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Studying an OC Curve of an Acceptance Sampling Plan: A Statistical Quality Control Tool

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Abstract: - The research problems under consideration are relationships between sampling risks and other elements inbuilt in a single acceptance sampling plan. Two levels of quality are considered: first, average quality level desired by the consumer AQL, and, second, quality level called lot tolerance percent defective LTPD, or the worst level of quality that the consumer may tolerate. The producer's risk α is the risk of incorrect rejection is the risk that the sampling plan will fail to verify an acceptable lot's quality set by AQL and, thus, reject it. The probability of acceptance a lot with LTPD quality is the consumer's risk β or the risk of incorrect accepting. Operating characteristic (OC) curve describes how well an acceptance plan discriminates between good and bad lots. Acceptance sampling plan consists of a sample size n, and the maximum number of defective items that can be found in the sample c. The OC curve pertains to a specific plan, i.e. to a combination of the sample size n and the acceptance criterion or level c. Using ExcelOM2 software, the following findings were found: (1) with c, AQL and LTPD fixed, the increasing sample size n resulted with the risk α increased and the risk β decreased; and (2) with n, AQL and LTPD fixed, the increasing critical value c implicated that the risk α decreased, but the risk β increased; (3) if AQL is increased, with all other components n, c, and LTPD fixed, then producer's risk α would increase, but consumer's risk β would remain the same; and, (4) if LTPD is increased, with all other components n, c and AQL fixed, α would remain the same, while β would decrease. In this paper some previous research results from studying α risk and β risk in audit sampling based on acceptance sampling applications from [1] are shortly presented.

Key-Words: - Statistical quality control, decision making, acceptance sampling plan, consumer's risk, producer's risk, acceptable quality level, lot tolerance percent defective.

1 Introduction

Acceptance sampling is an inspecting procedure applied in statistical quality control, see [3] and [6]. It is a method of measuring random samples of populations called "lots" of materials or products against predetermined standards. Acceptance sampling is a part of operations management or of accounting auditing and services quality supervision. It is important for industrial, but also for business purposes helping decision-making process for the purpose of quality management.

Sampling plans are hypothesis tests regarding product that has been submitted for an appraisal and subsequent acceptance or rejection. The products may be grouped into batches or lots or may be single pieces from a continuous operation. A random sample is selected and could be checked for various characteristics. For lots, the entire lot is accepted or rejected in the whole. The decision is based on the pre-specified criteria and the amount of defects or defective units found in the sample. Accepting or rejecting a lot is analogous to not rejecting or rejecting the null hypothesis in a hypothesis test. In the case of continuous production process, a decision may be made to continue sampling or to check subsequent product 100%.

The hypotheses for acceptance sampling plan as a kind of statistical test are:

$$H_0 ... \textit{The lot is of acceptable quality}$$
$$H_1 ... \textit{The lot is not of acceptable quality}. \quad (1)$$

Rejecting the lot is the same as rejecting the null hypotheses H_0.

If the quality controls have broken down, the sampling will prevent defective products from passing any farther. There is a number of different methods widely used for selecting a product for checking quality characteristics:

(1) No checking;
(2) 100% checking;
(3) Constant percentage sampling;
(4) Random spot checking;
(5) Audit sampling (with no acceptance and rejection criteria); and
(6) Acceptance sampling.

Acceptance sampling is based on probability and is the most widely used sampling technique all through industry. Many sampling plans are tabled and published and can be used with little guidance. The Dodge-Romig Sampling Inspection Tables are an example of published tables, see [6]. Some applications require special unique sampling plans, so an understanding of how a sampling plan is developed is important. In acceptance sampling, the risks of making a wrong decision are known.

In some previous research findings in [1] from studying an audit sampling based on acceptance sampling applications using other software are showed considering α - the management risk, and β - the risk of audit results users. The paper presents the author's research results achieved using sampling methods and methods of statistical quality control in the analysis of audit risks that are caused by sampling. Using the audit hypothesis testing model and substantive test based on hypothetical examples, the following relationships were recognized: inverse proportionality between the risk α and the risk β; inverse proportionality between β risk and specific audit risks called inherent, control and analytical procedures risks. The sample size was inversely proportionate to: the levels of the risk α, and of the acceptable precision (A), and to the size of tolerable misstatement (TM), as well. The value of precision *A* would increase if the risk β would increase. When analysing OC curves of an acceptance sampling plan selected, the conclusion arose that, with fixed values of other relevant factors (α, AQL and LTPD), an inverse proportionality between the risk of incorrect acceptance of an audit population, which is the risk β of audit results users, and the needed sample size n existed. When changing on low levels the management risk α, which is the risk of incorrect rejection of an audit population, with unchanged values of other relevant factors (β, AQL and LTPD), the needed sample size n does not change visibly.

2 Problem Formulation

2.1 Types of Risks in Acceptance Sampling

Because an entire lot of material is not being inspected, not everything is known, so, sampling will always incur certain risks, see in [7]. Only the sample is known.

This incurs the risk of making two types of errors in «the accept : not accept» decision.

- A lot may be rejected that should be accepted and the risk of doing this is the producer's risk.
- The second error is that a lot may be accepted that should have been rejected and the risk of doing this is called the consumer's risk.

But, it is a good thing that these two risks could be measured.

The Type I Error, called significance level, is preset on with quite low level, most at 5% (or 1% or 10%), to protect of this type of error. It is true that:

$$\alpha = P\{Type\ I\ Error\}$$
$$\alpha = P\{rejected\ H_0 \mid H_0\ is\ true\}, \quad (2)$$

and

$$\beta = P\{not\ rejected\ H_0 \mid H_0\ is\ false\}$$
$$\beta = P\{Type\ II\ Error\}. \quad (3)$$

The power of the test is equal to:

$$Power = 1 - \beta = P\{rejected\ H_0 \mid H_0\ is\ false\}. \quad (4)$$

Because the probability of committing a Type I Error (α) and the probability of committing Type II Error (β) have an inverse relationship and the letter is the complement of the power of the test (1-β), then α and the power of the test vary directly. An increase in the value of the level of significance (α) results in and increase in power, and a decrease in α results in a decrease in power. An increase in the size of the sample *n* chosen results in an increase in power and vice versa.

2.2. Designing an Acceptance Sampling Plan

Acceptance sampling is defined as an inspection procedure used to determine whether to accept or reject a specific quantity of goods or materials, [4]. The best sample plan minimizes producer's risk of rejecting an acceptable lot and customer's risk of receiving bad product. There are many possibilities to solve this problem, e.g. see computerized solutions in [5].

Nowadays, as more companies start to apply quality programs, such as Total Quality Management (TQM) approach, they work closely with suppliers to ensure high levels of quality and the need for acceptance sampling plans is decreasing. The goal is that no defect items should be entered into the production process, passed from a producer to a customer, which could be an external or an internal customer. In reality many firms must check their materials inputs.

The basic procedure for acceptance sampling is quite simple: (1) A random sample is taken from a large quantity of items and tested or measured relative to the quality characteristic of interest. (2) If the sample passes the test, the entire quantity called a lot of items is accepted. (3) If the sample fails the test, two scenarios are possible: either the entire quantity of items is subjected to 100 percent inspection and all defective items would be repaired or replaced, or the whole quantity is returned to the supplier.

Designing an acceptance sampling plan is making a decision about quality and risk. Acceptance sampling involves both the producer or supplier of materials, and the consumer or buyer. Consumers need acceptance sampling to limit the risk of rejecting good-quality materials or accepting bad-quality materials. Consequently, the consumer, sometimes in conjunction with the producer through contract specifications, determines the parameters of the plan. Any firm can be in a production chain, so can be both a producer of goods purchased by another firm and a consumer of goods or raw materials supplied by another firm.

When designing an acceptance sampling plan two levels of quality are considered: first, acceptable quality level, and, second, the unacceptable or worst quality level.

The first is the quality level desired by the consumer and is called the limit quality or the acceptable quality level (AQL). The producer's risk α is the risk that the sampling plan will fail to verify an acceptable lot's quality AQL and, thus, reject it. This kind of risk is also called a Type I Error of the plan. Most often the producer's risk is preset at α =0.05, or 5%. Both, producers and consumers also are interested in a low producer's risk, because of the high costs of sending back good materials or products, which could cause interruption and delay of a production process or make poor relations with the partners.

The second, unacceptable level of quality is the worst level of quality that the consumer can tolerate and it is called the lot tolerance proportion (or percent) defective (LTPD). The probability of accepting a lot with LTPD quality is the consumer's risk β, or the Type II Error of the plan. In the praxis a common value for the consumer's risk is set as LTPD=0.10, or 10%.

Three often used attribute sampling plans are the single-sampling plan, the double-sampling plan, and the sequential sampling plan. Analogous variable sampling plans also have been devised for variable measures of quality. Different types of acceptance sampling plans are designed to provide a specified producer's and consumer's risk. It is in the consumer's interest to keep the Average Number of Items Inspected (ANI) to a minimum because that keeps the cost of inspection low.

The single-sampling plan is a decision rule to accept or reject a lot based on the results of one random sample from the lot. The procedure is to take a random sample of size n and inspect each item. If the number of defects does not exceed a specified acceptance number c, the consumer accepts the entire lot. Any defects found in the sample are either repaired or returned to the producer. If the number of defects in the sample is greater than c, the consumer subjects the entire lot to 100 percent inspection or rejects the entire lot and returns it to the producer.

Accepted lots and screened rejected lots are sent to their destination. The rejected lots may be submitted for repeated inspection.

In a double-sampling plan: (1) management specifies two sample sizes (n_1 and n_2) and two acceptance numbers (c_1 and c_2); (2) If the quality of the lot is very good or very bad, the consumer can make a decision to accept or reject the lot on the basis of the first sample, which is smaller than in the single-sampling plan. To use the plan, the consumer takes a random sample of size n_1; (3) If the number of defects is less than or equal to c_1, the consumer accepts the lot; (4) If the number of defects is greater than c_2, the consumer would reject the lot; (5) If the number of defects is between c_1 and c_2, the consumer would take a second sample of size n_2; (6) If the combined number of defects in the two samples is less than or equal to c_2, the consumer would accept the lot. Otherwise, it is rejected. This plan is also called a «lot by lot double-sampling». Rejected lots are detailed or scrapped and accepted lots and detailed rejected lots are sent to their destination.

The sequential sampling plan is a further refinement of the double- and multiple-sampling concept. The inspector will select one part from the lot and check for the specified requirements.

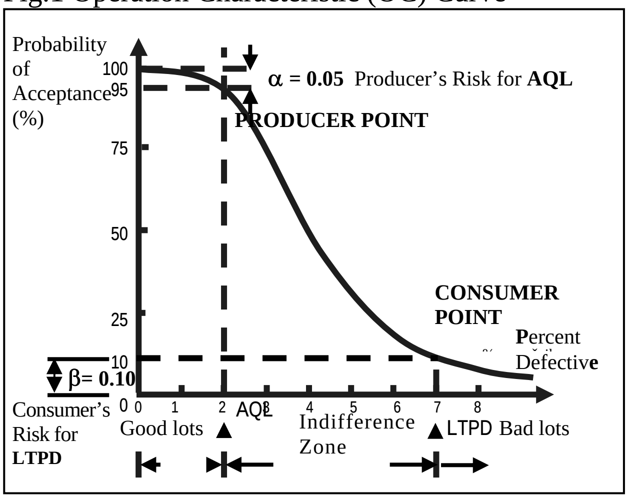
So called continuous sampling is used where product flow is continuous and not feasible to be formed into lots, described in [7].

2.3 Operating Characteristic (OC) Curve

Analysts create a graphic display of the performance of a sampling plan by plotting the probability of accepting the lot for a range of proportions of defective units. This graph, called an OC curve, describes how well a sampling plan discriminates between good and bad lots.

Undoubtedly, every manager wants a plan that accepts lots with a quality level better than the AQL 100 percent of the time and accepts lots with a quality level worse than the AQL zero percent of the time. An OC curve is developed by determining the probability of acceptance for several values of incoming quality. An OC curve showing producer's risk α and consumer's risk β is given in Fig.1.

Fig.1 Operation Characteristic (OC) Curve



On the vertical axis is the probability of acceptance and this is the probability that the number of defects or defective units in the sample is equal to or less than the acceptance number c of the sampling plan.

The units on the abscissa are in terms of percent defective. The AQL is the acceptable quality level in percentages and the LTPD is lot tolerance percent defective. The producer's risk α is the probability of rejecting a lot of AQL quality, i.e. Type I Error. The consumer's risk β is the probability of accepting a lot of LTPD quality, i.e. Type II Error.

Although the hypergeometric may be used when the lot sizes are small (finite), the binomial and Poisson are by far the most popular distributions to use when constructing sampling plans (for infinite lots from processes), compare to [6].

3 Problem Solution

The sampling distribution for the single-sampling plan is the binomial distribution because each item inspected is either defective or not. The probability of accepting the lot equals the probability of taking a sample of size n from a lot.

How can management change the sampling plan to reduce the probability of rejecting good lots and accepting bad lots? To answer this question, let us see how n and c affect the shape of the OC curves. A better single-sampling plan would have a lower producer's risk α and a lower consumer's risk β. Sampling plans may be constructed to meet certain criteria and to insure that the specified outgoing quality levels are met. In the construction of a lot by lot single-sampling plan, four parameters must be determined prior to determining the sample size n and acceptance number c. The parameters are: the acceptable quality level AQL; the risk α; the lot tolerance percent defective LTPD; and the risk β.

In most situations the objective is to find a sample size n and acceptance number c whose OC curve meets the above parameters. In this paper, first, the effect of sample size n and then the effect of acceptance number c on the shape of the OC curve will be discussed. After that, the effect of changing AQL and LTPD will be briefly overviewed.

3.1 Sample Size Effect on OC Curve

The question is what would happen if the sample size n would increase with the acceptance number left unchanged at c=1?

Different values of the producer's and consumer's risks are shown in the Table 1, where the results are calculated using the software ExcelOM2 from [2].

Table 1 The Producer's Risk and the Consumer's Risk in OC Curve for Given AQL and LTPD with Fixed c=1 and Changing Sample Size n

	Acceptance level c=1	
Sample Size n	Producer's Risk α (for a given AQL=1%)	Consumer's Risk β (for a given LTPD=5%)
30	0,0361	0,5535
40	0,0607	0,3991
50	0,0894	0,2794
60	0,1212	0,1916
70	0,1553	0,1292
80	0,1908	0,0861
90	0,2273	0,0567
100	0,2642	0,0371
110	0,3012	0,0241
120	0,3377	0,0155
130	0,3737	0,0100
140	0,4089	0,0064
150	0,4430	0,0041

Fig.2 OC Curve for n=30, c=1, AQL=1%, LTPD=5%, α =0,0361, β =0,5535, and Probability of Acceptance =(1-α)=0,9639.

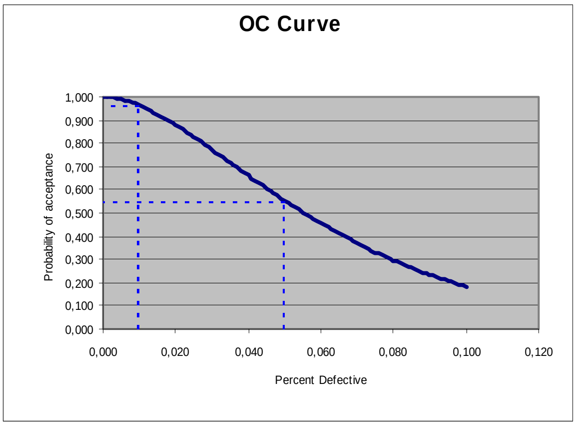


Fig.3 OC Curve for n=80, c=1, AQL=1%, LTPD=5%, α =0,1908, β =0,0861, and Probability of Acceptance =(1-α)=0,8092.

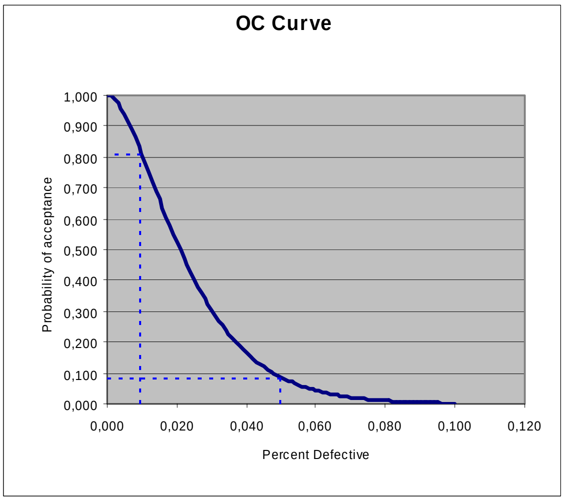


Fig.4 OC Curve for n=150, c=1, AQL=1%, LTPD=5%, α =0,4430, β =0,0041, and Probability of Acceptance = (1-α)=0,557.

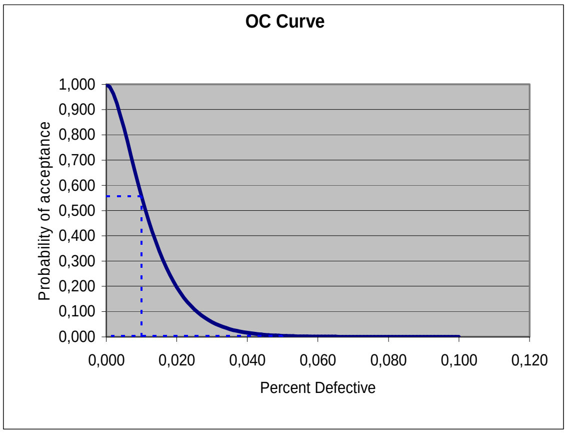


Table 1 presents an OC curve results for producer's risk α and consumer's risk β with desired values of AQL=1%, LTPD=5%. The sample size n is changing, while c=1. It could be seen how the OC curve responds. Effects of increasing sample size on the OC curve while holding acceptance number c=1 constant could be noticed in Fig.2, Fig.3 and Fig.4., created using ExcelOM2 software. Increasing n while holding c constant increases the risk α and reduces the risk β.

3.2 Acceptance Number Effect on OC Curve

The results of increasing acceptance number from c=1 to c=2, while holding sample size n on the same levels as in Table 1, are showed in Table 2.

Increases in the acceptance number from one to two lowers the probability of finding more than two defects and, consequently, lowers the producer’s risk α .

Table 2 The Producer's Risk and the Consumer's Risk in OC Curve for Given AQL and LTPD with Fixed c=2 and Changing Sample Size n

	Acceptance level c=2	
Sample Size n	Producer's Risk α (for a given AQL=1%)	Consumer's Risk β (for a given LTPD=5%)
30	0,0033	0,8122
40	0,0075	0,6767
50	0,0138	0,5405
60	0,0224	0,4740
70	0,0333	0,3137
80	0,0466	0,2306
90	0,0619	0,1664
100	0,0794	0,1183
110	0,0987	0,0829
120	0,1196	0,0575
130	0,1421	0,0395
140	0,1658	0,0269
150	0,1905	0,0182

However, raising the acceptance number for a given sample size increases the risk of accepting a bad lot β. An increase in the acceptance number from c=1 to c=2 increases the probability of getting a sample with two or less defects and, therefore, increases the risk β. Thus, to improve single-sampling acceptance plan, management should increase the sample size n, which reduces the risk β, and increase the acceptance number c, which reduces the risk α.

Comparison of Fig.2 with Fig.5 and Fig.3 with Fig.6, shows the following principle: Increasing the critical value for an acceptance number c, while holding the sample size n constant, decreases the producer's risk α, and increases the consumer's risk β. The results for risks in Table 1 and Table 2 support the respective images.

Fig.5 OC Curve for n=30, c=2, AQL=1%, LTPD=5%, α=0,0033, β=0,8122, and Probability of Acceptance =(1-α)=0,9967.

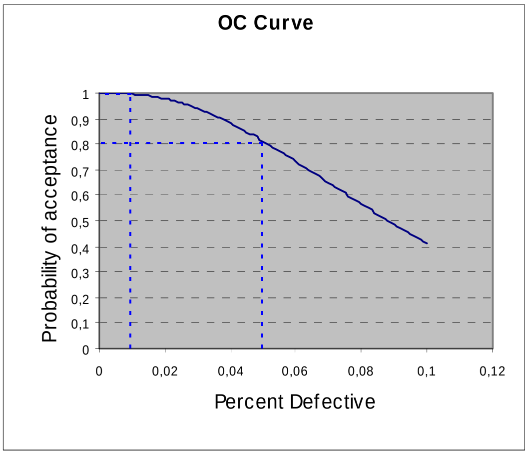
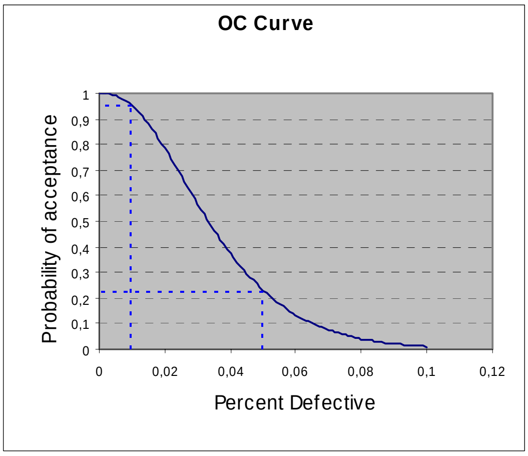


Fig.6 OC Curve for n=80, c=2, AQL=1%, LTPD=5%, α=0,0466, β=0,2306, and Probability of Acceptance =(1-α)=0,9534.



4 Conclusion

Acceptance sampling is concerned with the decision to accept or reject a lot (or batch) of goods. The design of the acceptance sampling process includes decisions about sampling versus complete inspection, attribute versus variable measures, AQL, α, LTPD, β, and sample size. Management can select the best plan (choosing sample size n and acceptance number c) by using an operating characteristic (OC) curve.

If the sample size n is increased, with c, AQL and LTPD fixed, the OC curve would change so that the producer's risk α increases while consumer's risk β decreases. Further, with increasing the critical value c, and with n, AQL and LTPD fixed, the probability being the producer's risk α would decrease, but the probability for consumer's risk β would increase.

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