the in-control ARL for an $\overline{X}$ chart based on $\widehat{\sigma}_{\bar{R}}$ rather than $\widehat{\sigma}_{S_p}$. Furthermore, Ng and Case (1992) showed that, as m increases, the in-control ARL for the estimated parameter case converges to the ARL value for the known-parameter case.

An influential paper by Quesenberry (1993) contained a more comprehensive simulation study that showed similar results for $\overline{X}$ charts based on $\widehat{\sigma}_{\bar{S}}$. When $n = 5$, he recommended $m \geq 100$ in order for the $\overline{X}$ chart to behave almost like the equivalent $\overline{X}$ chart with known parameters. Just as was shown for $\widehat{\sigma}_{\bar{R}}$ and $\widehat{\sigma}_{S_p}$, both the in-control and out-of-control ARL and SDRL were higher for the estimated parameters case. Quesenberry (1993) made the important observation that a higher in-control ARL is not necessarily better because the RL distribution will reflect an increased number of short RLs as well as an increased number of very long RLs. In other words, the distribution of the RL with estimated parameters will be flatter, with heavier tails than the RL distribution with known parameters (a geometric distribution) even though both distributions are right skewed.

Del Castillo (1996a) did a similar study and showed that improved performance of the $\overline{X}$ chart can be obtained by using $\widehat{\sigma}_{S_p}$ rather than $\widehat{\sigma}_{\bar{R}}$ or $\widehat{\sigma}_{\bar{S}}$. The $\overline{X}$ chart based on $\widehat{\sigma}_{S_p}$ performs more like the known-parameter case; thus the out-of-control ARL, as well as the quantiles of the RL distribution, were improved by using $\widehat{\sigma}_{S_p}$. Del Castillo (1996b) provided a program that can be used to numerically estimate the marginal ARL of $\overline{X}$ charts with estimated parameters.

Chen (1997) considered $\overline{X}$ charts based on $\widehat{\sigma}_{\bar{R}}$, $\widehat{\sigma}_{\bar{S}}$, and an adjusted version of $\widehat{\sigma}_{S_p}$ and made similar sample size recommendations as those of Quesenberry (1993). He found that the impact of parameter estimation on out-of-control performance is more severe for smaller shifts in the process than for larger shifts and that the impact is greater on the SDRL than the ARL. Thus, a larger sample size is required for the SDRL to be sufficiently close to the SDRL for known parameters. Similar results were obtained by Chakraborti (2000), but he recommended much larger sample sizes (about 500–1000) than did Quesenberry (1993) and Chen (1997).

In summary, for $\overline{X}$ charts, the in-control and out-of-control ARL and SDRL are higher when estimated parameters are used than when parameters are known, no matter the estimator of σ_0. The use of $\widehat{\sigma}_{S_p}$ is preferred because it results in chart behavior most like the known-parameter case, although it should be noted that use of $\widehat{\sigma}_{S_p}$ is simply choosing the lesser evil when m is small.

Burroughs et al. (1993) studied the effect of using runs rules on $\overline{X}$ charts. They considered three common rules, C_1 (one point beyond 3 standard deviations from the center line), $C_{12} = C_1 \cup C_2$ ($C_2 = 2$ out of 3 points beyond 2 standard deviations from the center line), and $C_{123} = C_1 \cup C_2 \cup C_3$ ($C_3 = 4$ out of 5 points beyond 1 standard deviation). Note that C_1 is the simple Shewhart chart with no runs rules while C_{12} and C_{123} are combinations of multiple rules. They used numerical integration to evaluate the marginal ARL and found, when C_1 or C_{12} are used, both the in-control and out-of-control ARLs are higher than the known parameter case, but when C_{123} is used, the in-control ARL is actually lower than would be expected and the out-of-control ARL is higher than expected. Thus, while additional runs rules improve the sensitivity of the chart (Champ and Woodall, 1987), they cause the chart performance to differ more from the known-parameter chart performance. However, the study of Burroughs et al. (1993) is not comprehensive enough to make a complete determination.

Wu et al. (2002) considered robust estimators to obtain the control limits for $\overline{X}$ charts. They studied, via simulation, seven different estimators of σ_0: $\widehat{\sigma}_{\bar{R}}$, $\widehat{\sigma}_{\bar{S}}$, $\widehat{\sigma}_{S_p}$, one based on the absolute deviations from the mean, and three others based on deviations from the median. The three estimators based on deviations from the median were found to be biased, but correction factors found via simulation were provided. For normally distributed data, the four estimators based on deviations give comparable in-control and out-of-control ARL performance to the three classical estimators. For data coming from a contaminated normal distribution (a mixture of two normal distributions), the in-control ARL performance for all seven estimators worsened, indicating an increased number of signals when the data are not normally distributed. This emphasizes the need to use some other appropriate control chart scheme when the data are not normally distributed. Out-of-control ARL performance for the estimators based on the median is slightly better than the classical estimators, but this performance is negated by the corresponding decrease in the in-control ARL.

Individuals (X) Charts ($n = 1$)

Quesenberry (1993) considered the effects of parameter estimation on Individuals charts. Through simulation he found that, just as in the case of $\overline{X}$ charts, the in-control ARL increases when parameters are estimated. The sample size recommended to achieve similar performance to that of a chart with known parameters is larger than the number of subgroups needed for $\overline{X}$ charts, although the total number of observations is smaller. Rigdon et al. (1994) did a similar study and found the RL distribution approaches the known-parameter case as the sample size increases. They concurred with the conclusions of Quesenberry (1993) and made a rough recommendation that at least 100 observations are needed in Phase I.

In a recent paper, Albers and Kallenberg (2004a) studied the Individuals chart using exceedance probabilities and the ARL as performance measures. Their strategy consisted of applying corrections to the control limits to reduce the number of samples needed to ensure that the exceedance probabilities are sufficiently small. However, they presented limited results on the out-of-control performance by studying only larger shifts in the mean. Because corrections to the control limits improve the in-control ARL, but likely have an adverse effect on the out-of-control ARL, these limited results do not give a complete picture of the performance of the control chart scheme. One who uses these corrections will have good in-control performance, but it is likely that ability to detect shifts quickly will be reduced. The related paper of Albers and Kallenberg (2004c) contained a larger study of out-of-control performance. However, their calculation of the ARL is based on an approximation that requires large sample sizes to be useful. For example, the approximate ARL was calculated to be a negative number for very small sample sizes, which is obviously problematic. Thus, one should be wary of their sample size recommendations because their determination of the needed Phase I sample size is based on an approximation of the ARL. It is not clear if the approximation is accurate unless the sample size is large enough. Corrections to the control limits are useful in maintaining in-control performance, but they also result in a negative impact on out-of-control performance that has not been studied adequately.

Maravelakis et al. (2002) studied Individuals charts to monitor changes in the variability. Their use of the Individuals chart to monitor variability instead of the traditional moving range chart is consistent with the recommendation of Rigdon et al. (1994) and others to use a single chart to monitor both the mean and the variability. Maravelakis et al. (2002) did not consider decreases in the standard deviation, as this cannot be detected by the Individuals chart unless a specific runs rule is used (such as 15 points in a row all within 1 standard deviation of the mean). For detecting increases in variation, a sample size of at least 300 was recommended because of the higher marginal in-control and out-of-control ARL and SDRL values than for the known-parameter case. This recommendation is consistent with that of Quesenberry (1993) for Individuals charts for monitoring the mean.

Charts for Dispersion (R, S, and S^2 Charts)

Chen (1998) studied the marginal RL distribution of R, S, and S^2 charts, where the R chart was based on $\widehat{\sigma}_{\bar{R}}$; the S chart on $\widehat{\sigma}_{\bar{S}}$; and the S^2 chart on $\widehat{\sigma}_{S_p}$. For all three charts, the in-control ARL is lower in the estimated parameter case, suggesting that more short RLs will occur, but the out-of-control ARL is higher, suggesting that changes in variability will not be detected very quickly. He concluded that this deteriorating performance is worse for increases in the variability than for decreases, although the deterioration is lessened as the magnitude of the shift gets larger. Thus, the impact of parameter estimation is more pronounced for Shewhart charts that are used to monitor the variability than for those that are used to monitor the mean. To achieve adequate performance for detecting changes in the standard deviation with $4 \leq n \leq 10$, he recommended that at least 75 Phase I samples be taken.

Maravelakis et al. (2002) performed a similar study on the S chart and made similar conclusions as those of Chen (1998). Although their sample size recommendations are larger ($m \geq 100$ for $n \geq 20$), this may be due to the fact that they used $\widehat{\sigma}_{\bar{S}}$ rather than $\widehat{\sigma}_{S_p}$. In addition, their results show that the S chart based on estimated parameters can be ARL biased. This agrees with the result of Champ and Lowry (1994), who showed that S charts can be ARL biased in the known-parameter case. Zhang et al. (2005) studied modifications of the S^2 chart, resulting in an ARL-unbiased chart and two versions of ARL-biased charts. They considered the appropriate sample size to achieve adequate performance and found that the ARL-biased charts have slightly smaller sample size requirements than the ARL-unbiased chart.

Building on the study of Burroughs et al. (1993), Burroughs et al. (1995) studied S charts with runs rules and found similar results. Just as with $\overline{X}$ charts, use of multiple runs rules worsens the performance more than a single runs rule.

Other Types of Charts

Much of the work on the effects of parameter estimation has been performed on Shewhart charts. Some of the conclusions will hold for other types of charts; such as, larger sample sizes are needed to ensure performance similar to charts based on known parameters. However, not all of the conclusions concerning Shewhart charts hold for other types of charts, so it is important to evaluate each type of chart individually.

EWMA Charts

Jones et al. (2001) studied the marginal and conditional RL performance of the EWMA chart with estimated mean and standard deviation. Based on properties of the conditional RL distribution, they showed that overestimating the variance leads to a chart that signals less frequently, whether the process is in or out of control. Similarly, underestimating the variance leads to a chart that signals more frequently in all cases. When the mean is estimated, the EWMA chart always signals more frequently when the process is in control; however, the out-of-control chart performance depends on the direction of the process shift relative to the estimated mean. The marginal RL distribution of the EWMA chart was used to make sample size recommendations in order for the chart based on estimates to perform similarly to one based on known parameters. A common recommendation is to make the smoothing constant (λ) small for EWMA charts in order make them more sensitive to small process shifts. However, the smaller the value of λ, the larger the sample size needs to be to ensure performance similar to that of a chart based on known parameters. Jones et al. (2001) recommended 100 samples of size 5 for $\lambda = 0.5$ and 400 samples of size 5 for $\lambda = 0.1$. Thus the required Phase

I sample size is strongly dependent on the value of λ used.

When it is difficult to obtain the large number of samples recommended, Jones (2002) considered modifying EWMA charts so that they have a specified in-control ARL. Rather than design optimal charts for given shifts with the parameters assumed to be known, it is better to design the charts so that they will perform well in the estimated-parameters case. The latter charts will have slightly worse performance than charts with known parameters for detecting shifts of a given size, but will be much better in terms of in-control performance.

The dependence of the out-of-control ARL performance of EWMA charts on the type of shift can also be seen in the results of Zhang and Chen (2002). They showed that EWMA charts are ARL unbiased when the variance is estimated; but when the mean is estimated, EWMA charts are ARL biased. They also gave a formula for the sample size needed to have a specified probability that the mean estimator will be within an acceptable range of the parameter value. For example, their worst-case scenario showed that, to have a 90% chance of the estimated mean being within 0.1 standard deviations of the parameter value, at least 55 samples of size 5 are needed.

CUSUM Charts

Bagshaw and Johnson (1975) studied the marginal and conditional ARL performance for CUSUM charts when the standard deviation is estimated. When the standard deviation is overestimated, they found that the conditional in-control ARL is larger than it would be if the standard deviation were known, and smaller when the standard deviation is underestimated. As the size of the mean shift increases, the difference in the out-of-control ARL from that of the known-parameter case becomes negligible. They also studied the effect of subtracting from the CUSUM statistic a reference value (k) equal to one half of the magnitude of the desired change in the mean to detect. The marginal ARL evaluation was only considered for $n = 10$, where there is a tremendous amount of variability, but they concluded that a positive reference value leads to a better chart on average. Because of the small size of their study and their focus only on the ARL, their conclusions are better supported by the extensive analytical study of Jones et al. (2004).

Yang (1990) found that the reference value impacts the performance of one-sided CUSUM charts when the process mean is estimated and the variance is known. He found that the marginal ARL is not a sufficient measure and proposed an alternative measure based on the ratio of the ARL to the SDRL.

Hawkins and Olwell (1998) discussed briefly the effect of parameter estimation on CUSUM charts for individual data points collected over time. For the particular one-sided CUSUM chart studied, it appears that estimation of the mean causes bigger changes in the conditional ARL than estimation of the standard deviation. They concluded that 100 observations in the Phase I sample are not sufficient. They noted that a chart tuned to be more sensitive to small shifts is affected by parameter estimation more than one tuned to large shifts and that a one-sided CUSUM is more severely impacted than a two-sided CUSUM chart.

Using an approach similar to that of Jones et al. (2001), Jones et al. (2004) studied CUSUM charts when the mean and standard deviation are estimated. Just as was found for EWMA charts, the ARL, SDRL, and percentiles can either increase or decrease relative to the known-parameter case, depending on the direction and magnitude of the estimation error. They found the same "flattening" of the marginal RL distribution that was found for $\bar{X}$ charts. Further, they showed that a two-sided CUSUM chart with estimated parameters does not have as skewed of a marginal RL distribution as a one-sided CUSUM chart.

Charts for Autocorrelated Data

When considering control charts for autocorrelated data, there are two basic approaches. The first approach (called the residuals control chart) is to model the data with an appropriate time-series model, such as an AR(1) or ARIMA(1,0,1), and then plot the residuals of the data from the one-step-ahead forecasts. If the model has been specified correctly, the in-control residuals will be i.i.d. observations, and standard control charts are used on the residuals rather than the observations themselves to detect changes in the process. The second approach (the modified control chart) uses the actual observations and adjusts the control limits to account for the autocorrelation. If autocorrelation is present, it is preferable to first try to remove the source of autocorrelation, if possible, or to use some form of feedback control to remove the variability in the process. Control charts for autocorrelated data (both the residuals and modified control chart) should only be used as a last resort.

Schmid (1997) considered briefly the effects of parameter estimation on EWMA and CUSUM charts for autocorrelated data. Based on simulation, he calculated the relative efficiency of the ARL of the estimated-parameters chart to the ARL of the known-parameters chart and found that the estimation impact is more severe for smaller shifts than for larger shifts. A modified EWMA chart seemed to be most robust to parameter estimation, but this is difficult to state generally because the specification of the smoothing constant has an impact on performance. The problem can be simplified by using an Individuals chart for autocorrected data, as was done by Kramer and Schmid (1997, 2000). Kramer and Schmid (1997, 2000) concluded that the ARL is robust to amounts of autocorrelation less than 0.5; thus, standard charts for uncorrelated data can be used when the autocorrelation is low. In the presence of positive autocorrelation, they recommended that the modified control charts be used and for negative autocorrelation the residuals control chart should be used. Larger Phase I sample sizes are needed to ensure that the in-control ARL is as desired.

Adams and Tseng (1998) considered the effect of parameter estimation on the residuals control charts for individual observations. In their study, there were four types of charts: Individuals, Individuals with runs rules, EWMA, and CUSUM. The conditional ARL was evaluated by simulation. For an AR(1) model, underestimation of the time series parameter leads to substantial decreases in the in-control ARL for all four charts. Overestimation leads to decreases in the in-control ARL for the two Individuals charts and dramatic increases in the in-control ARL for the EWMA and CUSUM charts. The reverse situation is true when an IMA(1, 1) model is used. The in-control ARL performance of the Individuals charts is more consistent than for the EWMA and CUSUM charts in the sense that underestimation or overestimation leads to a more drastic change in the ARL for EWMA and CUSUM charts. They did not study the out-of-control ARL performance nor did they consider the SDRL or percentiles of the RL distribution, but recommended that these charts only be used when a Phase I sample size of at least 400 is available.

Limited simulation studies of control chart performance of EWMA and CUSUM charts based on residuals for autocorrelated data using an ARMA(1, 1) model appeared in Lu and Reynolds (1999, 2001). They found that there is more variability in the time-series parameter estimates for smaller sample sizes. This causes the in-control ARL to be much higher than for the known-parameter case; therefore, larger amounts of data are needed to set good control limits when autocorrelation is present than when there is no autocorrelation.

Apley (2002) and Apley and Lee (2003) proposed widening the control limits for residual EWMA and Individuals charts for autocorrelated data. The widened control limits protect against an increase in the rate of false alarms at the expense of a small increase in the out-of-control ARL. This loss of performance that occurs when the control limits are widened is more drastic when the autocorrelation is higher. While an Individuals chart is more robust to parameter estimation (because it is equivalent to the EWMA chart with the maximum value of the smoothing constant), the EWMA chart with widened control limits still outperforms the Individuals chart for shifts of less than 4 standard deviations. The recommended sample size requirements for EWMA charts of autocorrelated data are dependent on the value of the smoothing constant but are larger than those of Jones et al. (2001) for EWMA charts based on independent data.

Multivariate Charts

Lowry and Montgomery (1995) made sample-size recommendations for multivariate T^2 control charts to ensure that control limits based on estimated parameters would be sufficiently close to the control limits for the known-parameter case. Their recommended sample sizes are too small, however, because they did not consider the performance measures of the RL distribution for the studied charts.

A more detailed study on the effects of parameter estimation on multivariate T^2 charts with χ^2-based control limits was done by Nedumaran and Pignatiello (1999). In considering shifts in the mean vector, they used simulation to show that the ARL and SDRL for both an in-control and out-of-control process are substantially lower than the ARL under the assumption of known parameters. They recommended Phase I sample sizes of at least 200 when $n = 5$ and the number of quality characteristics monitored is 3. Their recommended sample size is much larger than that of Lowry and Montgomery (1995), and it increases as the number of quality characteristics increases.

Champ et al. (2005) studied the T^2 chart with corrected control limits based on the F distribution. They showed that, when estimating the parameters used in the T^2 statistic, both the in-control and out-of-control ARLs are higher than in the known-parameter case. However, unlike univariate charts, there is not an increase in the number of short RLs because the lower percentiles of the RL distribution are similar to the known-parameter case. Thus, the increase in the ARL is due to an increase in the number of larger RLs. Their sample size recommendations to achieve in-control performance similar to charts with known parameters are closer to those of Nedumaran and Pignatiello (1999) than those of Lowry and Montgomery (1995), but in contrast, they decrease as the dimension of the data vector increases. This is due to the fact that Nedumaran and Pignatiello (1999) determined how large a sample is needed for the control limit to be close enough to the appropriate quantile of the χ^2 distribution, whereas Champ et al. (2005) determined how large a sample is needed for the control chart to behave like it would if the parameters were known.

Attribute Charts

Braun (1999) considered the effects of parameter estimation on c and p charts for attribute data. Conditional values of the ARL and SDRL were calculated, and just as is the case for $\overline{X}$ and Individuals charts, the in-control ARL and SDRL are higher when parameters are estimated. He states that the probability of a signal should be used as a measure of performance instead of the ARL when the process is in control, but there is not a clear explanation or justification of this statement. Nor is there any evaluation of the out-of-control performance, so the overall impact of parameter estimation is not clear. In addition, marginal performance measures were not considered; thus, he made no recommendation of the needed sample size to ensure adequate performance.

To justify the use of a Bayesian approach to set control limits for np, p, c, and u charts, Hamada (2002) briefly studied the effect of using parameter estimates. He concluded that the conditional probability of a signal for attributes charts can differ from the nominal value of the known-parameter case, but he did not evaluate the RL distribution of these charts nor their out-of-control performance.

Yang et al. (2002) considered geometric charts that involve plotting the number of samples between nonconforming units. These charts require a large number of observations and are most appropriate for high-quality situations where the proportion nonconforming is very low. While they did use the rate of false alarms as a performance measure, it is not very useful because of the dependence between the events of successive observations falling outside the control limits. Thus, it is more appropriate to base action on their conclusions from the ARL calculations where the in-control ARL for geometric charts with estimated parameters is higher than for the known-parameter case if the proportion nonconforming is overestimated and lower if underestimated. The geometric chart is ARL-biased, which means that they would not detect increases quickly until the magnitude of the shift in the proportion nonconforming becomes sufficiently large. This problem is minimized as the sample size increases, but very large Phase I samples (at least 50,000 observations) are needed to ensure adequate performance.

Charts for Other Situations

Most control charts for continuous data are based on the assumption that the quality characteristic of interest follows a normal distribution. While this assumption is reasonable for many cases, recent research on the effect of parameter estimation has considered charts for nonnormal distributions. For example, Sim (2003a) investigated the performance measures of a synthetic control chart when the data follow a gamma distribution. Sim (2003b) studied control charts for monitoring the variability of a process that follows an inverse Gaussian distribution, and Sim and Wong (2003) studied R charts to monitor changes in variability for data that follow an exponential, Laplace, or logistic distribution. Just as with charts for normally distributed data, larger Phase I sample sizes are needed, particularly for charts used to monitor variability.

Albers and Kallenberg (2004b) studied nonparametric charts that use a function of the largest order statistic as a control limit. Proposed corrections are made to the limits when estimated parameters are used. As would be expected, their examples demonstrated that these charts require much more data than the parametric charts to ensure proper performance. They should only be used when there is a clear justification for not assuming approximate normality and large amounts of Phase I data are available.

Shu et al. (2004) studied the effect of parameter estimation on regression control charts where the monitored response is adjusted based on a value of a covariate. At each time period, a simple linear regression model can be fit to account for the covariate, so in addition to estimating μ_0 and σ_0, there are estimators of the slope (β_0) and the intercept (β_1). The resulting residuals from the regression model can either be plotted individually (called SheREG) or used to construct an exponentially weighted moving average (EWMAREG) chart. If σ_0 is underestimated, the conditional in-control ARL will always be lower than expected, no matter the values of the estimates of β_0 and β_1. When σ_0 is overestimated, the change in the conditional in-control ARL depends on the error in estimating β_0 and β_1. The marginal performance measures suggest a Phase I sample size of at least 300 for the SheREG chart and an even larger sample size for the EWMAREG chart, especially if the smoothing constant is small. Shu et al. (2005) performed a related study.

Discussion

From this review, it is clear that more data in Phase I are needed than is typically recommended to achieve performance comparable with the known-parameters case. Every sample size recommendation discussed in this literature review is greater, and often much greater, than the typical recommendation of 20–30 small samples. This concurs with a statement made by Shewhart (1939, p. 63) that, "If we wish to reduce the chance of making an error in estimating the probability associated with chosen tolerance limits (control limits), there is no royal small-sample road for doing this". Sample size recommendations depend on the type of chart used.

Often it is difficult to make an assessment of the impact of parameter estimation because it depends on the direction of the estimation error and the particular control chart setting. However, this is a poor excuse for not making better comparisons of both the marginal and conditional RL distribution performance. Nor does it justify not using the SDRL and other percentiles of the RL distribution to supplement the ARL. The benefit of graphical representation of the empirical RL distribution for comparative purposes should not be ignored.

In general, as more parameters are estimated, larger sample sizes are needed. For example, multivariate charts require more Phase I data than univariate charts and charts for autocorrelated data will typically require more data than those based on i.i.d. data.

As noted by Hawkins and Olwell (1998, p. 161) and by Hawkins et al. (2003), a chart that has the desirable property of being sensitive to small shifts in a process has the undesirable property of being more severely impacted by parameter estimation. This is true in general of the charts studied here and nicely summarizes when charts with estimated parameters need to be used more cautiously. For example, Shewhart charts with runs rules, EWMA charts, and CUSUM charts are sensitive to small shifts and consequently more readily impacted by parameter estimation.

When it is prohibitive to obtain the recommended number of samples, other approaches are possible. These other approaches should be considered as temporary stopgap methods until the needed number of samples are obtained. Many of these methods avoid the distinction between Phase I and Phase II altogether. Hawkins (1987) proposed a self-starting CUSUM procedure and Quesenberry (1991) proposed Q charts, both of which are designed to be used early on in monitoring univariate processes before much data are available. Del Castillo and Montgomery (1994) proposed an alternative method to Q charts based on a Kalman filter that has better performance that improves as m increases. Nedumaran and Pignatiello (1999) considered a method for updating the limits of multivariate charts frequently as more data become available, and Nedumaran and Pignatiello (2001) and Tsai et al. (2004, 2005) considered a similar approach for $\overline{X}$ charts. In their methods, prospective control limits are calculated for a set number of future subgroups to maintain a nominal in-control ARL level. Sullivan and Jones (2002) considered a self-starting EWMA procedure for multivariate data. Hawkins et al. (2003) proposed a change-point method based on unknown parameters as an alternative to estimating parameters. In general, these methods can be used to achieve specified in-control performance until sufficient data are obtained. As a result, they have poorer detection capabilities if the process is out of control in the early stages of the monitoring scheme.

Future Research Ideas

In recent years, there has been much research considering the effects of parameter estimation on many types of charts, such as Shewhart, CUSUM, EWMA, and multivariate charts. More work is needed, however, to investigate the issues related to the effect of parameter estimation and to fill in the gaps of research that has already been done. For example, it was surprising to find papers that studied the impact of parameter estimation but made no sample size recommendations because the authors did not consider the marginal RL distribution. A summary of some unanswered questions and future research ideas are listed below.

(1) Control charts for attribute data have not been investigated very thoroughly, with only a few limited studies performed. In particular, the marginal distribution has not been considered in order to make useful sample size recommendations. It is believed that, due to the nature of attribute data, extremely large sample sizes will be needed when the parameters are estimated. Methods have not been developed to adjust the control limits to account for parameter estimation and obtain desired in-control performance.

(2) The effect of using robust or other alternative estimators has not been studied thoroughly. Most evaluations of performance have considered standard estimators based on the sample mean and standard deviation and have used the same estimators for both Phase I and II. However, in Phase I applications, it seems more appropriate to use an estimator that will be robust to outliers, step changes, and other data anomalies. Examples of robust estimation methods in Phase I control charts include Rocke (1989), Rocke (1992), Tatum (1997), Vargas (2003), and Davis and Adams (2005). The effect of using these robust estimators on Phase II performance is not clear, but it is likely to be inferior to the use of standard estimates because robust estimators are generally not as efficient.

(3) From the studies of Burroughs et al. (1993, 1995), it was shown that use of runs rules causes Shewhart control charts with estimated parameters to behave less like the known-parameter case. The disparity appears to increase as more rules are added to the standard charts; therefore, a more comprehensive study is needed to determine the properties of charts incorporating multiple runs rules. It is probable that a chart using multiple runs rules simultaneously will be affected more by parameter estimation than a chart using a single rule.

(4) Very little work has been done on multivariate charts and existing work has focused on detecting changes in the mean vector. Control chart performance under changes in the covariance matrix parameters has not been investigated, which would be analogous to changes in variance for univariate control charts. If the results of the multivariate case are consistent with the univariate case, performance will be worse for changes in the covariance matrix than for changes in the mean vector. Use of other estimators, such as the successive difference estimator or robust estimators, like those discussed in Vargas (2003), have not been studied. Furthermore, no attention has been given to other multivariate charts, such as the multivariate EWMA and multivariate CUSUM charts.

(5) In many applications, the control limits are updated as more data become available. However, most research is based on the assumption that there is a single sample of Phase I data that is used to calculate the Phase II limits. Nedumaran and Pignatiello (1999, 2001), Tsai et al. (2004, 2005) and the Q charts of Quesenberry (1991) are exceptions, but their focus is on updating the control limits more frequently during a startup period as data are available in order to improve initial performance. These papers addressed the question of whether it is best to start Phase II monitoring with fewer data points and then update the limits frequently or if it is better to delay the start of Phase II monitoring to preserve confidence in the performance of the control chart procedure. Investigation of these issues is needed. While updating control limits makes the problem more complex, it is not clear what its impact will be on the RL properties or detection capabilities.

(6) Related to the previous research question is the effect on control chart properties when the control limits are updated in some future time that is not necessarily during a start-up period. If the process is in control, it would be reasonable to use the data to update control limits during Phase II and not continue to use the original limits indefinitely. It is not clear how control chart performance is impacted, but it seems that making use of earlier Phase II data would lead to better control charts. Updating control limits would fit naturally in a Bayesian control chart scheme (Hamada, 2002) where prior estimates are updated resulting in posterior estimates that can continue to be updated over the life of the monitoring scheme.

(7) More work has been done on charts for monitoring shifts in the mean than charts to monitor the variance, even though the estimation effect appears to be more severe for charts monitoring changes in dispersion than for those monitoring changes in the mean. For example, EWMA or CUSUM charts for monitoring the variance have not been studied. It seems reasonable to expect that the effect of parameter estimation would be more severe on charts that

are used to monitor the variance than for charts to monitor the mean, as has been shown for Shewhart charts by Chen (1998).

(8) The effect of parameter estimation has not been considered for many other types of control charts. An incomplete list includes control charts for monitoring profiles discussed by Woodall et al. (2004), simultaneous charts (such as EWMA charts with Shewhart limits), and charts using variable sample sizes and/or sampling intervals (Reynolds et al. (1988), Prabhu et al. (1994), Costa (1997)). Based on the general conclusions mentioned earlier, it is hypothesized that charts for monitoring profiles will require larger Phase I sample sizes because of the multivariate nature of the data. It is also hypothesized that charts with variable sample sizes and/or sampling intervals will be more severely impacted by parameter estimation because they are more sensitive to smaller shifts. Research for these charts is needed in order to have good recommendations on how to best design the charts with estimated parameters.

(9) When there are not sufficient Phase I data, corrections to the control limits are often recommended. There has not been a comparison of these correction methods to determine which are better, nor has there been much study of corrections for charts other than Shewhart charts. A comparison is needed of the corrections proposed by Nedumaran and Pignatiello (2001), Tsai et al. (2004, 2005), and Albers and Kallenberg (2004a, c).

(10) In some cases, model misspecification is an important issue to consider when designing and setting up control charts. For example, control charts for autocorrelated data are strongly dependent on the correctness of the assumed time series model. The combined effect of model misspecification and parameter estimation has not been studied.

(11) While the issue of how to effectively complete a Phase I study seems basic, it is very important. There is little guidance in the literature on how to do a Phase I analysis, which is critical to the successful use of the chart in Phase II. See Champ and Jones (2004) for more discussion of this issue.

(12) It would be very useful to have good graphical methods to better assess the impact of parameter estimation rather than tables of values, as is typically done. A useful line of research would be to develop ways to graphically show the impact of parameter estimation in such a way that it can be more easily understood, especially for situations where the impact depends on a number of variables.

(13) The standard estimators that are often used in control charts are usually unbiased and optimal with respect to some criteria. However, it does seem reasonable that, if the restriction of only considering unbiased estimators were lifted, other estimators could be found that would yield better control chart performance. In addition, "optimal" estimators in Phase I may not continue to be optimal for Phase II control chart performance. It would be worthwhile to investigate other estimators than the standard estimators to determine if control chart performance can be improved.

The impact of parameter estimation on control chart performance should be studied as new types of control chart procedures are developed. Past research has developed methods under the assumption of known parameters and separate research in recent years has analyzed the properties of these methods when using estimated parameters. Future researchers who develop new control chart methods should include assessments of those methods with parameter estimation as part of their results, as did Lu and Reynolds (1999, 2001). In addition, any assessments should include multiple performance measures in their evaluations of control chart properties. There has been too much emphasis on the ARL and not enough on the SDRL and the percentiles of the RL distribution. Consideration of the effect of parameter estimation as new methods are being developed will increase the likelihood that new methods will be utilized and yield more successful applications of control chart monitoring schemes.

Conclusions

The effect of parameter estimation on control chart properties should not be ignored. It is preferable in Phase I to obtain as much in-control data as possible. To ensure adequate Phase II performance, many studies on different types of control charts contain Phase I sample size recommendations that are much larger than has been recommended previously. When adequate data are not available, the resulting charts will often signal more frequently when the process is in control and have reduced ability to detect process changes. The practitioner should be aware of this fundamental issue, especially early on in Phase II of the monitoring scheme. As more data become available and the process is still determined to be stable, the control limits should be updated. Much research has been done to determine the properties of control charts when the parameters are estimated, but additional work is needed.

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