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The Effect on the R Chart of Precision of Measurement

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ABSTRACT
In practice, data are almost always subject to rounding because of the lack of measurement precision. In this paper, we consider the effect of rounding normal data on the control factor limits for the R chart and the Type I risks. Guidance is given on what is an appropriate degree of precision when using the R chart on normal rounded data. We present, as far as we are aware, the most comprehensive set of results for Type I risks for the R chart that has been published to date. The results indicate that many of the "rules of thumb" suggested in the literature are not suitable.
Key Words: Average Run Length, R Charts, Simulation.

INTRODUCTION

IN this paper, we will investigate one particular aspect of measurement error, namely the effect of measurement "round-off" on control charts used in statistical process control. Many control charts assume a continuous characteristic of interest. In practice, however, it is impossible to measure or to record the values of a continuous variable using a continuous scale. We have to consider observations as being rounded from an underlying continuous distribution. Little is known about what happens to the control limits of charts which use rounded data.

Steiner, Geyer, and Wesolowsky (1996) present control chart techniques for grouped data. They point out that using the midpoint for each group value as a "representative" value (rounding) can adversely affect the working of any control charts that are designed for use with exact measurement. However, they give no information on how rounding may affect the control charts nor provide any guidance on what is an appropriate degree of precision to use. In the literature, various "rules of thumb" have been suggested for the degree of precision that should be used when recording data. To date, there has been little research into the suitability of these rules when applied to control charts. This paper is concerned with the extent to which data may be rounded without adversely affecting the control limits of a chart.

THE ROUNDING PROCESS: NOTATION AND TERMINOLOGY

If the values of a continuous random variable, X, are rounded, the result is a new discrete random variable, X[subR]. If the values of X are rounded into intervals of width, w, with midpoints, X[subR], and with the center of the interval containing zero, cw, then X[subR] has the following values: ... cw - 2w, cw - w, cw, cw + w, cw + 2w, ..., which is known as the rounding lattice. Hence, c determines the position of the rounding lattice relative to the origin (zero) and may be located anywhere between -0.5 and 0.5. The mathematical relationship between X and X[subR] is such that if X lies in the interval cw + (k - 1 / 2) w [less or equal] X < cw + (k + 1 / 2) w,

then its rounded value is the midpoint,

X[subR] = (c + k)w, k = 0, +/-1, +/-2,...

The values of X[subR] are called the rounded data. When data from X are rounded, the effect of the rounding does not depend solely on w, but also on the standard deviation, sigma, of X. Hence, the severity of rounding is better measured by the ratio r = w/sigma, which we call the degree of precision of the recorded data.

RULES OF THUMB

We are concerned about making measurements with sufficient precision for the application of Shewhart mean and range charts to be valid and useful. A selection of suggestions about precision that arise in the literature is given below, roughly in descending order of stringency. Some of these use the notation sigma[subR] for the standard deviation due to the measurement system, combining the effect of repeatability and reproducibility. We shall assume that there is no measurement error except that due to rounding. Tricker (1988) established that sigma[sup2[sub[subR] = sigma[sup2] (1 + r[sup2]/12), where r is the degree of precision of the recorded data (introduced earlier) and sigma[subR] is the standard deviation of X[subR]. To allow meaningful comparison we shall translate each rule of thumb into a condition on r.

(a) Tolerable Negative Error

We look first at the legal requirement in the Weights and Measures Act (Department of Trade and Industry (1979)) that overall measurement error should not exceed 20% of the Tolerable Negative Error. Bissell (1994) writes that this is usually taken to mean that sigma[subR] should not exceed 10% of the tolerance. If the gap between the specification limits is T, then for a process that is just capable, T = 6sigma, and we have sigma < 0.10T, which leads to r < 2.08.

(b) 10% of the Capability or 10% of the Gap Between the Specification Limits

The usual rule stated by engineers is that the measuring equipment should measure to within 10% of the tolerance spread. The reference manual from the Automotive Industry Action Group (1990) explains that the maximum size of the increment on any piece of measuring equipment, the resolution, should be 10% of the process capability of the dimension being measured or 10% of the specification limits, whichever is smaller. Thus, we have the following two conditions:

w < 6sigma / 10 = 0.6sigma or w < T / 10.

For a process that is just capable, T = 6sigma, so that these two conditions become the same, which leads to r < 0.6.

(c) Repeatability and Reproducibility

The required conditions are often expressed in terms of restrictons on the quantity 5.15sigma[subR] (chosen because 5.15 × the standard deviation gives the spread that includes 99% of the measurements if the distribution is Normal). The rule usually used in the automotive industry involves calculating 5.15sigma[subR] as a percentage of the specification (or of the process capability). If this percentage is less than 10%, then the system is considered to be good; if this percentage is between 10% and 30%, then the system may be acceptable; if this percentage is greater than 30%, then the system is thought to be in need of improvement. Taking the value of 5.15sigma[subR] as a percentage of the process capability (6sigma), we obtain the result that r < 0.40 is considered good and r > 1.21 is bad.

(d) Number of Distinct Categories

Another way is to find the number of distinct categories into which the measurement system divides the process capability. This is given by Bissell (1994) as [1.41sigma/sigma[subR]], where the square brackets mean "the next integer below" and where sigma is the true process standard deviation. Three or fewer categories is considered bad, while ten or more categories is considered good. These conditions lead to r < 0.49 being considered good and r > 1.22 being considered bad.

(e) Process Standard Deviation

Wheeler and Lyday (1990) argue that the increment in the measurement unit needs to be at least as small as the standard deviation of the process (i.e., r < 1.0) so that mean and range charts can be meaningfully set up. If the increment is smaller than this, the range chart gives inadequate discrimination because there are not enough different values for the range to take, and too many of the ranges are zero.

(f) 50% of the Standard Deviation

BS 2846 (1991) suggests that rounding observations to the most convenient unit below half the standard deviation of the observations is a useful rule (r < 0.5). This ensures that the increase in measured standard deviation due to rounding will be only about 1%.

(g) 10% of the Process Standard Deviation

The Automotive Industry Action Group (1990, p. 14) states, "a standard for adequate discrimination would be for the least count to be at most one-tenth of process standard deviation" (i.e., $r < 0.1$).

It is clear from the above that the "rules of thumb" lead to a wide variation in the value of r. This may be due to different purposes for measuring and the unknown effects of rounding. Table 1 gives the equivalent value of r for each of the above "rules of thumb" under the assumption that there is no other measurement error except that due to rounding.

THE INVESTIGATION

An appropriate measurement of variability is crucial in the use of all control charts. Testing whether or not process variability is in control is usually done by means of a control chart for the Range (R chart). When using X and R charts, practitioners are always advised to look at the R chart first. Only if the R chart is in control can the X chart interpretation be valid. Therefore, this papers studies the effect of rounding (measurement variability) on the R chart.

The process variability can be estimated using the mean of m sample ranges, $R = \sum_{i=1}^{m} R_i/m$, where R_i is the range of sample i. Assuming that the quality characteristic is normally distributed with unknown standard deviation sigma, then, following the usual U.S. practice, the three standard deviation control limits for the R chart are given by

$UCL = R D_4$,

Center Line = R, and

$LCL = R D_3$,

where

$D_3 = 1 - 3 (d_3 / d_2)$ and $D_4 = 1 + 3 (d_3 / d_2)$, with $d_2 = E [R / \sigma]$ and $d_3 = \sigma' / \sigma$,

and σ' is the standard deviation of R.

We shall examine the effect of rounding on

i) the control limit factors, D_3 and D_4, and

ii) the probability that the range lies outside of the three and two standard deviation control limits when the process is in statistical control.

Before carrying this out, we include the following numerical example. The example illustrates that inappropriate rounding can invalidate the R chart.

NUMERICAL EXAMPLE

The data used are typical weights in grams of product in containers to be marked "500 g". The process mean is 502 g with a standard deviation of 1 g. Values of some of the 25 samples of size 5 are given in Table 2. If the measuring equipment rounds to the nearest 0.5 g (i.e., if $r = 0.5$), these same values change to those given in Table 3. Figure 1 presents the range chart constructed using these values. This chart shows correctly that the process is in control for the range. However, if the equipment can measure only to the nearest 2.5 g (i.e., if $r = 2.5$), the values will be recorded as shown in Table 4. The range chart calculated using these values is shown in Figure 2. The process looks very out of control (for range). The apparent out-of-control nature of the process is due entirely to the rounding of the measurements.

CONTROL LIMIT FACTORS

The R chart uses the constants d_2 and d_3 in estimating the standard deviation of the range in calculating control limits. These constants, however, have been obtained under the assumption that the normal data are subject to no rounding. Therefore, when the underlying normal population is subject to rounding, these constants may be affected. If the difference between these constants for rounded data (d_{2R}, d_{3R}) and unrounded data (d_2, d_3) is considerable, the use of the control limit factors D_3 and D_4, which are functions of d_2 and d_3, may not be appropriate.

Tricker (1988) shows that the standard deviation of a distribution is more adversely affected by the rounding process than the mean of the distribution. Since d_3 is a function of the standard deviation of R and d_2 is a function of the mean of R, we would expect d_3 to be more distorted by rounding than d_2. In order to quantify the values of these constants for rounded data, d_{2R} and d_{3R} were obtained by simulation.

We generated random samples of size n = 2, 5, 8, 10, and 15 from a normal distribution. Each normal deviate was rounded according to the rounding lattice with interval w and lattice position c. The values of the constants under rounding were calculated for the various sample sizes. The degree of precision, r, ranged up to 1, and 11 lattice positions (c = -0.5, -0.4, ..., 0.4, 0.5) were used. The values of d[sub2R] and d[sub3R] are given in Table 5 for n = 2, 5, 8, 10, and 15 and c = 0.0.

The value of c was found to have a negligible effect on the control limit factors. Hence, the results in Table 5 are representative for all values of c. As expected, the results in this table show that d[sub3] is more distorted by the rounding process than d[sub2]. The values of d[sub2] are not affected for values of r as large as 1.0, while d[sub3] is sensitive to rounding for values as small as 0.5.

The relationship between the control limit factors for rounded data (D[sub3R], D[sub4R]) and unrounded data (D[sub3], D[sub4]) is as follows. When we round, we have d[sub2R] [approximate or equal to] d[sub2] and d[sub3R] = d[sub3] + Delta[subR], where Delta[subR] is the error in d[sub3] caused by rounding. Therefore,

D[sub3R] = 1 - 3d[sub3R] / d[sub2R] [approximate or equal to] 1 = 3(d[sub3] + Delta[subR]) / d[sub2] = D[sub3] = 3Delta[subR] / d[sub2] < D[sub3],

and

D[sub4R] = 1 + 3d[sub3R] / d[sub2R] [approximate or equal to] 1 + 3(d[sub3] + Delta[subR]) / d[sub2] = D[sub4] + 3Delta[subR] / d[sub2] > D[sub4].

Hence, rounding will cause an increase in D[sub4] and a decrease in D[sub3], as shown in the values of the control limit factors given in Table 6 for n = 2, 5, 8, 10, and 15 and c = 0.0. These results indicate that, in some cases, when we round, the published values of D[sub3] and D[sub4] may not be appropriate. This is particularly true for D[sub3] when r [greater or equal] 0.5 and for D[sub4] when r [greater or equal] 1.0. Referring to the "rules of thumb" given in Table 1, using rule (f) (r < 0.5) will ensure the suitability of using the published values of the control limit factors.

THREE AND TWO STANDARD DEVIATION CONTROL LIMITS

In this section, we consider the probability that the range lies outside of the three and two standard deviation control limits when the process is under control. These are often referred to as Type I risks. Of particular interest is how these risks are affected by the rounding process. The Type I risk for a particular chart has limited usefulness in assessing the overall performance of the R chart for rounded data as this can be different for each specific R chart. Therefore, to assess the performance of the R chart in general for rounded data, the properties of the probability distribution of Type I risks were considered. It was also thought useful to consider properties of the distribution of the average run length (ARL).

To study the effect of rounding on the R chart, a two-stage Monte Carlo simulation was used. In the first stage, following standard practice, the control limits are set using m samples of size n. In the second stage, the performance of the particular chart is simulated to obtain the Type I risk and the ARL. The whole process is repeated many times to obtain an estimate of the expected values of the Type I risk and of the ARL. The simulation process is given below.

Stage 1: Control Chart Selection.

1.1 Generate n deviates from a normal distribution with mean, μ, and standard deviation, sigma. Round these deviates according to the rounding lattice in Expression 1 with interval w and lattice position c.

1.2 Repeat 1.1 m times.

1.3 Compute the two and three standard deviation control limits for the R chart.

Stage 2: Control Chart Operation.

2.1 Obtain n rounded deviates as in 1.1.

2.2 Compute the sample statistic, R, and record whether R is within the control limits of 1.3.

2.3 Repeat 2.1 and 2.2 10,000 times, and estimate the Type I risk and ARL for limits in 1.3.

2.4 Repeat 1.1 to 2.3 10,000 times, and obtain the expected value and standard deviation (ET1 and SDT1) of the Type I risks and the expected value of the average run length (EARL).

The value of m was set to 25, the value often recommended to obtain reasonable estimates for the control limits. The values of ET1, SDT1, and EARL were calculated for the same values of n, r, and c used to obtain d[sub2R] and d[sub3R]. Once again, the value of c was found to have a negligible effect on the Type I risk and the ARL. Hence, Table 7 shows only a selection of results from the simulation when the value of c is zero.

The values of d[sub2] were found not to be affected for values of r as large as 1.0, but rounding did cause an increase in the values of d[sub3]. This implies that while rounding will cause very little change in the mean of R, there will be an increase in its variance. For the values of n considered, this increase in standard deviation was about 3% for r = 0.5 and 12% for r = 1.0. Generally, the positions of the control limits will be similar for rounded and unrounded data. However, the increase in the variance of R can result in an increase in the probability of R exceeding the control limits. This will cause a corresponding increase in the Type I risk, as shown in Table 7. Rounding is seen to cause an increase in ET1 with a corresponding increase in SDT1, with these increases being more pronounced for increasing r. The upper control limits were less sensitive to the rounding process than the lower control limits, as illustrated by the results in Table 7.

Like all distributions for rounded observations, the distribution of R for rounded data is discontinuous. Generally, as r increases, the discontinuities are less numerous in any given interval, and the steps increase in size. Hence, we would expect the Type I risk to become more unstable for increasing r. This is illustrated by the ET1 results in Table 7 for the two standard deviation control limits when r = 1.0.

The results for unrounded data (r = 0) are interesting in themselves. They indicate the expected Type I risk together with its variation. As far as we know, this is the most comprehensive set of results for Type I risks for the R chart that has been published.

CONCLUDING COMMENTS

Various rules have been suggested for the degree of precision that should be used when recording data, and as discussed in this paper, in many cases, the rules given in the literature were found to be unsuitable. The results of this study indicate the extent to which data may be rounded without adversely affecting the control limit factors and Type I risks. From our investigations we can make the following generalizations. From Table 7, for r less than 0.5, rounding will have negligible effect on the Type I risks for upper control limits. But for lower control limits, simulation results indicate that r less than 0.25 is more suitable. From Table 1, our results indicate that using rule (g) (r < 0.1) will cause rounding to have a negligible effect on Type I risks for lower control limits. However, for upper control limits, rule (f) (r < 0.5) is suitable.

Added materialADDED MATERIAL

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TABLE 1. Rules of Thumb with Equivalent Value of r

	Rule		Equivalent Value of r
(a)	20%	of Tolerable Negative Error	sigma[subR] < T/10
			For a Just Capable Process, r < 2.08
(b)	10%	of the Capability or	r < 0.6
	10%	of the Gap between the Specification Limits (T)	r < T/10sigma[subp]
			For a Just Capable Process, r < 0.6
(c)	Repeatability & Reproducibility		r < 0.40 is Good
			r > 1.21 is Bad
(d)	Number of Distinct Categories		r < 0.49 is Good
			r > 1.22 is Bad
(e)	Process Standard Deviation		r < 1.0
(f)	50%	of the Process Standard Deviation	r < 0.5
(g)	10%	of the Process Standard Deviation	r < 0.1

TABLE 2. Partial Data for the Example Measured in Grams

Sample						Range
1	500.28	499.31	502.07	500.76	500.70	2.76
2	500.39	499.61	500.10	499.06	499.88	1.33
.	.	.	.	.	.	.
.	.	.	.	.	.	.
25	499.94	499.44	498.34	499.91	500.39	2.05

TABLE 3: Partial Data for the Example Rounded to the Nearest 0.5 Gram

Sample						Range

1	500.5	499.5	502.0	501.0	500.5	2.5
2	500.5	499.5	500.0	499.0	500.0	1.5
.	.	.	.	.	.	.
.	.	.	.	.	.	.
25	500.0	499.5	498.5	500.0	500.5	2.0

TABLE 4. Partial Data for the Example Rounded to the Nearest 2.5 Grams

Sample						Range
1	500.0	500.0	502.5	500.0	500.0	2.5
2	500.0	500.0	500.0	500.0	500.0	0.0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
25	500.0	500.0	497.5	500.0	500.0	2.5

TABLE 5. Values of d[sub2R] and d[sub3R] for n = 2, 5, 8, 10 and 15, where r = 0.0 (No Rounding), 0.5, 1.0 and c = 0.0

	Values of d[sub2R]				
			n		
r	2	5	8	10	15
0.0	1.128	2.326	2.847	3.078	3.472
0.5	1.129	2.327	2.849	3.079	3.473
1.0	1.130	2.326	2.850	3.080	3.474

	Values of d[sub3R]				
			n		
r	2	5	8	10	15
0.0	0.853	0.864	0.820	0.797	0.756
0.5	0.856	0.864	0.847	0.823	0.785
1.0	0.949	0.953	0.914	0.897	0.873

TABLE 6. Values of D[sub3R] and D[sub4R] for n = 2, 5, 8, 10 and 15, where r = 0.0 (No Rounding), 0.5, 1.0 and c = 0.0

	Values of D[sub3R]				
			n		
r	2	5	8	10	15
0.0	0.000	0.000	0.136	0.223	0.348
0.5	0.000	0.000	0.109	0.198	0.322
1.0	0.000	0.000	0.035	0.126	0.246

	Values of D[sub4R]				
			n		
r	2	5	8	10	15
0.0	3.267	2.115	1.864	1.777	1.652
0.5	3.274	2.114	1.891	1.802	1.678
1.0	3.521	2.229	1.963	1.873	1.754

TABLE 7. Expected Value (ET1) and Standard Deviation (SDT1) of Type I Risk (%), and Expected Value of Average Run Length (EARL), for 3sigma and 2sigma Upper Control Limits and 2sigma Lower Control Limit

	3sigma Upper Control Limit								
	r = 0.0			r = 0.5			r = 1.0		
n	ET1	SDT1	EARL	ET1	SDT1	EARL	ET1	SDT1	EARL
2	1.5	1.7	300.0	1.6	1.9	288.7	1.9	2.7	304.4
5	0.7	0.6	369.0	0.7	0.8	380.1	1.1	1.2	400.0
8	0.6	0.5	344.8	0.7	0.6	309.3	1.0	1.4	395.1
10	0.6	0.4	330.6	0.7	0.6	316.8	0.9	1.3	301.0

15	0.6	0.4	303.7	0.7	0.6	302.0	0.9	0.7	365.6

2sigma Upper Control Limit

	r = 0.0			r = 0.5			r = 1.0		
n	ET1	SDT1	EARL	ET1	SDT1	EARL	ET1	SDT1	EARL
2	5.5	3.8	31.8	5.6	4.1	33.1	6.3	5.1	36.6
5	3.9	2.4	37.5	4.2	2.9	39.6	5.6	4.6	42.7
8	3.7	2.1	37.4	3.9	2.6	39.3	4.3	3.9	36.8
10	3.6	2.0	37.1	4.0	2.6	37.8	5.1	2.3	36.8
15	3.6	1.9	36.2	4.0	2.5	35.7	6.5	4.6	44.3

2sigma Lower Control Limit

	r = 0.0			r = 0.5			r = 1.0		
n	ET1	SDT1	EARL	ET1	SDT1	EARL	ET1	SDT1	EARL
2	-	-	-	-	-	-	-	-	-
5	0.7	0.2	158.5	1.8	0.2	73.6	1.0	0.0	100.4
8	1.1	0.4	106.6	1.5	0.1	69.3	4.6	0.2	26.0
10	1.1	0.4	100.6	1.9	1.8	153.0	1.8	0.1	56.2
15	1.2	0.5	98.5	1.7	1.8	145.8	6.1	6.0	305.4

FIGURE 1. Range Chart with r = 0.5.

FIGURE 2. Range Chart with r = 2.5.

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