SECTION 45
STATISTICAL PROCESS CONTROL[1]

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INTRODUCTION

This section presents the use of statistical techniques to control processes. Historically, the processes controlled statistically were manufacturing processes; however, in recent years, other processes such as those used by organizations dealing primarily in services have recognized the power of these techniques. The section starts with a few definitions and comments on notation. Where possible, the definitions used are those found in the most common terminology standards.

[1]In the Fourth Edition, material for the section on statistical process control was supplied by Dorian Shainin and Peter D. Shainin.

These standards are identified here as American National Standards; however, there are also international standards with the same numerical designations. The standards used are

- ANSI/ISO/ASQC A8402-1994, *Quality Management and Quality Assurance—Vocabulary*
- ANSI/ISO/ASQC A3534-1993, *Statistics—Vocabulary and Symbols*

  Part 1: *Probability and General Statistical Terms*

  Part 2: *Statistical Quality Control*

The reader is referred to these standards for definitions of additional terms. ANSI/ISO/ASQC A8402-1994 has replaced ANSI/ASQC A3 and ANSI/ISO/ASQC A3534-2 has replaced ANSI/ASQC A1 and A2 that were referenced in earlier editions of this handbook.

Definitions

- *Process:* Set of interrelated resources and activities that transform inputs into outputs. [Note: Resources may include personnel, finance, facilities, equipment, techniques, and methods (ANSI/ISO/ASQC A8402-1994, clause 1.2).]
- *State of statistical control:* State in which the variations among the observed sampling results can be attributed to a system of chance causes that does not appear to change with time. [Note: Such a system of chance causes generally will behave as though the results are simple random samples from the same population) (ANSI/ISO/ASQC A3534-2-1993, clause 3.1.5).]
- *Process in control, stable process:* Process in which each of the quality measures (e.g., the average and variability or fraction nonconforming or average number of nonconformities of the product or service) is in a state of statistical control (ANSI/ISO/ASQC A3534-2-1993, clause 3.1.6).
- *Chance causes:* Factors, generally many in number but each of relatively small importance, contributing to variation that have not necessarily been identified. [Note: Chance causes are sometimes referred to as *common causes* of variation (ANSI/ISO/ASQC A3534-2-1993, clause 3.1.9).]
- *Assignable causes:* Factors (usually systematic) that can be detected and identified as contributing to a change in a quality characteristic or process level. [Notes: (1) Assignable causes are sometimes referred to as *special causes* of variation; (2) many small causes of change are assignable, but it may be uneconomic to consider or control them; in this case they should be treated as chance causes (ANSI/ISO/ASQC A3534-2-1993, clause 3.1.8).]
- *Control chart:* Chart with upper and/or lower control limits on which values of some statistical measure for a series of samples or subgroups are plotted, usually in time or sample number order. The chart frequently shows a central line to assist detection of a trend of plotted values toward either control limit. [Note: In some control charts, the control limits are based on the within-sample or within-subgroup data plotted on the chart; in others, the control limits are based on adopted standard or specified values applicable to the statistical measures being plotted on the chart (ANSI/ISO/ASQC A3534-2-1993, clause 3.3.1).]

Notation. The standard statistical notation will be used wherever possible. That is, a measurement of a quality characteristic will be denoted by an x, parameters will be denoted by Greek letters, and statistics by Roman letters. An overbar will denote an average, and double overbars will indicate an average of averages. Other symbols will be defined as they are used.

THEORY AND BACKGROUND OF STATISTICAL PROCESS CONTROL

Concern over variation in manufactured products produced by the Western Electric Company and studies of sampling results led Dr. Walter A. Shewhart of the Bell Laboratories to the development of the control chart as early as 1924. An unpublished memorandum by Shewhart dated May 16, 1924 contains the first known control chart. In subsequent papers, Shewhart developed the concept more completely. See Shewhart (1926a, 1926b, 1927, and 1931) for more details about these early efforts. In 1931 Shewhart published his classic book, *Economic Control of Quality of Manufactured Product.* The first applications of the control chart by Shewhart were on fuses, heat controls, and station apparatus at the Hawthorne Works of the Western Electric Company. By 1927–1928, many other applications throughout the Western Electric Company were initiated by another member of the Quality Assurance Department of Bell Laboratories, Harold F. Dodge. These applications were as an adjunct to sampling inspection plans and as a basic device for demerits-per-unit charts for periodic quality assurance ratings.

In 1935, Dodge prepared the *ASTM Manual on Presentation of Data,* which addresses many types of control charts. This document is currently known as ASTM MNL 7 (1990). It has been used extensively by industry over the years. In 1941–1942, Dodge served as chairman of the American Standards Association Committee Z1, which published ASA standards Z1.1, Z1.2, and Z1.3 on control charts. Widespread use of control charts followed the initial publication of these standards. Revised versions of these three standards are now available from ANSI and ASQ (see ANSI/ASQC 1985a, 1985b, 1985c). Training courses on quality control principles (including the use of control charts and acceptance sampling plans) were sponsored by the Office of Production Research and Development of the War Production Board during World War II as a means for maintaining quality and promoting continual improvement. There have been many extensions and modifications of the basic control charts of Shewhart over the years. Control charts are now widely used in every industry. They are the principal tools of statistical process control (SPC).

In his studies, Shewhart observed that variation occurs in all things in nature as well as in manufactured goods. The study of this variation and its reduction are the principal vehicles of quality improvement. The control chart is the most important tool available to do this. Even though no two items are alike (variation), groups of observations form either predictable patterns or no patterns at all. The latter indicate that process improvement is necessary. This led Shewhart to conclude two important principles: (1) variation is inevitable, and (2) single observations form little or no basis for objective decision making. In order to determine patterns of observations, they may be plotted in several ways. One way is to form a histogram of the observations that presents a picture of the distribution. Another way is to plot the observations in the order in which they were obtained. This forms a line chart and is useful for observing trends or cycles in the data.

Shewhart recommended the use of a line chart to observe the data. He further indicated that there are two causes of variation. One he called *chance causes.* These are events that cause relatively minor fluctuations in the data. Each may be so small that its occurrence is not important or may be uneconomic to correct. Even though each contributes relatively minor fluctuations, together they form a pattern. Shewhart concluded from the central limit theorem and from empirical observations that they often formed approximately a normal distribution. The second cause of variation he called *assignable*; that is, a cause can be assigned for the fluctuations observed. These are sources of variation that cause a significant departure of the data from the pattern formed by the chance causes. The control chart developed by Shewhart contains a set of limits around the hypothesized normal distribution of chance causes. Any observation falling outside these limits indicates the presence of an assignable cause. In addition, because the observations are plotted in order of occurrence, trends or other unnatural patterns may be readily observed.

A control chart, then, is a graphic representation of the variation in the computed statistics being produced by the process. It has a decided advantage over presentation of the data in the form of a histogram in that it shows the sequence in which the data were produced. It reveals the amount and nature of variation by time, indicates statistical control or lack of it, and enables pattern interpretation and detection of changes in the process. The basic charts developed by Dr. Shewhart consisted of charts for averages and standard deviations. Other charts were developed for sample ranges, percentage nonconforming, and number of nonconformities per item or hundred items. Further improvements and modifications to the basic charts also have been made over the years. Such charts as cumulative sum charts and exponentially weighted moving average charts have allowed for quicker detection of small shifts in the parameter being followed. All these types of control charts will be reviewed briefly in this section.

STEPS TO START A CONTROL CHART

1. Choose the quality characteristic to be charted. In making this choice, there are several things to consider:
   a. Choose a characteristic that is currently experiencing a high number of nonconformities or items that do not conform. A Pareto analysis is useful to assist the process of making this choice.
   b. Identify the process variables contributing to the end-product characteristics to identify potential charting possibilities.
   c. Choose characteristics that will provide appropriate data to identify and diagnose problems. In choosing characteristics, it is important to remember that attributes provide summary data and may be used for any number of characteristics. On the other hand, variables data are used for only one characteristic on each chart but are necessary to diagnose problems and propose action on the characteristic.
   d. Determine a convenient point in the production process to locate the chart. This point should be early enough to prevent nonconformities and to guard against additional work on nonconforming items.

2. Choose the type of control chart.
   a. The first decision is whether to use a variables chart or an attributes chart. A variables chart is used to control individual measurable characteristics, whereas an attributes chart may be used with go no-go type of inspection. An attributes chart is used to control percentage or number of nonconforming items or number of nonconformities per item. A variables chart provides the maximum amount of information per item inspected. It is used to control both the level of the process and the variability of the process. An attributes chart often provides summary data that can be used to improve the process by then controlling individual characteristics.
   b. Choose the specific type of chart to be used. If a variables chart is to be used, decide whether the average and range or the average and standard deviation are to be charted. If small shifts in the mean are important, a cumulative sum or exponentially weighted moving average chart may be used. The disadvantage of these two latter charts is that they are more difficult for the practitioner to use and understand. If subgroups are not possible, individual readings may be used, but these are to be avoided if possible. For attributes charts, the percentage nonconforming or number of nonconforming items may be charted. In some cases, the number of nonconformities per inspection item may be preferable. All these charts will be discussed later.

3. Choose the center line of the chart and the basis for calculating the control limits. The center line may be the average of past data, the average of data yet to be collected, or a desired (standard) value. The limits are usually set at ±3 standard deviations, but other multiples of the standard deviation may be used for other risk factors. The use of 3 standard deviations results in a negligible risk of looking for problems that do not exist, i.e., false alarms. However, this multiple may result in an appreciable risk of failing to detect a small shift in the parameter being studied. Smaller multiples increase the risk of looking for a false alarm but reduce the risk of failing to detect a small shift. The fact that it is usually much more expensive to look for problems that do not exist than to miss some small problems is the reason that the $\pm 3\sigma$ limits are usually chosen.

4. Choose the rational subgroup or sample. It should be pointed out that the term *sample* is usually used, but *sample* could mean an individual value, and samples of more than one are desirable for control charts if feasible. For variables charts, samples of size 4 or 5 are usually used, whereas for attributes charts, samples of 50 to 100 are often used. Attributes charts in fact may be used with 100 percent inspection as a reflection of the underlying process involved. In addition to the size of the sample, the samples should be selected in such a way that the chance of a shift in the process is minimized during the taking of the sample (thus a small sample should be used); whereas the chance of a shift, if it is going to occur, is at a maximum between samples. This is the concept of *rational subgrouping.* Thus it is better to take small samples periodically than to take a single large sample. Experience is usually the best method for deciding on the frequency of taking samples. That is, the known rate of a chemical change or the known rate of tool wear should be considered when making these decisions. If such experience is not available, samples should be taken frequently until such experience is gained.

5. Provide a system for collecting the data. If control charts are to become a shop tool, the collection of data must be an easy task. Measurement must be made simple and relatively free of error. Measuring instruments must give quick and reliable readings. If possible, the measuring instrument actually should record the data, since this will eliminate a common source of errors. Data sheets should be designed carefully to make the data readily available. The data sheets must be kept in a safe and secure place, free from dirt or oil.
6. Calculate the control limits and provide adequate instruction to all concerned on the meaning and interpretation of the results. Production personnel must be knowledgeable and capable of performing corrective action when the charts indicate it.

CONSTRUCTING A CONTROL CHART FOR VARIABLES FOR ATTAINING A STATE OF CONTROL (NO STANDARD GIVEN CHARTS)

In this case we assume that we know nothing about the process, but we wish to determine if it is in a state of statistical control, i.e., to see if there are only chance causes of variation present. The procedure for all types of charts is

1. Take a series of 20 to 30 samples from the process.
2. During the taking of these samples, keep accurate records of any changes in the process such as a change in operators, machines, or materials.
3. Compute trial control limits from these data.
4. Plot the data on a chart with the trial limits to determine if any of the samples were out of control, i.e., if any plotted points are outside the control limits.

If none of the plotted points are outside the trial control limits, we can say the process is "in control," and these limits may be used for maintaining control. If, on the other hand, some of the plotted points are outside the trial control limits, we say the process is "not in control." That is, there are assignable causes of variation present. In such a case we must determine, from the records in step 2 above if possible, the cause of each out-of-control point, eliminate these samples from the data, and recalculate the trial control limits. If some points are outside these new limits, this step must be repeated until no points are outside the trial control limits. These final limits may then be used for future control.

$\bar{X}$ and R Charts. This set of two charts is the most commonly used statistical process control procedure. It is used whenever we have a particular quality characteristic that we wish to control, since we can use the charts with only one characteristic at a time. In addition, the data must be of a measurement or variables type. Most users of process control are interested in individual items of product and the values of a few quality characteristics on these items. Averages and ranges computed from small samples or subgroups of individual items provide very good measures of the nature of the underlying universe. They permit us to control and otherwise make decisions about the process from which the items came. The chart for averages is used to control the mean or central tendency of the process, whereas the chart for ranges is used to control the variability. In place of the range, the sample standard deviation is sometimes used, but the range (the largest minus the smallest values in the sample) is easier to calculate and is easier to understand by the operators.

Using the convention of control limits set at $\pm 3\sigma$, the control limits for the $\bar{X}$ chart will be set at

$$\mu \pm 3\sigma_{\bar{X}} = \mu \pm 3\sigma/\sqrt{n}$$

where μ = the process mean
σ = the process standard deviation
n = the sample size

Since the parameters μ and σ are unknown, we must estimate them from the data. The best estimate of μ is $\bar{\bar{X}}$, where $\bar{\bar{X}}$ is the average of the sample averages $\bar{X}$, and $\bar{X}$ is calculated as

$$\bar{X} = \sum x/n$$

The process standard deviation may be estimated by a function of the average of the sample ranges $\bar{R}$. The average range provides a good estimate of the standard deviation for small samples, say, less than 12. The estimate of the standard deviation is the average range divided by a constant d_2, where d_2 is a function of the sample size. Thus the control limits for the $\bar{X}$ chart are calculated as

$$\text{UCL}_{\bar{X}} = \bar{\bar{X}} + 3\,\bar{R}/(d_2\sqrt{n}) = \bar{\bar{X}} + A_2\,\bar{R}$$

$$\text{LCL}_{\bar{X}} = \bar{\bar{X}} - 3\bar{R}/(d_2\sqrt{n}) = \bar{\bar{X}} - A_2\,\bar{R}$$

where $A_2 = 3/(d_2\sqrt{n})$.

The range chart will be calculated similarly as $\mu_R \pm 3\sigma_R$, where μ_R and σ_R are the mean and standard deviation of the distribution of the sample ranges, respectively. The mean range is estimated by $\bar{R}$, and the standard deviation of the range is $\sigma_R = d_3\sigma \cdot$ This may be estimated by $d_3\bar{R}/d_2$. Therefore, we calculate the control limits for the range chart as

$$\text{UCL}_R = \bar{R} + 3d_3(\bar{R}/d_2) = (1 + 3d_3/d_2)\bar{R} = D_4\bar{R}$$

$$\text{LCL}_R = \bar{R} - 3d_3(\bar{R}/d_2) = (1 - 3d_3/d_2)\bar{R} = D_3\bar{R}$$

where the factors A_2, D_3, and D_4 are functions of the sample size n and are tabulated in Table 45.1 along with d_2, d_3, and some other factors that will be used later in this section. A more extensive table will be found in Appendix II, Table A.

To use these two charts, 20 to 25 samples of the same size n are taken from the process. Samples of 4 or 5 are usually used for these charts. The average and range are calculated for each sample. The 20 to 25 averages and ranges are then averaged to find the grand average and the average range. The upper and lower control limits may then be calculated for each of the charts. It will be noticed in Table 45.1 that for $n = 6$ or less, the value for D_3 is 0. This is so because if the lower control limit were to be calculated, it would be negative. This does not make sense because the range is always positive. The fact that the value for this factor is 0 does not mean that the lower control limit for the range is 0. It means that there is no lower control limit for the range for these small samples.

There are many examples of these charts in the literature. The book *Statistical Quality Control Handbook* (AT&T 1984) contains many illustrations of control charts used by the Western Electric

TABLE 45.1 Control Chart Factors

n	d_2	d_3	c_4	c_5	A	A_2	A_3	B_3	B_4	B_5	B_6	D_3	D_4
2	1.13	0.85	0.80	0.60	2.12	1.88	2.66	0	3.27	0	2.61	0	3.27
3	1.69	0.89	0.89	0.46	1.73	1.02	1.95	0	2.57	0	2.28	0	2.58
4	2.06	0.88	0.92	0.39	1.50	0.73	1.63	0	2.27	0	2.09	0	2.28
5	2.33	0.86	0.94	0.34	1.34	0.58	1.43	0	2.09	0	1.96	0	2.11
6	2.53	0.85	0.95	0.30	1.23	0.48	1.29	0.03	1.97	0.03	1.87	0	2.00
7	2.70	0.83	0.96	0.28	1.13	0.42	1.18	0.12	1.88	0.11	1.81	0.08	1.92
8	2.85	0.82	0.97	0.26	1.06	0.37	1.10	0.19	1.82	0.18	1.75	0.14	1.86
9	2.97	0.81	0.97	0.25	1.00	0.34	1.03	0.24	1.76	0.23	1.71	0.18	1.82
10	3.08	0.80	0.97	0.23	0.95	0.31	0.98	0.28	1.72	0.28	1.67	0.22	1.78

Company. Other books with excellent descriptions of control charts and examples of their use are Grant and Leavenworth (1996), Ott and Schilling (1990), and Wadsworth et al. (1986).

Example 45.1: Figures 45.1 and 45.2 illustrate the use of $\overline{X}$ and R charts. The figures are self-explanatory in that they follow all the procedures outlined earlier. It may be observed that the range chart is in control, but the sequence of points in the $\overline{X}$ chart starts low and after an adjustment exhibits good control for awhile. However, the last eight points are above the center line, and three of these points are above the upper control limit.

***s* Charts.** The sample standard deviation s may be used with the $\overline{X}$ chart in place of the sample range to measure the process dispersion. In such cases we would calculate the standard deviation of each sample, find the average of the sample standard deviations, and calculate the control limits from the equations below:

$$\text{UCL}_s = B_4\bar{s}$$

$$\text{LCL}_s = B_3\bar{s}$$

where values of B_3 and B_4 may be found in Table 45.1 and in Appendix II, Table A.

The standard deviation has superior mathematical properties over the range, and with present hand calculators and computers, it is relatively simple to calculate. However, the range is much easier for the operator with limited statistical training to understand, and for this reason, the range chart is the most often used chart to control process variability.

INTERPRETATION OF CONTROL CHARTS

It is customary to place the $\overline{X}$ chart directly above the range (or standard deviation) chart. Thus the two statistics computed from each sample are easily located, and the relationship between them is obvious. We look for any unusual points or patterns in the plotted data for either chart. In order to consider this, let us first understand natural patterns for control chart data. A stable process, i.e., one under statistical control, generally will not produce a discernible unnatural pattern. It will produce a random array of data that possesses several underlying characteristics. We have observed previously that the control limits are established at the extremities of the underlying distribution of the statistic being studied. Hence the data from a controlled process should have the following characteristics when plotted on a control chart:

- Most of the plotted points occur near the centerline.
- A few of the points occur near the control limits.
- Only an occasional rare point occurs beyond the control limits.
- The plotted points occur in a random manner with no clustering, trending, or other departure from a random distribution.

Having considered natural patterns, we now consider unnatural patterns of plotted points on a control chart. Certainly, a pattern lacking one or more of the preceding characteristics would be an unnatural pattern, but let us consider each of them in turn and consider possible reasons for data to not behave according to these rules.

Most of the data on an $\overline{X}$ chart may not occur near the centerline for several reasons. If the mean changes from time to time, the data may cluster around two lines above and below the centerline. This might occur when there are, for example, two machines or two persons operating at slightly different levels feeding into the production stream.

The absence of the second of the preceding characteristics of a controlled process, the occurrence of a few points near the control limits of an $\overline{X}$ chart, may be caused by two processes having

CONTROL CHART DATA SHEET

PART NUMBER: *102J*		DESCRIPTION: *SHAFT*	
LOT NO.: *195*	ORDER NO.: *7-18*	MACHINE NO.: *SBL - 20*	DEPT.: *M*
OPERATION: *FACING TO LENGTH*	OPERATOR: *BROWN*	SHIFT: *1* DATE *12-17-48*	INSPECTOR: *SMITH*

SAMPLE #	1	2	3	4	5	6	7	8	9	10	11	12	13	NO.	$\bar{X}$	R
	.9382	.9382	.9385	.9379	.9384	.9385	.9387	.9387	.9388	.9381	.9386	.9385	.9386	1	.93800	.0010
	.9378	.9380	.9382	.9380	.9385	.9385	.9385	.9386	.9382	.9385	.9387	.9384	.9386	2	.93810	.0002
	.9385	.9380	.9383	.9384	.9387	.9385	.9385	.9385	.9386	.9383	.9387	.9385	.9386	3	.93822	.0006
	.9375	.9382	.9379	.9384	.9386	.9385	.9387	.9382	.9386	.9386	.9385	.9384	.9381	4	.93818	.0005
														5	.93855	.0003
TOTAL	3.7520	3.7524	3.7529	3.7527	3.7542	3.7540	3.7544	3.7540	3.7542	3.7535	3.7545	3.7538	3.7539	6	.93850	.0000
$\bar{X}$	.93800	.93810	.93822	.93818	.93855	.93850	.93860	.93850	.93855	.93838	.93862	.93845	.93848	7	.93860	.0002
R	.0010	.0002	.0006	.0005	.0003	.0000	.0002	.0005	.0006	.0005	.0002	.0001	.0005	8	.93850	.0005
														9	.93855	.0006
SAMPLE #	14	15	16	17	18	19	20	21	22	23	24	25		10	.93838	.0005
	.9382	.9386	.9388	.9385	.9387	.9388	.9385	.9386	.9384	.9387	.9387	.9390		11	.93862	.0002
	.9387	.9387	.9387	.9387	.9387	.9385	.9387	.9390	.9386	.9388	.9387	.9389		12	.93845	.0001
	.9388	.9387	.9388	.9385	.9383	.9386	.9386	.9390	.9386	.9388	.9389	.9389		13	.93848	.0005
	.9388	.9386	.9386	.9384	.9389	.9386	.9385	.9389	.9388	.9386	.9390	.9390		14	.93862	.0006
														15	.93865	.0001
TOTAL	3.7545	3.7546	3.7549	3.7541	3.7538	3.7545	3.7538	3.7483	3.7544	3.7549	3.7553	3.7558		16	.93872	.0002
$\bar{X}$	.93862	.93865	.93872	.93852	.93865	.93862	.93865	.93888	.93860	.93872	.93882	.93895		17	.93852	.0003
R	.0006	.0001	.0002	.0003	.0006	.0003	.0003	.0004	.0004	.0002	.0003	.0001		18	.93865	.0006
														19	.93862	.0003
														20	.93865	.0003
														21	.93888	.0004
														22	.93860	.0004
														23	.93872	.0002
														24	.93882	.0003
														25	.93895	.0001
														TOTAL	23.46353	0.0090

$\bar{\bar{X}} = \frac{23.46353}{25}$ $\quad \bar{R} = \frac{0.0090}{25}$

$\bar{\bar{X}} = .938541$ $\quad \bar{R} = .00036$

LIMITS FOR AVERAGES CHART

UPPER CONTROL LIMIT $= \bar{\bar{X}} + A_2\bar{R}$

LOWER CONTROL LIMIT $= \bar{\bar{X}} - A_2\bar{R}$

$\bar{\bar{X}} = .938541$, $\bar{R} = .00036$ } COMPUTED FROM INSPECTION DATA. SEE LAST TWO COLUMNS ON THIS SHEET.

CONSTANT A_2 IS OBTAINED FROM TABLES (SEE FIGURE 22). FOR A SAMPLE SIZE = 4, $A_2 = .729$

$A_2\bar{R} = .729 \times .00036 = .000262$

U.C.L = .938541 + .000262 = .938803

L.C.L = .938541 − .000262 = .938279

LIMITS FOR RANGES CHART

UPPER CONTROL LIMIT $= D_4\bar{R}$

LOWER CONTROL LIMIT $= D_3\bar{R}$

$\bar{R} = .00036$ (SEE LAST COLUMN ON THIS SHEET)

CONSTANTS D_3, D_4 ARE OBTAINED FROM TABLES (SEE FIGURE 22). FOR A SAMPLE SIZE = 4, $D_3 = 0.000$, $D_4 = 2.282$

U.C.L. = 2.282 × .00036 = .00082

L.C.L. = 0 × .00036 = 0

FIGURE 45.1 Data sheet for determining if a process is in control.

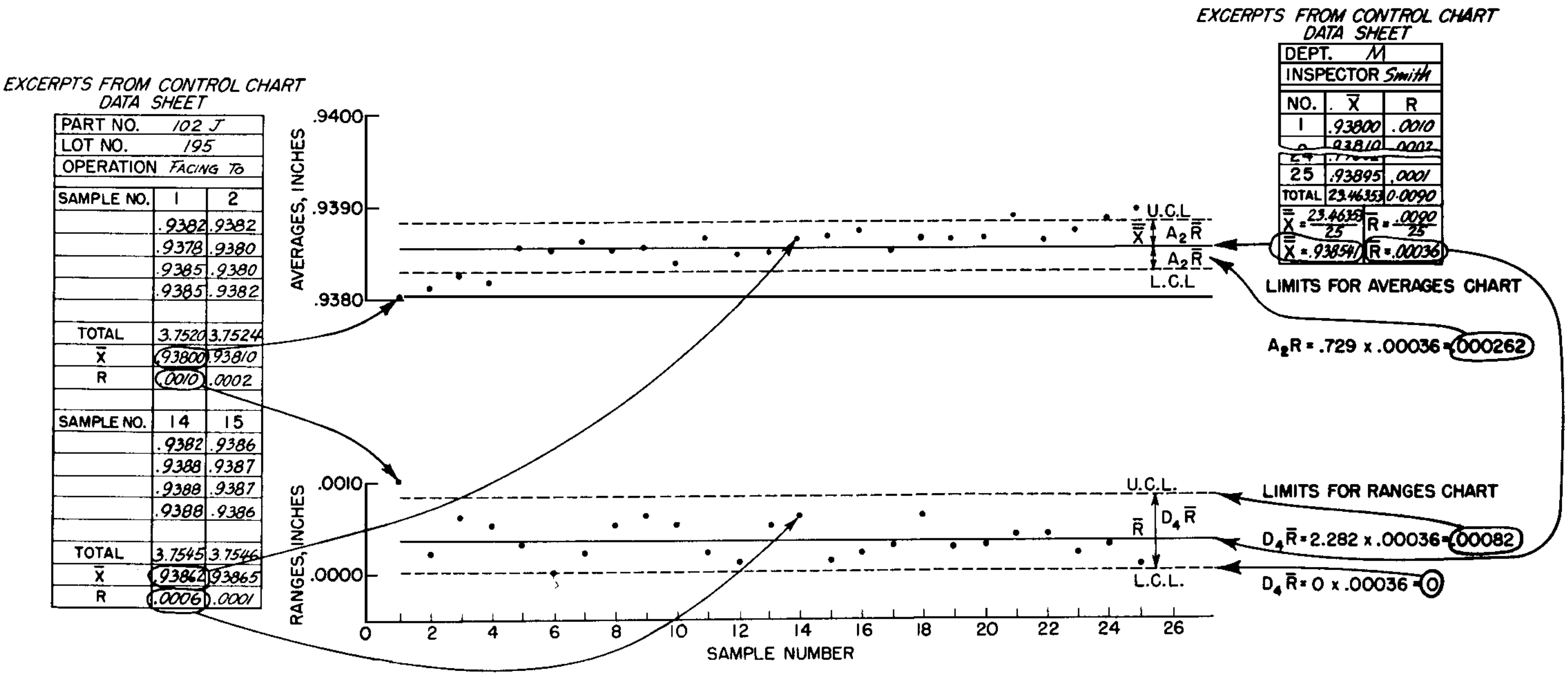

FIGURE 45.2 How to construct the Shewhart control chart.

different means being mixed together in the same sample. The sample averages will then tend to fluctuate about the centerline, but the range of each sample will be inflated, causing the control limits to be spread apart. In this case, the chart for averages might seem to be in very good control, but the range chart will show excessive variation. Data patterns lacking the third of the preceding characteristics, an occasional, rare point beyond the control limits, may indicate either frequent changes in the process average or an increased dispersion.

For the fourth characteristic of natural patterns, examples of unnatural patterns are sudden shifts in level, trends, bunching, or clustering. A detailed discussion of this topic may be found in the AT&T *Statistical Quality Control Handbook* (1984), where a dictionary of control chart patterns is developed.

If the process is "in control," there is approximately a risk of 0.0013, or 0.13 percent, of investigating a change in the process when there is none because a point falls outside one of the control limits. There is an equal risk of a point falling outside the other control limit, making a total risk of 0.0026, or 0.26 percent, of searching for an assignable cause that does not exist. This is often called the *risk of a type I error* or an α risk. It is also called the *producer's risk.* If the process is really out of control, the occurrence of a point outside a control limit is much more likely. How likely depends on how far the process is out of control. The risk of a point not falling outside the control limits and thus failing to detect a shift of a given size is the *risk of a type II error* or a β risk. This risk is also called the *consumer's risk.*

The control limits are usually set for a low α risk. Limits much wider than 3σ would save only a few needless investigations and might cause a delay in investigating out-of-control situations. Limits much narrower would increase the incidence of needless investigations while not appreciably reducing delay in the detection of out-of-control situations. The rule of investigating each point that falls outside the 3σ control limits is therefore a compromise.

Once the control limits have been set, there is information that may be obtained from the chart that is not exploited by the preceding rule. It is rare and about equally likely to have the following events occur when the process is in control:

1. One point outside one of the 3 standard deviation control limits
2. Two of three successive points outside 2 standard deviations
3. Four of five successive points outside 1 standard deviation
4. Eight successive points on the same side of the centerline

Any one of these should be regarded as a signal of an out-of-control condition that requires attention.

The use of all four of the preceding rules is optional and has proved practical in many cases. They were advocated in 1956 in the Western Electric Company's quality control program. The last three apply only to control charts with symmetric limits. Collectively, their use increases the α risk when the process is in control but decreases the β risk when it is not in control.

CONTROL CHARTS FOR INDIVIDUALS

Sometimes it is not practical to take a subgroup from a process. This is particularly true for chemical or other continuous processes or when studying variables such as temperature or pressure. Other situations for which a single observation makes the most sense are accounting data, efficiency, ratios, expenditures, or quality costs. In such a situation, if we took several samples from the process at the same time, we would really only be checking our measuring device and not the manufacturing process itself. In this instance we would only take one observation each time we sample. A control chart based on one observation is called a *chart for individuals* or an *X chart.* Since we are not using sample averages, we do not have the benefit of the central limit theorem, which tells us that the distribution of averages is approximately normal regardless of the underlying distribution. Therefore, an X chart is much more sensitive to a lack of normality of the underlying distribution than is an $\bar{X}$ chart.

Since we are only taking one observation, we do not have an obvious source of an estimate of the standard deviation that we can use to determine the control limits for an X chart. We have two possible choices: We can use a moving range, e.g., the difference between two successive observations, or we can calculate the standard deviation of 20 or more successive observations. If the process does not shift, both statistics will give about the same answer. However, if the process does shift, the moving range will minimize the effect of the shift. Therefore, that method is usually used.

The basic procedure for developing *X* charts is as follows:

1. Select the measurable characteristic to be studied.
2. Collect enough observations (20 or more) for a trial study. The observations should be far enough apart to allow the process to be potentially able to shift.
3. Calculate control limits and the centerline for the trial study using the formulas given later.
4. Set up the trial control chart using the centerline and limits, and plot the observations obtained in step 2. If all points are within the control limits and there are no unnatural patterns, extend the limits for future control.
5. Revise the control limits and centerline as needed (by removing out-of-control points or observing trends, etc.) to assist in improving the process.
6. Periodically assess the effectiveness of the chart, revising it as needed or discontinuing it.

Computing the Control Limits. Control limits for the *X* chart are calculated similarly to those for the $\overline{X}$ chart. That is, we set the control limits at the centerline ±3 standard deviations. They would then be set at

$$\mu \pm 3\sigma$$

The average of the observations $\overline{X}$ is commonly used to estimate the process mean μ. For the standards-given case, the known standard deviation would be used in the preceding equation. For the no-standards-given case, we must estimate the standard deviation. As mentioned earlier, we can use either the moving range or the standard deviation for this estimate.

Moving Range. The moving range is the difference between the largest and smallest of two successive observations. Thus for a total of n observations there will be $n - 1$ moving ranges. We would average the moving ranges to get an average moving range $\overline{R}$. The control limits would thus be set in a similar manner to those for the charts using subgroups as

$$UCL_x = \overline{X} + 3\overline{R}/d_2 = \overline{X} + E_2\overline{R}$$

$$LCL_x = \overline{X} - 3\overline{R}/d_2 = \overline{X} - E_2\overline{R}$$

where values of $E_2 = 3/d_2$ will be found in Appendix II, Table Y. If we employ the common rule of using moving ranges of size 2, E_2 from Appendix II, Table Y, is 3 divided by 1.13, or 2.66. The control limits for the individuals chart would thus be set at

$$UCL_x = \overline{X} + 2.66\overline{R}$$

$$LCL_x = \overline{X} - 2.66\overline{R}$$

Standard Deviation. As an alternative to the moving range method, the standard deviation s of all the trial observations is sometimes calculated. In this case, the control limits would be set at

$$\overline{X} \pm 3s$$

If the process average shifts, e.g., in the case of a trend, the standard deviation method will tend to overstate the variability. If the process average remains relatively constant, both methods will result in approximately the same control limits.

CONSTRUCTING CONTROL CHARTS FOR VARIABLES WHEN A STANDARD IS GIVEN

If standard values of the parameters are given, we have what is commonly called a *standards-given chart.* In this case, the standard for the mean is denoted as $\bar{X}_0$, and the standard for the standard deviation is denoted as σ_0. The control limits for the $\bar{X}$ chart will be at

$$\bar{X}_0 \pm 3\sigma_0/\sqrt{n} = \bar{X}_0 \pm A\sigma_0$$

where A is a function of the sample size n and is tabulated in Table 45.1 and Appendix II, Table A. In the case of control charts for individuals, the X chart, the control limits would be the same, except that since the sample size is 1, the limits would be merely $\bar{X}_0 \pm 3\sigma_0$.

CONTROL CHARTS FOR ATTRIBUTES

Control charts for variables require actual measurements, such as length, weight, tensile strength, etc. Thus go no-go data cannot be used for such charts. Charts for attributes, on the other hand, can be used in situations where we only wish to count the number of nonconforming items or the number of nonconformities in a sample. There are several advantages of attributes charts over variables charts.

1. Attributes charts can be used to cover many different nonconformities at the same time, whereas a separate chart must be used for each quality characteristic with variables charts.
2. The inspection required for attributes charts may be much easier than that for variables charts. We merely need to know if the item being inspected meets the specified requirements.
3. Attributes charts may be used for visual inspections for such attributes as cleanliness, correct labeling, correct color, and so on.
4. Attributes charts do not depend on an underlying statistical distribution.

On the other hand, variables charts need a much smaller sample size. In the case of the charts discussed earlier, we only used sample sizes of 4 or 5, whereas attributes charts would require sample sizes of at least 50. Attributes control charts are often used for 100 percent inspection, whereas this would be difficult for variables charts. The most common control charts for attributes are the *p* chart for percentage nonconforming, the *np* chart for number of nonconforming items, the *c* chart for number of nonconformities, and the *u* chart for number of nonconformities per item. We will discuss each of these charts in order.

Control Charts for Percentage Nonconforming (*p*). The variable to be controlled here is the percentage or fraction of each sample that is nonconforming to the quality requirements. Thus the number of inspected items containing one or more nonconformities is divided by the number of items inspected. This is the fraction nonconforming. Sometimes this ratio is multiplied by 100, and the variable plotted is the percentage nonconforming. Assuming that the process is constant, the underlying distribution would be the binomial distribution. The details of this distribution are covered in Section 44. For relatively large samples, the binomial distribution can be approximated adequately by the normal distribution, and as with the control charts for variables, virtually all the data should then fall within 3 standard deviations of the mean. If data fall outside these 3 standard deviation limits, this would indicate a lack of statistical control. Therefore, the control limits are set at these values.

Recall that the standard deviation of a binomial variable is

$$\sigma_p = \sqrt{\frac{p(1-p)}{n}}$$

Therefore, the upper and lower control limits for a *p* chart will be at

$$\bar{p} \pm 3 \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

Although the *p* chart is usually used for go no-go types of inspection, it also may be used for measurement inspection. Here a piece being inspected is considered nonconforming if its measurements are outside a set of specified limits. However, this use of the *p* chart is not recommended because it is less able to diagnose causes of nonconformities. A chart for determining control limits for a *p* chart will be found in Appendix II, Chart Z.

A *p* chart may be used when the sample size is constant or not constant. Since the *n* in the preceding expression for the control limits is the sample size, if *n* varies from subgroup to subgroup, as is often the case when the chart is used to plot 100 percent inspection data, the control limits will vary. They will be wider for small subgroups than for large ones. If the subgroup size varies, we have three possibilities.

1. We can calculate the average size of the subgroups. This is appropriate when the sizes are similar or all data lie near the centerline.
2. We can calculate separate control limits for each subgroup. This might lead to a rather confusing appearing chart.
3. We can find the average subgroup size and use the resulting control limits, but when a point falls near the limits, calculate the actual limits using the actual subgroup size.

The third approach is the recommended one. In using it, we must remember that a subgroup size larger than the average will mean the limits move in toward the centerline, so if a point lies outside the limits based on the average *n,* there is no point in calculating a new limit.

In setting up a *p* chart, we would, as for variables charts, collect 20 to 25 samples over enough time to allow the process to change. If the sample sizes are equal, we would determine the number of nonconforming items in each sample, divide each by the subgroup size, and average them. This is the $\bar{p}$ in the preceding expression for the control limits. In finding this average *p,* we first add all the numbers of nonconforming items and the numbers of items inspected in each subgroup. Then we divide the first total by the second to get the average. This procedure is essential if the subgroup size changes. As an example to show the importance of this procedure, suppose that we have five lots of a finished product as shown in Table 45.2. *N* is the lot size, *x* is the number of nonconforming items in each lot, and *p* is the fraction nonconforming. We are inspecting all items in each lot.

If we simply average the fraction nonconforming values in the right hand column, we would get a value for $\bar{p}$ of 0.590/5 = 0.118. This would give us upper control limits, using the actual lot sizes of 0.161, 0.255, 0.152, 0.215, and 0.197. None of the lots are out of control. However, if we instead find $\bar{p}$ as 90/1600 = 0.056, we get upper control limits of 0.087, 0.154, 0.081, 0.125, and 0.113. In this case, three of the five lots (lots 2, 4, and 5) are above the upper control limit, indicating that this does not represent a stable process. The true average fraction nonconforming is 0.056 not 0.118. The latter gives equal representation to each lot despite their widely differing sizes.

TABLE 45.2 Inspection Results of 5 Lots

Lot	*N*	*x*	*p*
1	500	32	0.064
2	50	10	0.200
3	800	10	0.013
4	100	18	0.180
5	150	20	0.133
Totals	1600	90	0.590

For all types of control charts for attributes, the lower control limit, when calculated using the appropriate expressions, may turn out to be negative. This, of course, makes no sense, so we simply do not have a lower control limit in these cases. In the preceding example, this applies to lots 2, 4, and 5. Note that lot 3 is actually below its lower control limit, which is, in this case, 0.032. This means that the quality, measured in terms of fraction nonconforming, is better than the average of the other lots. Some people use zero as a lower control limit when the calculated limit is negative. This might lead to a mistaken notion that a sample value of zero means that the subgroup is not in control.

Example 45.2: Table 45.3 and Figure 45.3 illustrate a p chart. Table 45.3 contains the result of final testing of permanent magnets used in electrical relays. There were a total of 14,091 magnets tested, and 1030 were found to be nonconforming. The average number of magnets tested per week was 14,091/19=741.6. The average fraction nonconforming was 1030/14,091=0.073. Figure 45.3 illustrates the control chart with limits based on these values of $\overline{n}$ and $\overline{p}$. Using the preceding formulas, the upper and lower control limits are set at 0.073 ± 0.0287, or 0.102 and 0.044.

Control Charts for Number of Nonconforming Items (*np*). In this case we plot the number of nonconforming items in each subgroup. Since p is x/n, x is *equal to np,* where x is the number of nonconforming items in a sample, and the chart is often called an *np chart.* For this type of chart, we must have a constant subgroup size. In this case the control limits are set at

$$n\,\overline{p} \pm 3\sqrt{n\overline{p}(1-\overline{p})}$$

TABLE 45.3 *p* Chart Data

Subgroup: Production during week		Inspected at: Final assembly and test		
Week No.	Week ending	No. magnets inspected	No. defective magnets	Fraction defective p
1	12/3	724	48	0.067
2	12/10	763	83	0.109
3	12/17	748	70	0.094
4	12/31	748	85	0.114
5	1/7	724	45	0.062
6	1/14	727	56	0.077
7	1/21	726	48	0.066
8	1/28	719	67	0.093
9	2/4	759	37	0.049
10	2/11	745	52	0.070
11	2/18	736	47	0.064
12	2/25	739	50	0.068
13	3/4	723	47	0.065
14	3/11	748	57	0.076
15	3/18	770	51	0.066
16	3/25	756	71	0.094
17	4/1	719	53	0.074
18	4/8	757	34	0.045
19	4/15	760	29	0.038
Totals		14,091	1030	
Averages		741.6	54.2	0.073

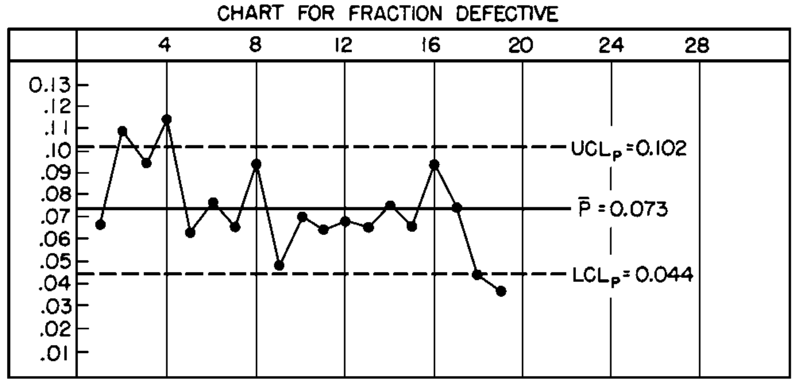

FIGURE 45.3 *p* chart for permanent magnets.

or, using $\overline{X}$ as the average number of nonconformities per subgroup, the control limits may be stated equivalently as

$$\overline{X} \pm 3\sqrt{\overline{X}(1 - \overline{X}/n)}$$

Control Charts for Number of Nonconformities (*c*). If we wish to plot the number of nonconformities, where each item inspected may have several nonconformities and each nonconformity is counted, we have a *c* chart, where *c* is the number of nonconformities in each sample. In this case, the underlying distribution is the Poisson distribution, discussed in Section 44. Recall that in this distribution the standard deviation is the positive square root of the mean, so if we again take 20 to 25 samples and calculate the average number of nonconformities, $\bar{c}$, the control limits will be set at

$$\bar{c} \pm 3\sqrt{\bar{c}}$$

A chart for determining control limits for a *c* chart will be found in Appendix II, Chart AA.

A *c* chart requires an *equally large* number of opportunities for a nonconformity to occur in each subgroup inspected. Thus, for example, if we are inspecting the number of defective solder connections in each circuit board, they must all have the same number of connections. If not, we must use the *u* chart, to be discussed next.

Example 45.3: A control chart for *c* is used to control nonconformities in sheeted material. Table 45.4 shows the results of a series of pinhole tests of paper intended to be impervious to oils. Specimen sheets 11 by 17 inches in size were taken from production, and colored ink was applied to one side of each sheet. Each inkblot that appeared on the other side of the sheet within 5 minutes was counted as a nonconformity.

The centerline of the chart is set at $\bar{c} = 200/35 = 8.0$ nonconformities per sheet. Control limits are set at $8.0 \pm 3\sqrt{8} = 8.0 \pm 8.5$, or 0 and 16.5. That is, there is only an upper limit set at 16.5. The resulting *c* chart is found in Figure 45.4.

Control Chart for Number of Nonconformities per Item (*u*). This chart is sometimes called a *standardized c chart.* It is used when more than one item makes up a sample but each item may have more than one nonconformity. The variable plotted on the chart is the number of nonconformities per item. Thus the number of items in a sample does not need to remain constant, as it does for the *c* chart. The control limits are calculated similarly to those for a *c* chart except that the variable is *c*/*n,* where *n* is the number of items in the sample. For example, if we are inspecting for defective solder joints on printed circuit boards where the boards have different numbers of solder connections, we would divide the number of nonconforming solder connections on each board by

TABLE 45.4 *c* Chart Data

Sheet number	Number of pinholes	Sheet number	Number of pinholes
1	8	14	6
2	9	15	14
3	5	16	6
4	8	17	4
5	5	18	11
6	9	19	7
7	9	20	8
8	11	21	18
9	8	22	6
10	7	23	9
11	6	24	10
12	4	25	5
13	7	Total	200

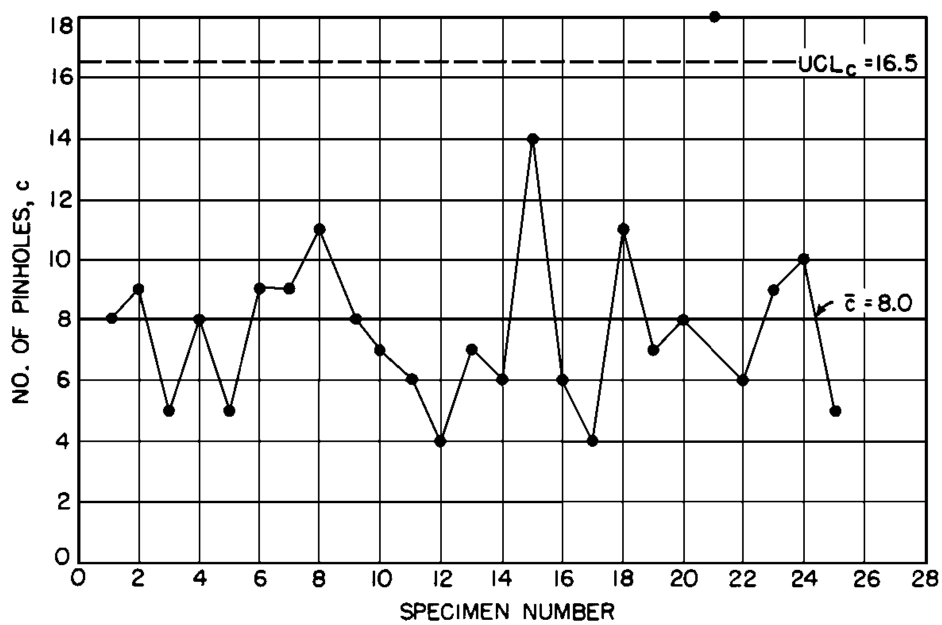

FIGURE 45.4 c chart for pinholes in paper.

the number of connections on that board.

The control limits for a *u* chart are set at

$$\bar{u} \pm 3\sqrt{\bar{u}/n}$$

Example 45.4: As an example of the use of a *u* chart, the data in Table 45.5 are the results of the inspection of 10 samples of cloth for imperfections introduced during processing. From the table, we determine that $\bar{u} = 59/1360 = 0.043$. We might then calculate the average sample size as 1360/10 = 136. Using this figure for *n* in the preceding expression for the control limits, we get

$$UCL_u = 0.043 + 3\sqrt{0.043/136} = 0.043 + 0.054 = 0.097$$

$$LCL_u = 0.043 - 3\sqrt{0.043/136} = 0.043 - 0.054 = <0$$

TABLE 45.5 Inspection Results of Cloth

Square meters of cloth inspected	Nonconformities found	u
200	5	0.025
80	7	0.088
100	3	0.030
300	15	0.050
120	4	0.033
90	6	0.067
250	10	0.040
50	1	0.020
100	6	0.060
70	2	0.029
Totals 1360	59	

There is no lower control limit, and only the second sample is close to the upper control limit; however, since the size of this sample is 80, less than the average sample size of 136, the correct upper control limit will be greater than 0.097, and this sample is in control.

CUMULATIVE SUM (CUSUM) CONTROL CHARTS

Another type of control chart is the *cumulative sum control chart* (often abbreviated as *CUSUM chart*). This chart is particularly useful for detecting small changes (between 0.5σ and 2.5σ) in the parameter being studied. For shifts larger than approximately 2.5σ, the Shewhart-type charts discussed previously are just as good or somewhat better and are easier to understand and use. A CUSUM chart is a plot of the cumulative sum of the deviations between each data point, e.g., a sample average, and a reference value T. Thus this type of chart has a memory feature not found in the previous types of charts. It is usually used to plot the sample average $\overline{X}$, although any of the other statistics, such as s or p, may be used. It is also often used for individual readings, particularly for chemical processes. This section will only discuss the CUSUM chart for averages. For a discussion of other types of CUSUM charts, the reader is referred to Wadsworth et al. (1986). For CUSUM charts, the slope of the plotted line is the important thing, whereas for the previous types of charts it is the distance between a plotted point and the centerline.

CUSUM charts, like other control charts, are interpreted by comparing the plotted points to critical limits. However, the critical limits for a CUSUM control chart are neither fixed nor parallel. A mask in the shape of a V is often constructed. It is laid over the chart with its origin over the last plotted point. If any previously plotted point is covered by the mask, it is an indication that the process has shifted.

Construction of a CUSUM Control Chart for Averages. The following steps may be followed to develop a CUSUM control chart for averages:

1. Obtain an estimate of the standard error of the statistic being plotted; e.g., $\sigma_{\overline{X}}$ may be obtained from a range chart or from some other appropriate estimator. If a range chart is used, the estimate is $\overline{R}/(d_2\sqrt{n})$ or $A_2\overline{R}/3$.
2. Determine the smallest amount of shift in the mean D for which detection is desired. Calculate $\delta = D/\sigma_{\overline{X}}$.
3. Determine the probability level at which decisions are to be made. For limits equivalent to the standard 3σ limits, this is $\alpha = 0.00135$.

4. Determine the scale factor k. This is the change in the value of the statistic to be plotted (vertical scale) per unit change in the horizontal scale (sample number). Ewan (1963) recommends that k be a convenient value between $1\sigma_{\bar{X}}$ and $2\sigma_{\bar{X}}$, preferably closer to $2\sigma_{\bar{X}}$.
5. Obtain the lead distance d from Table BB in Appendix II using the value of δ obtained in step 2.
6. Obtain the mask angle θ from Table BB in Appendix II by setting D/k equal to δ in the table and reading θ from the table. Straight-line interpolation may be used if necessary.
7. Use d and θ to construct the V mask.
8. The sample size for a CUSUM chart for averages is usually the same as for the $\bar{X}$ chart. However, Ewan (1963) suggests, for best results, that one use

$$n = 2.25s^2/D$$

where s is an estimate of the process standard deviation.

Operation of the CUSUM V Mask. The mask is placed over the last point plotted. If any of the previously plotted points are covered by the mask, a shift has occurred. Points covered by the top of the mask indicate a decrease in the process average, whereas those covered by the bottom of the mask indicate an increase. The first point covered by the mask indicates the approximate time at which the shift occurred. If no previous points are covered by the mask, the process is remaining in control.

Some Cautions

1. Periodically check the process variability with an R or s chart before drawing conclusions about the average level of the process.
2. Watch for gradual changes (trends) or changes that come and go in a short period of time. Such changes are not as apparent on a CUSUM chart as they are on an $\bar{X}$ chart.
3. The individual measurements are assumed to follow the normal distribution.

Example 45.5: The data in Table 45.6 summarize measurements of 20 samples of 4 each taken on the percentage of water absorption in common building brick. A reference value of $T = 10.0$ was used. To illustrate the CUSUM chart, the original data were modified to introduce a decrease in the average level of 2.0 percent, starting with subgroup 11. A range chart shows a value of $\bar{R}$ of 8.08.

1. $\sigma_{\bar{X}}$ is estimated as $\bar{R}/d_2\sqrt{n} = 8.08/2.059\sqrt{4} = 1.96$.
2. We wish to detect a shift in the process mean of $D = 1\sigma_{\bar{X}}$; therefore, $\delta = 1.96/1.96 = 1.0$.
3. We wish to use $\alpha = 0.00135$, the value corresponding to the standard control chart.
4. The scale factor k is calculated as $2\sigma_{\bar{X}} = 2(1.96) = 3.92 \approx 4$.
5. We enter Table BB in Appendix II to get the lead distance of the mask as $d = 13.2$.
6. We substitute $D/k = 2/4 = 0.5$ for δ in Table BB in Appendix II to get $\theta = 14°$.

Figure 45.5 shows the CUSUM chart with the mask at sample number 17. The mask was moved from left to right with the zero point on the mask over each plotted point. For the first 16 points, the mask did not cover any of the previously plotted points. At point 17 the shift was detected, and the mask indicated that the shift occurred at point 10 or 11.

Figure 45.5 also shows the same data plotted on a standard $\bar{X}$ chart. Such a chart would not have detected the shift based on a single point outside the control limits. However, it would have detected the shift at point 17 based on the fact that four of five successive points were outside the 1σ limits. A comparison of the CUSUM chart and the standard $\bar{X}$ chart usually involves the average run length (ARL). This is the average number of sample points plotted at a specified quality level before the chart detects a shift from a previous level. Ewan (1963) compares the CUSUM and $\bar{X}$ charts for various

TABLE 45.6 Data on Percentage Water Absorption

Sample no.	$\bar{X}$	$\bar{X} - 10.0$	$\sum (\bar{X} - 10.0)$
1	15.1	5.1	5.1
2	12.3	2.3	7.4
3	7.4	−2.6	4.8
4	8.7	−1.3	3.5
5	8.8	−1.2	2.3
6	11.7	1.7	4.0
7	10.2	0.2	4.2
8	11.5	1.5	5.7
9	11.2	1.2	6.9
10	10.2	0.2	7.1
11	7.6	−2.4	4.7
12	6.2	−3.8	0.9
13	8.2	−1.8	−0.9
14	7.8	−2.2	−3.1
15	6.8	−3.2	−6.3
16	6.1	−3.9	−10.2
17	4.3	−5.7	−15.9
18	8.5	−1.5	−17.4
19	7.7	−2.3	−19.7
20	9.7	−0.3	−20.0

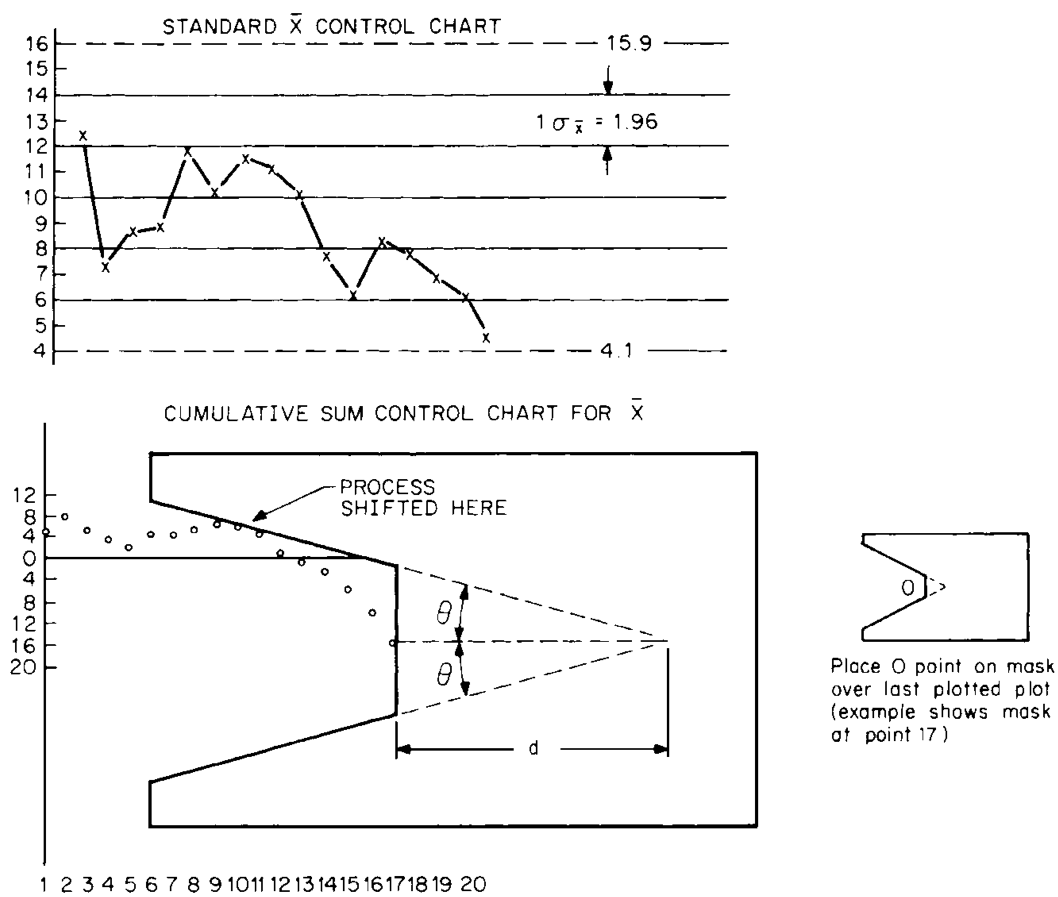

FIGURE 45.5 Comparison of cumulative sum and standard control charts.

amounts of shift in the process mean. For shifts between $0.5\sigma_{\overline{X}}$ and $2.0\sigma_{\overline{X}}$ the CUSUM chart detects the shift with many fewer samples than are needed by the $\overline{X}$ chart. For larger shifts there is no longer any advantage to the CUSUM chart. The preceding statement assumes that the $\overline{X}$ chart uses only the one point outside the control limits rule. As indicated in the preceding example, if the other rules for detecting out-of-control conditions are used, the advantage of the CUSUM chart is diminished.

CUSUM Limits Using the Tabulation Method. For some processes it may not be convenient to use a V mask. An alternative tabulation method may be used that is particularly well suited for computer applications. This method is equivalent to the charting method with the mask. The procedure is as follows:

1. Form the CUSUM as $C_1 = \sum (\overline{X}_i - K_1)$, where $K_1 = T + D/2$, to detect a shift upward.
2. Form the CUSUM as $C_2 = \sum (\overline{X}_i - K_2)$, where $K_2 = T - D/2$, to detect a shift downward.
3. Tabulate these quantities sequentially with $\overline{X}_i$ ignoring negative values of C_1 and positive values of C_2. That is, reset the upper CUSUM to zero when it is negative and the lower CUSUM to zero when it is positive.
4. Watch the progress of the C_1 and C_2 values. When either value equals or exceeds $Dd/2$ in absolute value, a signal is produced.

Example 45.6: We will use the same data as in Example 45.5 to illustrate this technique. Recall that for that example, we had $T = 10.0$, $D = 2$, and $d = 13.2$. Then we have for the preceding steps,

1. $K_1 = T + D/2 = 10.0 + 2/2 = 11.0$ and $C_1 = \sum (\overline{X}_i - 11.0)$
2. $K_2 = T - D/2 = 10.0 - 2/2 = 9.0$ and $C_2 = \sum (\overline{X}_i - 9.0)$
3. Decision limit $= Dd/2 = 13.2$

This procedure is illustrated by Table 45.7.

THE EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART

Another type of control chart is the exponentially weighted moving average (EWMA) control chart. It was first introduced by Roberts (1959) and later by Wortham and Ringer (1971), who proposed it for applications in the process industries as well as for applications in financial and management control systems for which subgroups are not practical. Like the CUSUM charts, it is useful for detecting small shifts in the mean. Single observations are usually used for this type of chart. The single observations may be averages (when the individual readings making up the average are not available), individual readings, ratios, proportions, or similar measurements. A brief discussion of the chart is presented here. The interested reader should consult the preceding references or other more recent publications such as Hunter (1986), Lowry et al. (1992), Lucas and Saccucci (1990), Ng and Case (1989), Crowder (1989), and Albin et al. (1997).

The plotted statistic is the weighted average of the current observation and all previous observations, with the previous average receiving the most weight, that is,

$$Z_t = \lambda x_t + (1 - \lambda)Z_{t-1} \qquad 0 < \lambda < 1$$

where $Z_0 = \mu$
Z_t = the exponentially weighted moving average at the present time t
Z_{t-1} = the exponentially weighted moving average at the immediately preceding time

TABLE 45.7 Tabulation of Data on Water Absorption

Sample no.	$\overline{X}$	$\overline{X} - 11.0$	C_1	$\overline{X} - 9.0$	C_2	Remarks
1	15.1	4.1	4.1	6.1	>0	
2	12.3	1.3	5.4	3.3	>0	
3	7.4	−3.6	1.8	−1.6	−1.6	
4	8.7	−2.3	<0	−0.3	−1.9	
5	8.8	−2.2	<0	−0.2	−2.1	
6	11.7	0.7	0.7	2.7	>0	
7	10.2	−0.8	<0	1.2	>0	
8	11.5	0.5	0.5	2.5	>0	
9	11.2	0.2	0.7	2.2	>0	
10	10.2	−0.8	<0	1.2	>0	
11	7.6	−3.4	<0	−1.4	−1.4	
12	6.2	−4.8	<0	−2.8	−4.2	
13	8.2	−2.8	<0	−0.8	−5.0	
14	7.8	−3.2	<0	−1.2	−6.2	
15	6.8	−4.2	<0	−2.2	−8.4	
16	6.1	−4.9	<0	−2.9	−11.3	
17	4.3	−6.7	<0	−4.7	−16.0	Lower signal
18	8.5	−2.5	<0	−0.5	−16.5	
19	7.7	−3.3	<0	−1.3	−17.8	
20	9.7	−1.3	<0	0.7	−17.1	

x_t = the present observation
λ = the weighting factor for the present observation

The x_t are assumed to be independent, but the sample statistics Z_t are autocorrelated. However, Wortham and Ringer (1971) demonstrated that for large t, the sample statistic is normally distributed when the x_t are normally distributed with mean μ and variance σ^2. That is,

$$E(Z_t) = \mu$$

and

$$\text{Var}(Z_t) = \sigma^2[\lambda/(2-\lambda)][1 - (1 - \lambda)^{2t}]$$

As t increases, the last term bracketed on the right-hand side converges rapidly to one, and the corresponding expression for the variance becomes

$$\text{Var}(Z_t) \approx \sigma^2\,[\lambda/(2 - \lambda)]$$

By choosing $\lambda = 2/(t + 1)$, the variance approximation becomes

$$\text{Var}(Z_t) = \sigma^2/t \qquad \text{(the variance of averages of sample size } t\text{)}$$

Under these conditions, the control limits become $\hat{\mu} \pm 3\sqrt{\hat{\sigma}^2/t}$. For other values of λ, the control limits are

$$\text{UCL} = \hat{\mu} + 3\hat{\sigma}\sqrt{\lambda/(2 - \lambda)}$$

$$\text{LCL} = \hat{\mu} - 3\hat{\sigma}\sqrt{\lambda/(2-\lambda)}$$

For the first few observations, the first equation for the variance should be used. This can be illustrated by the following example. If a good estimate of σ is not available, a range chart should be used with $\hat{\sigma}$ estimated by $\overline{R}/d_2$. In the case of individuals, the average moving range can be used as with the control chart for individuals.

Example 45.7: To illustrate the EWMA control chart, we will use the same data as used in Example 45.5. Recall from that example that we have results of 20 samples of size 4 each taken on the percentage of water absorption in common building brick. The averages are shown in Table 45.6 and again in Table 45.8. The range chart gave us an estimate of $\sigma_{\bar{X}}$ of 1.96. For the first few samples we will use the complete formula for the standard deviation, and we will use the target value of 10 for our estimate of μ. For this example, a λ of 0.2 was used.

The resulting Minitab output with the data plotted is shown in Figure 45.6 It may be observed from the table and the chart that the control limits have stabilized after the eighth or ninth sample. The plot drops below the lower control limit on the sixteenth sample and stays there for the rest of the run.

The design parameters of this chart are the multiple of σ and the value of λ used. It is possible to choose these parameters to closely approximate the performance of the CUSUM chart. There have been several theoretical studies of the average run length of EWMA charts; see, for example, Lucas and Saccucci (1990). They provide average run length tables for a large range of values of λ and control chart widths. In general, Montgomery (1991) recommends values of λ between 0.05 and 0.25, with $\lambda = 0.08$, $\lambda = 0.10$, and $\lambda = 0.15$ being popular choices. Small values of λ should be used to detect small shifts. Three standard deviation limits, as with the Shewhart charts, seem to work well in most cases; however, if λ is less than 0.10, a value of 2.75σ works somewhat better.

Like the CUSUM chart, the EWMA chart is more effective than the $\bar{X}$ chart in detecting small shifts (less than 2.5σ) in the mean; however, both charts perform worse than the $\bar{X}$ chart for larger shifts. In order to overcome this difficulty, some authors have suggested plotting both the Shewhart limits and the EWMA limits on the same chart (see, for example, Albin et al. 1997).

TABLE 45.8 Data and Calculations for EWMA Chart

			Control limits for Z_t	
Sample t	$\bar{X}_t$	Z_t	LCL	UCL
0		10.00		
1	15.1	11.02	8.8	11.2
2	12.3	11.28	8.5	11.5
3	7.4	10.50	8.3	11.7
4	8.7	10.14	8.2	11.8
5	8.8	9.87	8.2	11.9
6	11.7	10.24	8.1	11.9
7	10.2	10.23	8.1	11.9
8	11.5	10.48	8.1	11.9
9	11.2	10.62	8.1	11.9
10	10.2	10.54	8.1	12.0
11	7.6	9.95	8.1	12.0
12	6.2	9.20	8.0	12.0
13	8.2	9.00		
14	7.8	8.76		
15	6.8	8.37		
16	6.1	7.92		
17	4.3	7.20		
18	8.5	7.46		
19	7.7	7.51		
20	9.7	7.95		

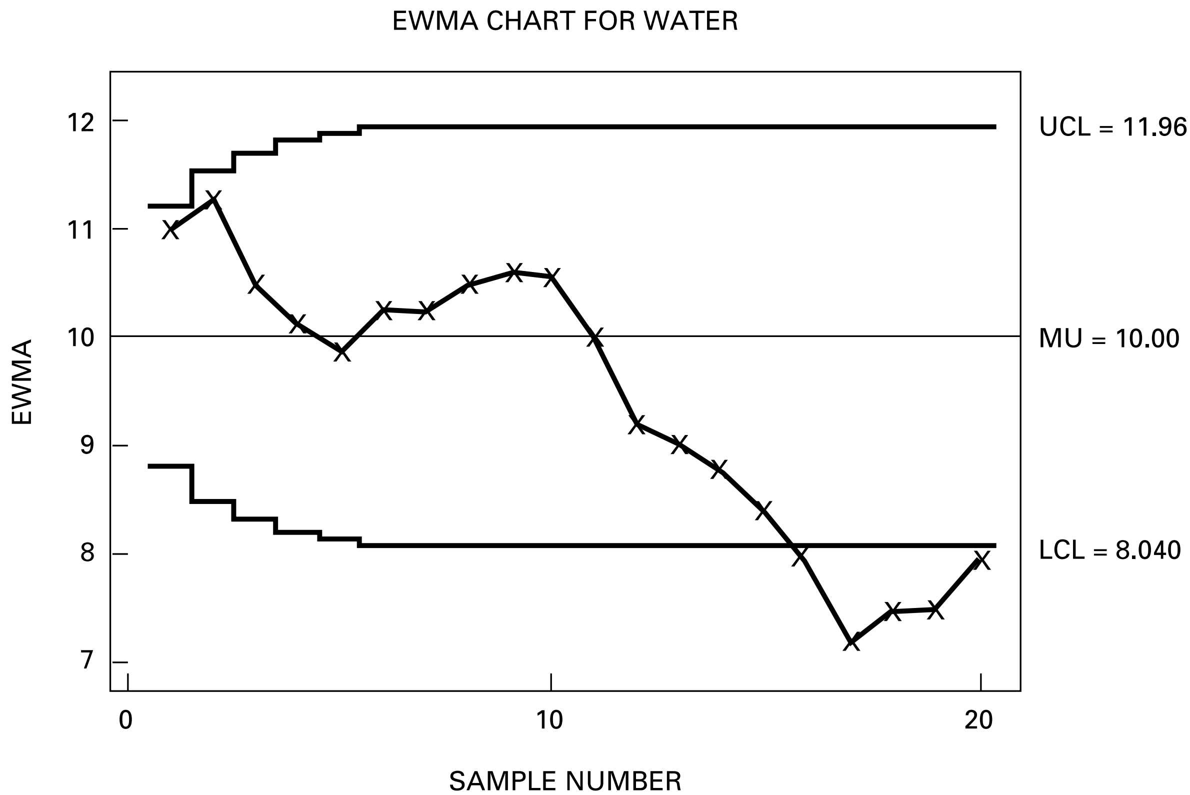

FIGURE 45.6 EWMA chart.

SHORT-RUN CONTROL CHARTS

Some processes are carried out in such short runs that the usual procedure of collecting 20 to 30 samples to establish a control chart is not feasible. Sometimes these short runs are caused by previously known assignable causes that take place at predetermined times. Hough and Pond (1995) discuss four ways to construct control charts for such situations:

1. Ignore the systematic variability, and plot on a single chart.
2. Stratify the data, and plot them on a single chart.
3. Use regression analysis to model the data, and plot the residuals on a chart.
4. Standardize the data, and plot the standardized data on a chart.

The last option has received the most consideration. It involves the use of the transformation

$$Z = \frac{X - \mu}{\sigma}$$

to remove any systematic changes in level and variability. This standardization of Shewhart charts has been discussed by Nelson (1989), Wheeler (1991), and Griffith (1996). Control charts of this form have control limits of ±3.0. These charts are sometimes called *Z charts.*

There are several variations of these charts, some of which are as follows:

1. *Difference charts,* in which a constant such as a known mean (μ), an average from past data ($\overline{X}$), a target value (T), or the specification limit is subtracted from each observation. Burr (1989) discussed this chart and included its use to determine process capability.
2. *Standardized charts,* in which a constant as above is subtracted from each observation and the result is divided by a second constant. When the divisor is the standard deviation, the resulting Z values have a standard deviation of 1, and the control limits are ±3 for $\alpha = 0.0027$. The divisor

can be a known standard deviation (σ) or an estimate from past data, such as $\overline{R}/d_2$. Sometimes the divisor is not a standard deviation but some other value. For example, Burr (1989) discussed a measure used at Kodak in which the nominal was subtracted from the observation and the result divided by half the tolerance. This had the advantage of indicating the fraction of the tolerance used by the process as well as the closeness to the nominal.

3. *Short-run* $\overline{X}$, *R charts,* in which the statistic

$$Z = (\overline{X} - \overline{\overline{X}})/\overline{R}$$

is plotted against control limits for the mean set at $\pm A_2$. The statistic $R/\overline{R}$ is plotted on a range chart with control limits set at D_3 and D_4. The result is a set of dimensionless charts that are suitable for plotting different parts or runs on the same chart.

Another chart for short runs is the Q chart discussed by Quesenberry (1991, 1995a, 1995b). This technique allows the chart to be constructed from initial data without the need of previous estimates of the mean or variance. It allows charting to begin at the start of a production run. Furthermore, the probability integral transformation is used to achieve normality for Q from nonnormal data such as the range or standard deviation. Quesenberry explains how the method may be used for charts for the mean or standard deviation when those parameters are either known or unknown.

To illustrate the Q chart, consider the case where normally distributed measurements are to be plotted when the mean is unknown and the variance is known. The chart for the process location is constructed as follows:

1. Collect individual measurements: $x_1, x_2, x_3, \ldots, x_r$.
2. For the rth point to be plotted, compute

$$Q_r(X_r) = [(r-1)/r]^{1/2}\,[(x_r - \overline{X}_{r-1})/\sigma] \qquad r = 1, 2, 3, \ldots$$

where $\overline{X}_{r-1}$ is the average of the previous $r - 1$ points.

3. Plot $Q_r(X_r)$ for each of the data points against control limits of ± 3.0.

The chart for process variation is constructed similarly:

1. Use the same measurements: $x_1, x_2, \ldots, x_r$.
2. For the rth data point to be plotted (plot for only even r)
   a. Compute

$$[(x_r - x_{r-1})^2/2\sigma^2], \qquad r = 2, 4, 6, \ldots$$

   b. Find the percentile of the χ^2 distribution with 1 degree of freedom for the value computed in ***a***.
   c. Find the normal deviate value for the percentile determined in ***b*** and set it equal to $Q(R_r)$.
3. Plot $Q(R_r)$ for even data points against control limits of ± 3.0.

BOX-JENKINS MANUAL ADJUSTMENT CHART

J. S. Hunter has suggested an important addition to the charting tools discussed earlier (Hunter 1997). Whereas the Shewhart, CUSUM, and EWMA charts for variables data *monitor* processes, the charts Hunter calls the *Box-Jenkins Manual Adjustment Charts* may be used to *regulate* them. These charts are based on the early work of Box and Jenkins (1962, 1970). Box and Luceño (1997) have recently published a text with a thorough explanation of these techniques. Hunter (1997) includes a worked example accompanied by many graphs.

The Shewhart, CUSUM, and EWMA charts discussed earlier require an operator to plot the time history of the statistic of interest and to leave the process alone as long as the plotted points fall within the control limits and satisfy the run rules. This assumes that the process mean μ is constant and that departures of the data from the mean are independent and normally distributed with a constant variance σ^2. The charts thus *monitor* the process. The objective is to reduce variability by the elimination of special (assignable) cause events identified by the charts.

The Box-Jenkins manual adjustment charts provide a mechanism to forecast and *regulate* the process after each observation. The procedure assumes the process to be *nonstationary*; i.e., the process level is changing constantly, and the variance is increasing. The objective of the procedure is to keep the variance as small as possible.

It is important to note that if the observed deviations from the mean are independent, this method should not be used, and if it is used, it may tend to inflate the process variance. However, if the deviations are not independent, a Box-Jenkins manual adjustment chart can provide both a forecast and the adjustments necessary to force the process to be on target with minimum variance. For a complete discussion of this technique, the reader is referred to the paper by Hunter (1997).

MULTIVARIATE CONTROL CHARTS

When there are two or more correlated quality characteristics that must be controlled simultaneously, such as both the length and weight of a part, the use of multivariate control charts is necessary. If we construct both the dimensions with separate control charts, the region of control will be rectangular, whereas the region using multivariate methods will be elliptical. This means that some observations will be in control in one case and out of control in the other.

Multivariate control charts are beyond the scope of this section, but the reader is referred to Alt et al. (1998), Jackson (1985), or Woodall and Neube (1985) for a detailed discussion of this topic. Multivariate versions of Shewhart, CUSUM, and EWMA charts are available in the literature. They are all discussed with references in Alt et al. (1998).

PRE-CONTROL

The control charts discussed so far in this section provide limits based solely on the observed variation of the process and therefore provide a means to detect process changes. In some situations it is important to detect the presence of only those changes which might incur the presence of nonconforming items. For such cases, control limits may be derived using a combination of the observed frequency and the product specifications. The process is then permitted to change as long as nonconforming items do not become imminent. The chart gives alarm signals only when nonconforming items threaten. Modified control limits [see Wadsworth et al. (1986, pp. 256–266)] and acceptance control charts are two examples of such charts. Acceptance control charts are described in Section 46. Another example of this approach is the PRE-control technique.

PRE-Control starts with a process centered between the specification limits and detects shifts that might result in making some parts outside of the specification limits. PRE-Control requires no plotting and no computations from the sample data. It only requires the inspection of two items to give control information. It uses the normal distribution to determine significant changes in the process that might result in nonconforming items. The principle of PRE-control assumes that the process uses up the entire tolerance spread. That is, the difference between the upper and lower specification limits is 6σ with the process exactly centered.

Two PRE-control (PC) limits are set one-fourth of the way in from each specification limit, as in Figure 45.7. This results in five regions that are often colored for practical applications. The two regions outside the specification limits are colored red. The two regions between the PC limits and the specification limits are colored yellow, and the middle region between the PC limits is colored green. If the distribution were as indicated in Figure 45.7*a*, 86 percent of the parts will be in the green zone, whereas 7 percent or one part in 14 will be in each yellow zone. The probability that two successive parts will fall in the same yellow zone will be 1/14 times 1/14, or 1/196 (approximately 1 in 200). Therefore, we can say that if two parts in a row fall in the same yellow zone, there is a much greater chance (195/196) that the process mean has shifted than not. It is advisable then to adjust the process toward the center.

It is equally unlikely to get one part in one yellow zone and the next in the other yellow zone. This would not indicate that the process has shifted but that some new factor has been introduced that has caused the variability to increase to an extent that nonconforming pieces are inevitable. An immediate study of the process and some corrective action must be made before it is safe to continue its operation.

These principles lead to the following set of rules that summarize the PRE-control procedure:

1. Divide the specification band with PC lines located one-fourth of the way in from each specification limit. If desired, color the zones appropriately, as indicated above, red, yellow, and green.
2. Start the process.
3. If the first piece is in the red (nonconforming) zone, adjust the process.
4. If the first piece is in the yellow (caution) zone, check the next piece.
5. If the second piece is in the same yellow zone, adjust the process toward the center.
6. If the second piece is in the green (good) zone, continue the process, and adjust the process only when two pieces in a row are in the same yellow zone.
7. If two successive pieces are in opposite yellow zones, stop the process and take action to reduce the variability.
8. When five successive pieces fall in the green zone, *frequency gaging* may start and continue as long as the average number of checks to an adjustment is 25. While waiting for five pieces in the green zone, if a piece falls in the yellow zone, restart the count.
9. During frequency gaging, make no process adjustments until a piece exceeds the PC line (yellow zone). If this occurs, check the next piece and continue as in step 6.
10. When the process is adjusted, five successive pieces in the green zone must again be made before returning to frequency gaging.
11. If the operator checks more than 25 pieces without having to adjust the process, the frequency of checking may be reduced so that more pieces are produced between checks. If, on the other hand, adjustment is needed before 25 pieces are checked, the frequency of checking should be increased. An average of 25 checks between adjustments is an indication that the frequency is correct.

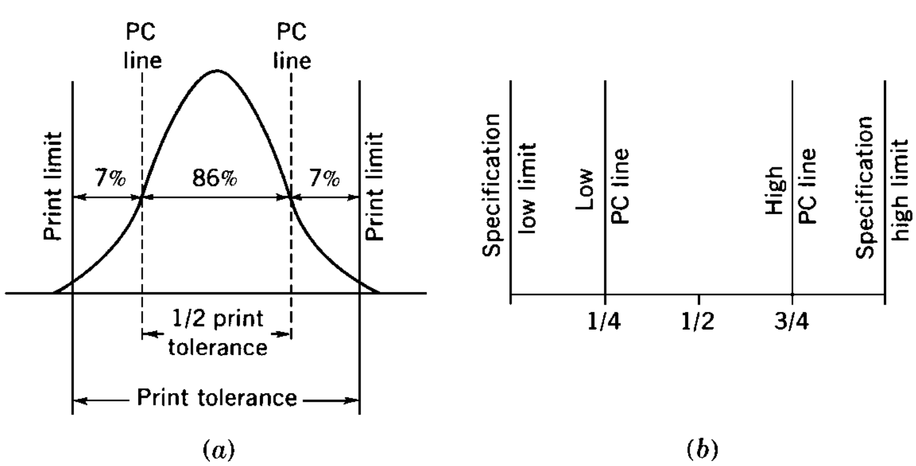

FIGURE 45.7 (*a*) Assumptions underlying PRE-control. (*b*) Location of PRE-control lines.

For one-sided specification limits, a single PC limit is established halfway between the tolerance and zero (in the case of flatness or concentricity) or one-fourth the distance between the specification limit and the best piece (in the case of a variable like yield or strength). In these cases there is but one yellow zone, one red zone, and one green zone.

The PRE-control technique indicates changes in the process mean or variability. The technique is simple to use and understand, it can make use of go no-go gages, and it can guarantee a certain percentage nonconforming if adjustments are made when indicated. On the other hand, process control techniques such as this, which make use of narrow limit gaging, must be introduced to the shop with care. Unless fully explained, the limits appear to be tightening up the tolerances.

Further discussion of this technique and the statistics behind it may be found in Brown (1966) and Shainin (1984). A comparison of PRE-control with $\bar{X}$ and R charts may be found in Sinibaldi (1985). Ledolter and Swersey (1997) evaluate the PRE-control procedure and conclude that PRE-control "is not an adequate substitute for control charts." They go on to explain that control charts are useful for identifying assignable causes of variation and distinguishing them from chance causes. This is particularly important in the early stages of a process when the process capability is likely to be low ($C_p < 1$). PRE-Control is poorly suited for this situation, and its use is likely to lead to excessive tampering with the process.

STATISTICAL CONTROL OF AUTOMATED PROCESSES

Other sections in this handbook have discussed automated manufacturing processes and the role of computers in both manufacturing and nonmanufacturing processes. The march toward automation has sparked some important innovations in data analysis for the control of processes:

1. Continuous-reading instrumentation that yields large amounts of data on process variables and quality characteristics.
2. Automation of statistical analyses (e.g., calculation of averages, standard deviations, and control limits). This is discussed in the next paragraph.
3. Comparison of process results with preset numerical standards. This comparison may result in
   - ***a.*** The generation of a document giving pertinent information on a nonstandard condition.
   - ***b.*** The generation of an error signal that automatically makes a process adjustment.
4. In recent years a great improvement in size control devices has been realized. While an item is being ground, a measuring pair of contacts monitors the change in size and electronically instructs the grinding wheel to prepare to stop as the desired size is being approached.

Software for Statistical Process Control. Many personal computer programs are available for statistical analysis of data, including all types of control charts. Gages with direct digital output into hand-held data-collection devices are available. These devices will store the gage readings collected on the shop floor and download them into a personal computer in the office. As an alternative, gage readings may be entered into the collection device with its keyboard.

The software will calculate the sample statistics, initial control limits, and the control chart. Control limits may be easily recalculated periodically, and $\pm 2\sigma$ limits can be calculated as an additional guide. Most software will provide additional summaries and analyses such as listing of the raw data, out-of-specification values, histograms, checks for runs and other patterns within control limits, tests for normality, process capability calculations, Pareto charts, and trend analyses.

The relatively low cost of personal computers and the availability of software have contributed substantially to the renewed interest in $\bar{X}$ and R and other control charts. This computer software combination also has made it practical to collect large quantities of data and subject them to complex analyses. However, process improvement still requires the identification of the vital few and often of unexpected variables causing excess variation. Computer analysis of previous data are no substitute for such tools as design of experiments. Computers are most helpful when the diagnostician has created a template for a spreadsheet program and least helpful when a packaged statistical analysis program is used. Users of such programs are rarely familiar with the detailed logic of the prepackaged programs and can be led to erroneous conclusions.

The American Society for Quality annually publishes a directory of software for process control and other statistical uses (ASQC 1996). In addition, the *Journal of Quality Technology* (ASQ) and *Quality* (Hitchcock Publishing Company) have computer columns describing programs for process control.

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