exceeds three sigma limits is 0.0027. But the joint probability that both variables exceed their control limits simultaneously when they are both in control is (0.0027)(0.0027) or 0.00000729, which is much smaller than 0.0027. The situation becomes more distorted as the number of characteristics increases. For this and other reasons, monitoring several characteristics independently can be misleading. Multivariate control charts and statistics (e.g., Hotelling's T^2 charts, multivariate EWMA) address this issue. See Montgomery (2000, Section 8.4) for a highly useful discussion.

Process Capability

In planning the quality aspects of operations, nothing is more important than advance assurance that the processes will meet the specifications. In recent decades, a concept of process capability has emerged to provide a quantified prediction of process adequacy. This ability to predict quantitatively has resulted in widespread adoption of the concept as a major element of quality planning. Process capability is the measured, inherent variation of the product turned out by a process.

Basic definitions. Each key word in this definition must itself be clearly defined because the concept of capability has an enormous extent of application, and nonscientific terms are inadequate for communication within the industrial community.

- Process refers to some unique combination of machine, tools, methods, materials, and people engaged in production. It is often feasible and illuminating to separate and quantify the effect of the variables entering this combination.
- Capability refers to an ability, based on tested performance, to achieve measurable results.
- Measured capability refers to the fact that process capability is quantified from data that, in turn, are the results of measurement of work performed by the process.
- Inherent capability refers to the product uniformity resulting from a process that is in a state of statistical control (i.e., in the absence of time-to-time "drift" or other assignable causes of variation). "Instantaneous reproducibility" is a synonym for inherent capability.
- The product is measured because product variation is the end result.

Uses of process capability information. Process capability information serves multiple purposes:

- Predicting the extent of variability that processes will exhibit. Such capability information, when provided to designers, provides important information in setting realistic specification limits.
- Choosing from among competing processes that are most appropriate to meet the tolerances.
- Planning the interrelationship of sequential processes. For example, one process may distort the precision achieved by a predecessor process, as in hardening of gear teeth. Quantifying the respective process capabilities often points the way to a solution.
- Providing a quantified basis for establishing a schedule of periodic process control checks and readjustments.
- Assigning machines to classes of work for which they are best suited.

- Testing theories of causes of defects during quality improvement programs.
- Serving as a basis for specifying the quality performance requirements for purchased machines.

These purposes account for the growing use of the process capability concept.

Planning for a process capability study. Capability studies are conducted for various reasons, for example, to respond to a customer request for a capability index number or to evaluate and improve product quality. Prior to data collection, clarify the purpose for making the study and the steps needed to ensure that it is achieved.

In some cases, the capability study will focus on determining a histogram and capability index for a relatively simple process. Here the planning should ensure that process conditions (e.g., temperature, pressure) are completely defined and recorded. All other inputs must clearly be representative (i.e., specific equipment, material, and, of course, personnel).

For more complex processes or when defect levels of 1 to 10 parts per million are desired, the following steps are recommended:

1. Develop a process description, including inputs, process steps, and output quality characteristics. This description can range from simply identifying the equipment to developing a mathematical equation that shows the effect of each process variable on the quality characteristics.
2. Define the process conditions for each process variable. In a simple case, this step involves stating the settings for temperature and pressure. But for some processes, it means determining the optimum value or aim of each process variable. The statistical design of experiments provides the methodology. Also, determine the operating ranges of the process variables around the optimum because the range will affect the variability of the product results.
3. Make sure that each quality characteristic has at least one process variable that can be used to adjust it.
4. Decide whether measurement error is significant. This can be determined from a separate error of measurement study. In some cases, the error of measurement can be evaluated as part of the overall study.
5. Decide whether the capability study will focus only on variability or will also include mistakes or errors that cause quality problems.
6. Plan for the use of control charts to evaluate the stability of the process.
7. Prepare a data collection plan, including adequate sample size that documents results on quality characteristics along with the process conditions (e.g., values of all process variables) and preserves information on the order of measurements so that trends can be evaluated.
8. Plan which methods will be used to analyze data from the study to ensure that before starting the study, all necessary data for the analysis will be available. The analyses should include process capability calculations on variability and also analysis of attribute or categorical data on mistakes and analysis of data from statistically designed experiments built into the study.
9. Be prepared to spend time investigating interim results before process capability calculations can be made. These investigations can include analysis of optimum values and ranges of process variables, out-of-control points on control charts, or other unusual results. The investigations then lead to the ultimate objective, that is, improvement of the process.

Note that these steps focus on improvement rather than just on determining a capability index.

Standardized process capability formula. The most widely adopted formula for process capability is:

$$\text{Process capability} = \pm 3\sigma \text{ (a total of } 6\sigma)$$

where σ is the standard deviation of the process under a state of statistical control (i.e., under no drift and no sudden changes). If the process is centered at the nominal specification and follows a normal probability distribution, 99.73 percent of production will fall within 3σ of the nominal specification.

Relationship to product specifications. A major reason for quantifying process capability is to compute the ability of the process to hold product specifications. For processes that are in a state of statistical control, a comparison of the variation of 6*s* to the specification limits permits ready calculation of percentage defective by conventional statistical theory.

Planners try to select processes with the 6*s* process capability well within the specification width. A measure of this relationship is the capability ratio:

$$C_p = \text{capability ratio} = \frac{\text{specification range}}{\text{process capability}} = \frac{\text{USL} - \text{LSL}}{6s}$$

where USL is the upper specification limit and LSL is the lower specification limit.

Note that 6*s* is used as an estimate of 6σ.

Some companies define the ratio as the reciprocal. Some industries now express defect rates in terms of parts per million. A defect rate of one part per million requires a capability ratio (specification range over process capability) of about 1.63.

Figure 19.30 shows four of many possible relations between process variability and specification limits and the likely courses of action for each. Note that in all of these cases, the average of the process is at the midpoint between the specification limits.

Table 19.20 shows selected capability ratios and the corresponding level of defects, assuming that the process average is midway between the specification limits. A process that is just meeting specification limits (specification range ±3σ) has a C_p of 1.0. The criticality of many applications and the reality that the process average will not remain at the midpoint of the specification range suggest that C_p should be at least 1.33. Note that a process operating at $C_p = 2.0$ over the short term (and centered midway between the specification limits) will correspond to a process sigma capability measure of $3C_p$, or 6 sigma (allowing for a 1.5s shift over the long term. This corresponds to $6s - 1.5s = 4.5s$, which is expected to produce 3.4 ppm outside of the two-sided specification limits over the long term).

Note that the C_p index measures whether the process variability can fit within the specification range. It does not indicate whether the process is actually running within the specification because the index does not include a measure of the process average (this issue is addressed by another measure, C_{pk}).

Three capability indexes commonly in use are shown in Table 19.21. Of these, the simplest is C_p. The higher the value of any indexes, the lower the amount of product outside the specification limits.

Pignatiello and Ramberg (1993) provide an excellent discussion of various capability indexes. Bothe (1997) provides a comprehensive reference book that includes extensive discussion of the mathematical aspects. These references explain how to calculate confidence bounds for various process capability indexes.

The C_{pk} capability index. Process capability, as measured by C_{pk}, refers to the variation in a process about the average value. This concept is illustrated in Figure 19.31. The two processes have equal capabilities (C_p) because 6σ is the same for each distribution, as indicated by the

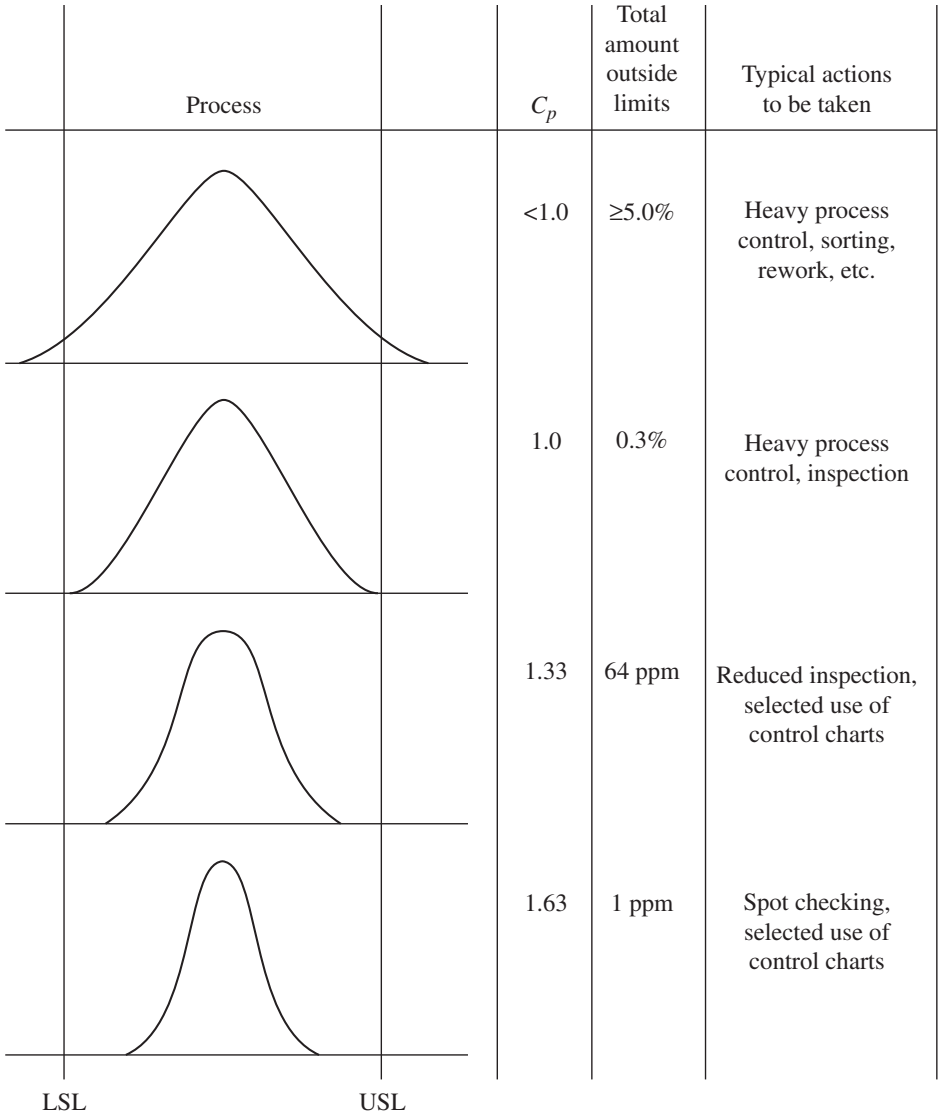

Figure 19.30 Four examples of process variability. (*Quality Planning and Analysis, Copyright 2007. Used by permission.*)

widths of the distribution curves. The process aimed at μ2 is producing defectives because the aim is off center, not because of the inherent variation about the aim (i.e., the capability).

Thus, the C_p index measures potential capability, assuming that the process average is equal to the midpoint of the specification limits and the process is operating in statistical control; because the average is often not at the midpoint, it is useful to have a capability index that reflects both variation and the location of the process average. Such an index is C_{pk}.

C_{pk} reflects the current process mean's proximity to either the USL or LSL. C_{pk} is estimated by

$$\hat{C}_{pk} = \min\left[\frac{\bar{X} - \text{LSL}}{3s}, \frac{\text{USL} - \bar{X}}{3s}\right]$$

In an example from Kane (1986),

$\text{USL} = 20 \quad \bar{X} = 16$

$\text{LSL} = 8 \quad s = 2$

Process Capability Index (C_p)	Total Product Outside Two-Sided Specification Limits*
0.5	13.36%
0.67	4.55%
1.00	0.3%
1.33	64 ppm
1.63	1 ppm
2.00	0

*Assuming that the process is centered midway between the specification limits.
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TABLE 19.20 Process Capability index (C_p) and Product Outside Specification Limits

Process Capability	Process Performance
$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}$	$P_p = \frac{\text{USL} - \text{LSL}}{6s}$
$C_{pk} = \min\left[\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right]$	$P_{pk} = \min\left[\frac{\text{USL} - \bar{X}}{3s}, \frac{\bar{X} - \text{LSL}}{3s}\right]$
$C_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}}$	$P_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{s^2 + (\bar{X} - T)^2}}$

(*Source: Quality Planning and Analysis*, Copyright 2007. Used by permission.)

TABLE 19.21 Process Capability and Process Performance Indexes

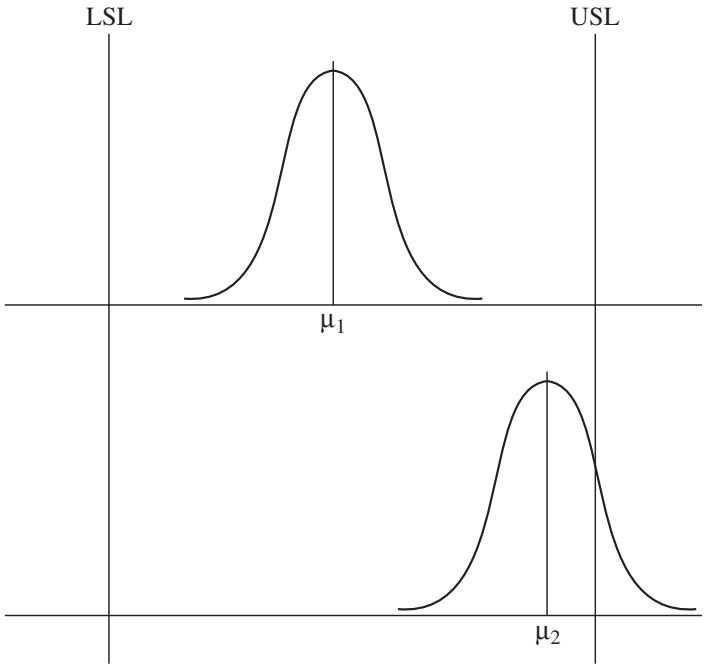


FIGURE 19.31 Process with Equal Process Capability but Different Aim. (*Quality Planning and Analysis. Copyright 2007. Used by permission.*)

The standard capability ratio is estimated as

$$\frac{\text{USL} - \text{LSL}}{6\sigma} = \frac{20 - 8}{12} = 1.0$$

which implies that if the process were centered between the specification limits (at 14), then only a small proportion (about 0.27 percent) of product would be defective.

However, when we calculate C_{pk}, we obtain

$$\hat{C}_{pk} = \min\left[\frac{16 - 8}{6}, \frac{20 - 16}{12}\right] = 0.67$$

which indicates that the process mean is currently nearer the USL. (Note that if the process were centered at 14, the value of C_{pk} would be 1.0.) An acceptable process will require reducing the standard deviation and/or centering the mean. Also note that if the actual average is equal to the midpoint of the specification range, then $C_{pk} = C_p$.

The higher the value of C_p, the lower the amount of product outside specification limits. In certifying suppliers, some organizations use C_{pk} as one element of certification criteria. In these applications, the value of C_{pk} desired from suppliers can be a function of the type of commodity purchased.

A capability index can also be calculated around a target value rather than the actual average. This index, called C_{pm} or the Taguchi index, focuses on reduction of variation from a target value rather than reduction of variability to meet specifications.

Most capability indexes assume that the quality characteristic is normally distributed. Krishnamoorthi and Khatwani (2000) propose a capability index for handling normal and nonnormal characteristics by first fitting the data to a Weibull distribution.

Two types of process capability studies are as follows:

1. *Study of process potential*. In this study, an estimate is obtained of what the process can do under certain conditions (i.e., variability under short-run defined conditions for a process in a state of statistical control). The C_p index estimates the potential process capability.
2. *Study of process performance*. In this study, an estimate of capability provides a picture of what the process is doing over an extended period. A state of statistical control is also assumed. The C_{pk} index estimates the performance capability.

Estimating inherent or potential capability from control chart analysis. In a process potential study, data are collected from a process operating without changes in material batches, workers, tools, or process settings. This short-term evaluation uses consecutive production over one time period. Such an analysis should be preceded by a control chart analysis in which any assignable causes have been detected and eliminated from the process.

Because specification limits usually apply to individual values, control limits for sample averages cannot be compared to specification limits. To make a comparison, we must first convert R to the standard deviation for individual values, calculate the 3s limits, and compare them to the specification limits. This process is explained below.

If a process is in statistical control, it is operating with the minimum amount of variation possible (the variation due to chance causes). If, and only if, a process is in statistical control, the following relationship holds for using s as an estimate of σ:

$$s = \frac{\bar{R}}{d_2}$$

Tables 19.22 and 19.23 provide values of d_2. If the standard deviation is known, process capability limits can be set at $\pm 3\sigma$, and this value used as an estimate of 3σ.

Factors for $\bar{X}$ and R Control Charts;* Factors for Estimating s from R†				
Number of Observations in Sample	A_2	D_3	D_4	Factor for Estimate from $\bar{R}$: $d_2 = \bar{R}/s$
2	1.880	0	3.268	1.128
3	1.023	0	2.574	1.693
4	0.729	0	2.282	2.059
5	0.577	0	2.114	2.326
6	0.483	0	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
11	0.285	0.256	1.744	3.173
12	0.266	0.284	1.717	3.258
13	0.249	0.308	1.692	3.336
14	0.235	0.329	1.671	3.407
15	0.223	0.348	1.652	3.472

$$\begin{cases}\text{Upper control limit for } \bar{X} = \text{UCL}_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} \\ \text{Lower control limit for } \bar{X} = \text{LCL}_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}\end{cases}$$

$$\begin{cases}\text{Upper control limit for } R = \text{UCL}_R = D_4\bar{R} \\ \text{Lower control limit for } R = \text{LCR}_R = D_3\bar{R}\end{cases}$$

$$s = \bar{R}/d_2$$

From *1950 ASTM Manual on Quality Control of Materials* and *ASTM Manual on Presentation of Data, 1945.* American Society for Testing and Materials. Copyright ASTM International. Reprinted with permission. (*Source: Quality Planning and Analysis*, Copyright 1997. Used by permission.)

TABLE 19.22 Factors for $\bar{X}$ and R Control Charts

n	A_2	D_3	D_4	d_2
2	1.880	0	3.268	1.128
3	1.023	0	2.574	1.693
4	0.729	0	2.282	2.059
5	0.577	0	2.114	2.326
6	0.483	0	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.079

(*Source: Quality Planning and Analysis*, Copyright 2007. Used by permission.)

TABLE 19.23 Constants for $\bar{X}$ and R Chart

For the data shown in Figure 19.32 (machine N-5),

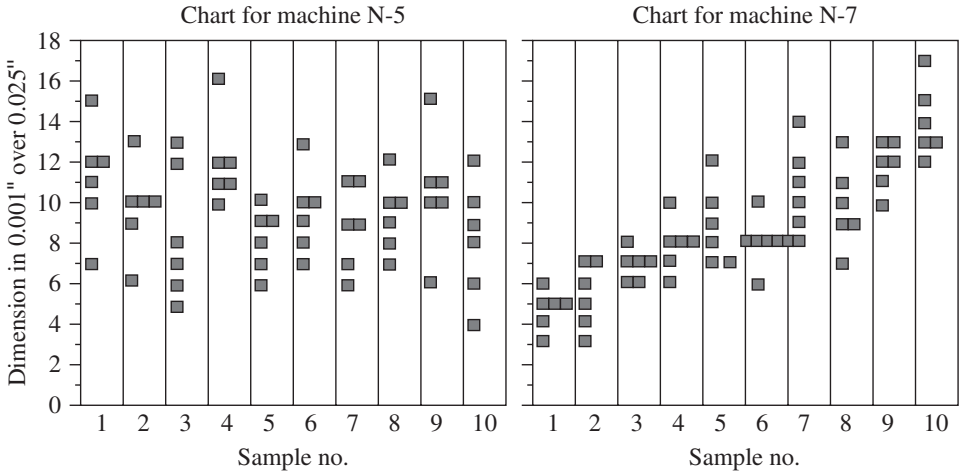
$$s = \frac{\bar{R}}{d_2} = \frac{6.0}{2.534} = 2.37$$

and

$$\pm 3s = \pm 3(2.37) = 7.11$$

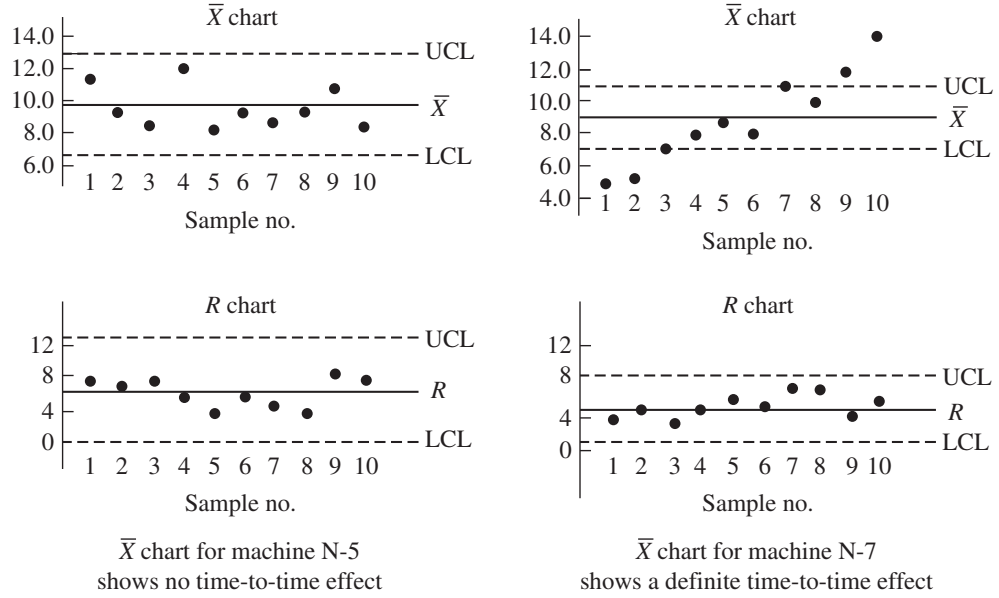
or

$$6s = 14.22 \text{ (or } 0.0124 \text{ in the original data units)}$$



For machine N-5:

Sample	1	2	3	4	5	6	7	8	9	10
$\bar{X}$	11.2	9.7	8.5	12.0	8.2	9.5	8.8	9.3	10.5	8.2
R	8.0	7.0	8.0	6.0	4.0	6.0	5.0	5.0	9.0	8.0



$\bar{X}$ chart for machine N-5 shows no time-to-time effect

$\bar{X}$ chart for machine N-7 shows a definite time-to-time effect

FIGURE 19.32 $\bar{X}$ and R charts confirm. (*Quality Planning and Analysis. Copyright 2007. Used by permission.*)

The specification limit was 0.258 ± 0.005.

Thus,

USL = 0.263

LSL = 0.253

Then

$$C_p = \frac{\text{USL} - \text{LSL}}{6s} = \frac{0.263 - 0.253}{0.0142} = 0.72$$

Even if the process is perfectly centered at 0.258 (and it was not), it is not capable.

The assumption of statistical control and its effect on process capability. All statistical predictions assume a stable population. In a statistical sense, a stable population is one that is repeatable (i.e., a population that is in a state of statistical control). The statistician rightfully insists that this be the case before predictions can be made. The manufacturing engineer also insists that the process conditions (feeds, speeds, etc.) be fully defined.

In practice, the original control chart analysis will often show that the process is out of statistical control. (It may or may not be meeting product specifications.) However, an investigation may show that the causes cannot be economically eliminated from the process. In theory, process capability should not be predicted until the process is in statistical control. However, in practice, some kind of comparison of capability to specifications is needed. The danger in delaying the comparison is that the assignable causes may never be eliminated from the process. The resulting indecision will thereby prolong interdepartmental bickering on whether "the specification is too tight" or "manufacturing is too careless."

A good way to start is by plotting individual measurements against specification limits. This step may show that the process can meet the product specifications even with assignable causes present. If a process has assignable causes of variation but is able to meet the specifications, usually no economic problem exists. The statistician can properly point out that a process with assignable variation is unpredictable. This point is well taken, but in establishing priorities of quality improvement efforts, processes that are meeting specifications are seldom given high priority.

If a process is out of control and the causes cannot be economically eliminated, the standard deviation and process capability limits can nevertheless be computed (with the out-of-control points included). These limits will be inflated because the process will not be operating at its best. In addition, the instability of the process means that the prediction is approximate.

It is important to distinguish between a process that is in a state of statistical control and a process that is meeting specifications. A state of statistical control does not necessarily mean that the product from the process conforms to specifications. Statistical control limits on sample averages cannot be compared to specification limits because specification limits refer to individual units. For some processes that are not in control, the specifications are being met and no action is required; other processes are in control, but the specifications are not being met, and action is needed.

In summary, we need processes that are both stable (in statistical control) and capable (meeting product specifications).

The increasing use of capability indexes has also led to the failure to understand and verify some important assumptions that are essential for statistical validity of the results. Five key assumptions are:

1. *Process stability*. Statistical validity requires a state of statistical control with no drift or oscillation.
2. *Normality of the characteristic being measured*. Unless nonparametric methods or alternative distributions are used, normality is needed to draw statistical inferences about the population.

3. *Sufficient data*. Sufficient data are necessary to minimize the sampling error for the capability indexes.
4. *Representativeness of samples*. Random samples must be included.
5. *Independent measurements*. Consecutive measurements cannot be correlated.

These assumptions are not theoretical refinements—they are important conditions for properly applying capability indexes. Before applying capability indexes, readers are urged to read the paper by Pignatiello and Ramberg (1993). It is always best to compare the indexes with the full data versus specifications depicted in a histogram.

Measuring process performance. A process performance study collects data from a process that is operating under typical conditions but includes normal changes in material batches, workers, tools, or process settings. This study, which spans a longer term than the process potential study, also requires that the process be in statistical control.

The capability index for a process performance study is

$$C_{pk} = \min\left[\frac{\bar{X} - \text{LSL}}{3s}, \frac{\text{USL} - \bar{X}}{3s}\right]$$

Problem Consider a pump cassette used to deliver intravenous solutions (Baxter Travenol Laboratories, 1986). A key quality characteristic is the volume of solution delivered in a predefined time. The specification limits are

$$\text{USL} = 103.5 \quad \text{LSL} = 94.5$$

A control chart was run for one month, and no out-of-control points were encountered. From the control chart data, we know that

$$\bar{\bar{X}} = 98.2 \text{ and } s = 0.98$$

Figure 19.33 shows the process data and the specification limits.

Solution The capability index is

$$C_{pk} = \min\left[\frac{98.2 - 94.5}{3(0.98)}, \frac{103.5 - 98.2}{3(0.98)}\right]$$

$$C_{pk} = 1.26$$

For many applications, 1.26 is an acceptable value of C_{pk}.

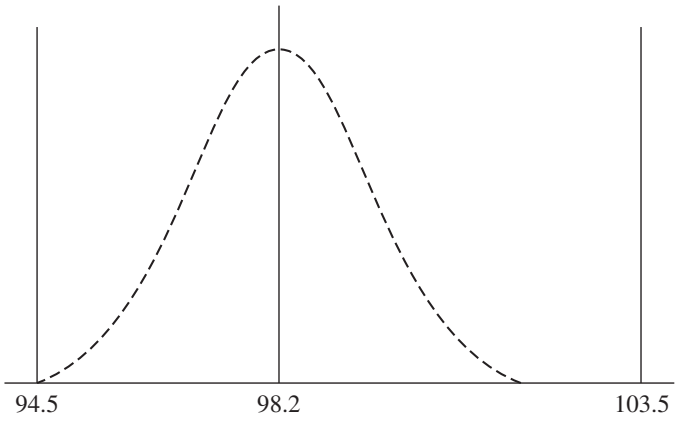


Figure 19.33 Delivered volume of solution.

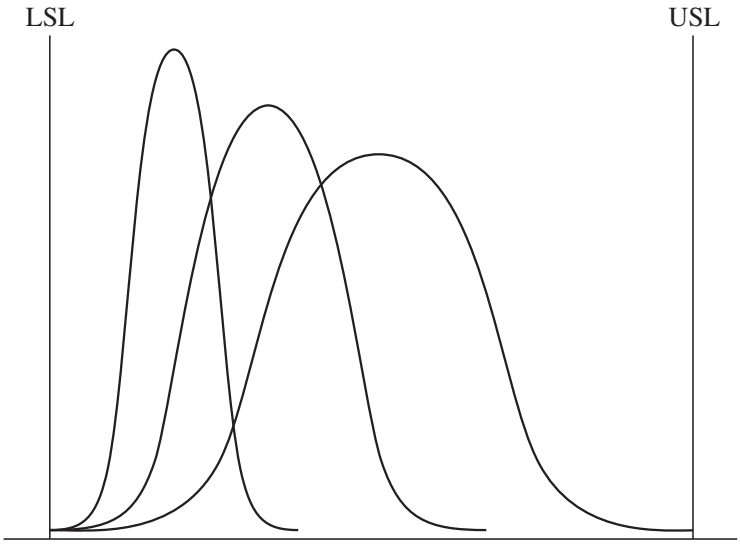


Figure 19.34 Three Processes with $C_{pk} = 1$. (*Quality Planning and Analysis. Copyright 2007. Used by permission.*)

Interpretation of C_{pk}. In using C_{pk} to evaluate a process, we must recognize that C_{pk} is an abbreviation of two parameters—the average and the standard deviation. Such an abbreviation can inadvertently mask important detail in these parameters. For example, Figure 19.34 shows that three extremely different processes can all have the same C_{pk} (in this case $C_{pk} = 1$).

Increasing the value of C_{pk} may require a change in the process average, the process standard deviation, or both. For some processes, increasing the value of C_{pk} by changing the average value (perhaps by a simple adjustment of the process aim) may be easier than reducing the standard deviation (by investigating the many causes of variability). The histogram of the process should always be reviewed to highlight both the average and the spread of the process.

Note that Table 19.21 also includes the capability index C_{pm}. This index measures the capability around a target value T rather than the mean value. When the target value equals the mean value, the C_{pm} index is identical to the C_{pk} index.

Attribute (or categorical) data analysis. The methods discussed earlier assume that numerical measurements are available from the process. Sometimes, however, the only data available are in attribute or categorical form (i.e., the number of nonconforming units and the number acceptable).

The data in Table 19.24 on errors in preparing insurance policies also can be used to illustrate process capability for attribute data. The data reported 80 errors from six policy writers, or 13.3 errors per writer—the current performance. The process capability can be calculated by excluding the abnormal performance identified in the study—type 3 errors by worker B, type 5 errors, and errors of worker E. The error data for the remaining five writers becomes 4, 3, 5, 2, and 5, with an average of 3.8 errors per writer. The process capability estimate of 3.8 compares with the original performance estimate of 13.3.

This example calculates process capability in terms of errors or mistakes rather than the variability of a process parameter. Hinckley and Barkan (1995) point out that in many processes, nonconforming product can be caused by excessive variability or by mistakes (e.g., missing parts, wrong parts, wrong information, or other processing errors). For some processes, mistakes can be a major cause of failing to meet customer quality goals. The actions required to reduce mistakes are different from those required to reduce variability of a parameter.

Readers are directed to DeVor et al. (1992) for a good background in process control charting.

	Policy Writer						
Error Type	**A**	**B**	**C**	**D**	**E**	**F**	**Total**
1	0	0	1	0	2	1	4
2	1	0	0	0	1	0	2
3	0	(16)	1	0	2	0	(19)
4	0	0	0	0	1	0	1
5	2	1	3	1	4	2	(13)
6	0	0	0	0	3	0	3
.							
.							
.							
27							
28							
29							
Total	6	(20)	8	3	(36)	7	80

(*Source: Quality Planning and Analysis*, Copyright 2007. Used by permission.)

TABLE 19.24 Matrix of Errors by Insurance Policy Writers

Software

While many of the tools mentioned in this chapter can be applied using programs such as Microsoft Excel, numerous software packages are available that provide more specialized assistance. Some of these packages and vendors are listed here, according to their primary emphasis. Most vendors have multiple software options.

Basic statistics:

- QI Macros
- SigmaXL
- StatPlus

Advanced statistics:

- JMP
- Minitab
- Systat

Design of experiments:

- StatSoft STATISTICA
- Stat-Ease
- STRATEGY
- Statgraphics

Monte Carlo, discrete event simulation:

- @Risk
- Crystal Ball
- iGrafx

Reliability, availability:

- Isograph
- Relex 2009
- ReliaSoft

Control charting:

- CHARTRunner
- Statit

References

Anand, K. N. (1996), The Role of Statistics in Determining Product and Part Specifications: A Few Indian Experiences, *Quality Engineering*, vol. 9, no. 2, pp. 187–193.

Automotive Industry Action Group (2003). *Measurement Systems Analysis* (3rd ed.). Southfield, MI.

Barnett, V., and Lewis, T. (1994). *Outliers in Statistical Data* (3rd ed.). John Wiley & Sons, New York.

Bender, A. (1975). Statistical Tolerancing as It Relates to Quality Control and the Designer, *Automotive Division Newsletter of ASQC*, April, p. 12

Bothe, D. R. (1997). *Measuring Process Capability*. McGraw-Hill, New York.

Box, G. E. P., and Luceno, A. (1997). *Statistical Control by Monitoring and Adjustment*. Wiley, New York.

Box, G. E. P., and Draper, N. R. (1969). *Evolutionary Operation: A Statistical Method for Process Improvement*. John Wiley & Sons, New York.

Box, G. E. P., Hunter, J. S., and Hunter, W. G. (2005). *Statistics for Experimenters: Design, Innovation and Discovery* (2nd ed.). Wiley-Interscience, Hoboken, NJ.

Burdick, R. K., and Larsen, G. A. (1997). Confidence Intervals on Measures of Variability in R&R Studies, *Journal of Qualitiy Technology*, vol. 29, no. 3, pp. 261–273.

Carter, C. W. (1996). Sequenced Levels Experimental Designs, *Quality engineering*, vol. 8, no. 1, (pp. 181 -188), no. 2 (pp. 361-366), no. 3 (pp. 499–504), no.4 (pp. 695–698).

Case, K. E., Bennett, G. K., and Schmidt, J. W. (1975). "The Effect of Inspector Error on Average Outgoing Quality," *Journal of Quality Technology*, vol. 7, no. 1, pp. 1–12.

Coleman, S., Greenfield, T., Stewardson, D., and Montgomery, D. C. (2008). *Statistical Practice in Business and Industry*. John Wiley & Sons, Hoboken, NJ. (See Chapter 13).

Davison, A. C., and Hinkley, D. (2006). *Bootstrap Methods and Their Applications* (8th ed.). Cambridge: Cambridge Series in Statistical and Probabilistic Mathematics, Davison Hinkley, Cambridge University Press, Cambridge.

del Castillo, E. (2007). *Process Optimization: A Statistical Approach*. Springer Science and Business Media, New York.

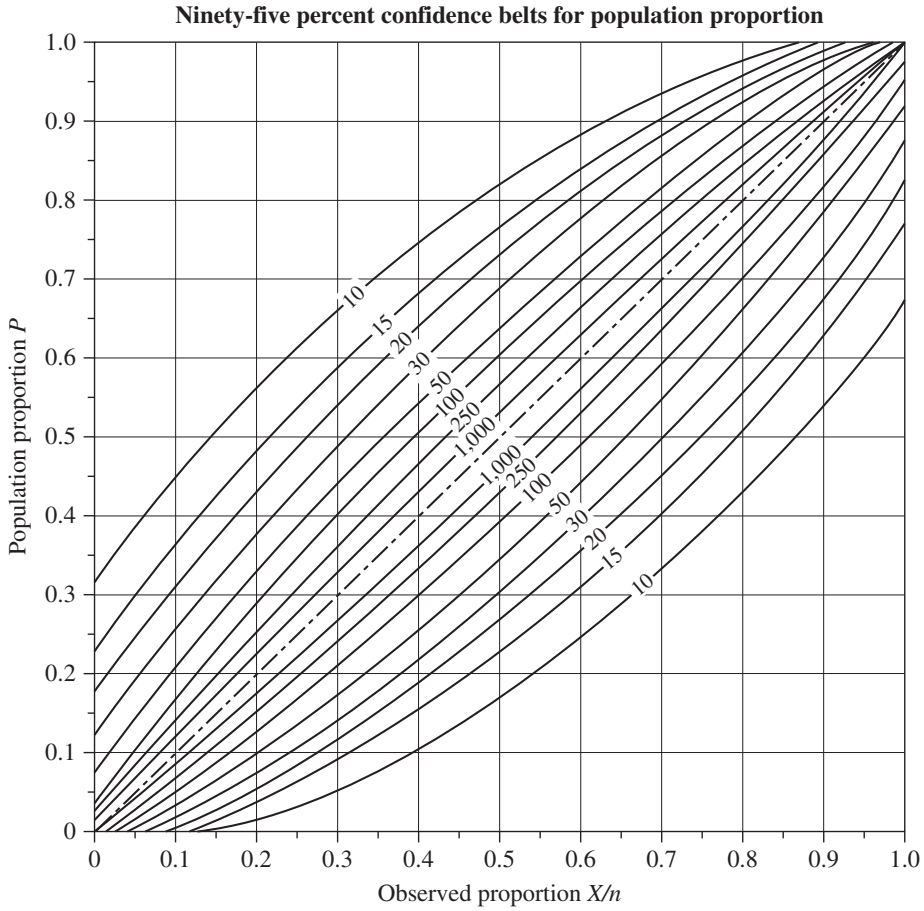
DeVor, R. E., Chang, T., and Sutherland, J. W. (1992). *Statistical Quality Design and Control: Contemporary Concepts and Methods*. Prentice Hall, Upper Saddle River, NJ.

Dodson, B. (1999). Reliability Modeling with Spreadsheets, *Proceedings of the Annual Quality Congress*, ASQ, Milwaukee, pp. 575–585.

Eagle, A. R. (1954). A Method for Handling Errors in Testing and Measurement, *Industrial Quality Control*, March, pp. 10–14.
Emory, W. C., and Cooper, D. R. (1991). *Business Research Methods* (4th ed.). Boston: Irwin/McGrawHill.
Engel, J., and DeVries, B. (1997). Evaluating a Well-Known Criterion for Measurement Precision, *Journal of Quality Technology*, vol. 29, no. 4, pp. 469–476.
Gomer, P. (1998). Design for Tolerancing of Dynamic Mechanical Assemblies, *Annual Quality Congress Proceedings*, ASQ, Milwaukee, pp. 490–500.
Graves, S. B. (1997). How to Reduce Costs Using a Tolerance Analysis Formula Tailored to your Organization, Report no. 157, Center for Quality and Productivity Improvement, University of Wisconsin, Madison.
Griffith, G. K. (1996). *Statistical Process Control Methods for Long and Short Runs*, 2nd ed., ASQ Quality Press, Milwaukee.
Gryna, F. M., Chua, R. C., and De Feo, J. A. (2007). *Juran's Quality Planning and Analysis* (5th ed.). McGraw Hill, New York.
Hinckley, C. M., and Barkan, P. (1995). The Role of Variation, Mistakes, and Complexity in Producing Nonconformities, *Journal of Quality Technology*, vol. 27, no. 3, pp. 242–249.
Hoag, L. L., Foote, G. L., and Mount-Cambell, C. (1975). The Effect of Inspector Accuracy on Type I and II Errors of Common Sampling Techniques, *Journal of Quality Technology*, vol. 7, no. 4, pp. 157–164.
Hough, L. D., and Pond, A. D. (1995). Adjustable Individual Control Charts for Short Runs. *Proceedings of the 40th Annual Quality Congress*, ASQ, Milwaukee, pp. 1117–1125.
Ireson, W. G., Coombs, C. F., Jr., and Moss, R. Y. (1996). *Handbook of Reliability Engineering and Management*, 2nd ed., McGraw-Hill, New York.
Early, J. F. Quality Improvement Tools, *The Power of Quality*, The Health Care Forum, June 1989.
Jones, J., and Hayes, J. (1999). A Comparison of Electronic Reliability Prediction Models, IEEE Transactions of Reliability, vol. 48, no. 2, pp. 127–134.
Kane, V. E. (1986). Process Capability Indices, *Journal of Quality Technology*, vol. 18, no. 1, pp. 41-52.
Krishnamoorthi, I. S., and Khatwani, S. (2000). *Statistical Process Control for Health Care*, Duxbury, Paciric Grove, CA.
Kutner, M., Nachtsheim, C., Neter, J., and Li, W. (2004). *Applied Linear Statistical Models* (2nd ed.). Irwin/McGraw-Hill, New York.
Kvam, P. H., and Vidakovic, B. (2007). *Nonparametric Statistics with Applications to Science and Engineering*. John Wiley & Sons, Hoboken, NJ.
Law, A. M., and Kelton, W. D. (2000). *Simulation Modeling and Analysis* (3rd ed.). McGraw-Hill.
Ledolter, J., and Swersey, A. (1997). An Evaluation of Pre-Control, *Journal of Quality Technology*, vol. 29, no. 1, pp. 163–171.
Ledolter, J., and Swersey, A. J. (2007). *Testing 1-2-3: Experimental Design with Applications in Marketing and Service Operation*s. Stanford University Press, Palo Alto, CA.
Meeker, W. Q., and Escobar, L. A. (1998). *Statistical Methods for Reliability Data*. John Wiley & Sons, 1998.
Meeker, W. Q., and Escobar, L. A. (1998). *Statistical Methods for Reliability Data*. John Wiley & Sons, New York.
Montgomery, D. C. (2000). *Introduction to Statistical Quality Control*, 4th ed., Wiley, New York, NY.
Myers, R. H., Montgomery, D. C., and Anderson-Cook, C. M. (2009). *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*. John Wiley & Sons, Hoboken, NJ.
Nelson, L. S. (1989). Standardization of Shewhart Control Charts, *Journal of Quality Technology*, vol. 21, 287–289.
O'Connor, P. D. T. (1995). *Practical Reliability Engineering*, 3rd Ed. rev., John Wiley and Sons, New York.
Pignatiello, J. H., Jr., and Ramberg, J. S. (1993). Process Capability Indices: Just Say No, *ASQC Quality Congress Transactions 1993*, American Society for Quality, Milwaukee.

Pyzdek, T. (1993). Process Control for Short and Small Runs, *Quality Progress*, April, pp. 51–60.
Ross, P. J. (1996). *Taguchi Techniques for Quality Engineering*. McGraw- Hill, New York.
Sprent, P., and Smeeton, N. C. (2001). *Applied Nonparametric Statistical Methods* (3rd ed.). Chapman and Hall/CRC Press, Boca Raton, FL.
Steiner, S. H. (1997). Pre-Control and some Simple Alternatives, *Quality Engineering*, vol. 10, no. 1, pp. 65–74.
Tsai, P. (1988). Variable Gauge Repeatability and Reproducibility Study Using the Analysis of Variance Method, *Quality Engineering*, vol. 1, no. 1, pp. 107–115.
Wheeler, D. J. (1991). *Short Run SPC*. SPC Press, Inc, Knoxville, TN.
Wong, D., and Baker, C. (1988). Pain in Children: Comparison of Assessment Scales, *Pediatric Nursing*, vol. 14, no. 1, pp. 9–17, 1988.
Young, F., Malero-Mora, P., and Friendly, M. (2007). *Visual Statistics: Seeing Data with Dynamic Interactive Graphs*. John Wiley & Sons, Hoboken, NJ.

Reference Charts for Table 19.3



Example in a sample of 10 items, 8 were defective (X/n – 8/10). The 95% confidence limits on the population proportion defective are read from the two curves (for n – 10) as 0.43 and 0.98.

–Ninety-five percent confidence belts for population proportion" from *Selected Techniques of Statistical analysis–OSRD* by C. Eisenhart, M.W. Hastay, and W.A. Wallis. Copyright 1947 by the McGraw-Hill Companies, Inc. Reprinted by permission of The McGraw-Hill Companies, Inc.

Binomial Distribution*

Probability of r or fewer occurrences of an event in n trials, where p is the probability of occurrence on each trial.

		P									
n	r	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.9928	0.9720	0.9392	0.8960	0.8438	0.7840	0.7182	0.6480	0.5748	0.5000
	2	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125
	2	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875
	3	1.0000	0.9999	0.9995	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375
5	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0312
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
	2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
	3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
	4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
6	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
	1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
	2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
	3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6562
	4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
7	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
	1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
	2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
	3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
	4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
	5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.0994	0.9984	0.9963	0.9922
8	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
	2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
	3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
	4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367

	5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
9	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
	2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
	3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
	4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
	5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
	6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
10	0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0232	0.0107
	2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
	3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
	4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
	5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
	6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
	7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990