INTRODUCTION

Acceptance Sampling—General. Acceptance sampling refers to the application of specific sampling plans to a designated lot or sequence of lots. Acceptance sampling procedures can, however, be used in a program of *acceptance control* to achieve better quality at lower cost, improved control, and increased productivity. This involves the selection of sampling procedures to continually match operating conditions in terms of quality history and sampling results. In this way the plans and procedures of acceptance sampling can be used in an evolutionary manner to supplement each other in a continuing program of acceptance control for quality improvement with reduced inspection. It is the objective of acceptance control in any application to eventually phase out acceptance sampling in favor of supplier certification and process control. After explaining a variety of specific sampling procedures, this section concludes with suggestions on how and when to progress from sampling inspection toward reliance on process control and check inspection and eventually to no inspection at all, depending on the stage of the life cycle of the product and the state of control.

The disposition of a lot can be determined by inspecting every unit ("100 percent inspection") or by inspecting a sample or portion of the lot. Economy is the key advantage of acceptance sampling as compared with 100 percent inspection. However, sampling has additional advantages:

1. Economy due to inspecting only part of the product
2. Less handling damage during inspection
3. Fewer inspectors, thereby simplifying the recruiting and training problem
4. Upgrading the inspection job from piece-by-piece decisions to lot-by-lot decisions
5. Applicability to destructive testing, with a quantified level of assurance of lot quality
6. Rejections of entire lots rather than mere return of the defectives, thereby providing stronger motivation for improvement

Sampling also has some inherent disadvantages:

1. There are risks of accepting "bad" lots and of rejecting "good" lots.
2. There is added planning and documentation.
3. The sample usually provides less information about the product than does 100 percent inspection.

Acceptance sampling under a system of acceptance control provides assurance that lots of product are acceptable for use. It is assumed that acceptable product is normally being submitted by honest, conscientious, capable suppliers. The plans provide warnings of discrepancies as a means of forestalling use of the product while providing signals of the presence of assignable causes for both the producer and the consumer. If this is not so, other means of dealing with the supplier should be used—such as discontinuance or 100 percent inspection. This is why it is called acceptance sampling.

Why Is Sampling Valid? The broad scheme of use of sampling is shown in Figure 46.1. It is universally realized that inspection of a sample gives information then and there about the quality of the pieces in the lot. But that is only the beginning. Beyond this, a vast area of knowledge is unfolded because *the product can tell on the process.*

The sample, being the result of the variables present in the process at the time of manufacture, can give evidence as to those variables. Thereby it is possible to draw conclusions as to whether the process was doing good work or bad at the time it produced the samples.

From these conclusions about the process, it is then possible to reverse the reasoning, so that the known *process now can tell on the product.* We already know about the inspected product, but the knowledge of the process gives information about the uninspected pieces. In this way acceptance sampling of a lot is valid not only because the uninspected pieces are neighbors of the inspected

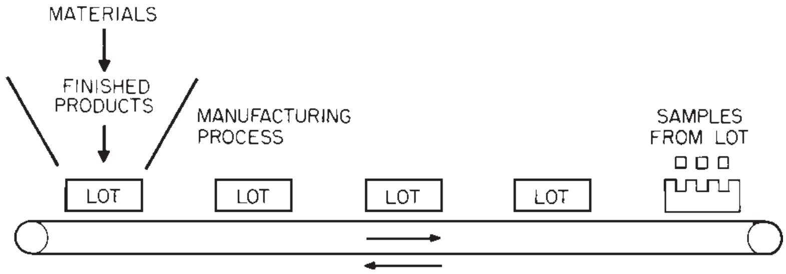

FIGURE 46.1 Measurement of a sample tells (1) whether the pieces in the sample are good or bad; (2) whether the process, at the time the sample was made, was doing good work or bad; (3) whether the uninspected pieces made at the same time by the same process are good or bad (principle of sampling inspection); (4) whether the process is stable; (5) whether the unmanufactured pieces are going to be good or bad (control inspection).

pieces; acceptance sampling is also valid because the uninspected pieces may be derived from the same process which the inspected pieces have labeled as "good."

When we go a step further and examine a series of samples, we learn whether the process is stable or not. Once the series of samples tells on the process by certifying stability, we can use this knowledge of stability to predict the quality of unmanufactured product.

Cowden (1957) has summarized the characteristics of a good acceptance plan. It will:

1. Protect the producer against having lots rejected when the product is in a state of control, and satisfactory as to level and uniformity
2. Protect the consumer against the acceptance of bad lots
3. Provide long-run protection to the consumer
4. Encourage the producer to keep the process in control
5. Minimize sampling costs, inspection, and administration
6. Provide information concerning product quality

Types of Sampling. Any acceptance sampling application must distinguish whether the purpose is to accumulate information on the immediate *product* being sampled or on the *process* which produced the immediate lot at hand. Accordingly, two types of sampling have been distinguished:

Type A: Sampling to accept or reject the immediate lot of product at hand

Type B: Sampling to determine if the process which produced the product at hand was within acceptable limits

The type of sampling will determine the appropriate probability distribution to be used in characterizing the performance of the plan. In addition, the type of data generated will also play a role. In acceptance sampling, data can be of the following types:

- Attributes—go no-go information

  Defectives—usually measured in proportion or percent defective. This refers to the acceptability of units of product for a wide range of characteristics.

  Defects—usually measured by actual count or as a ratio of defects per unit. This refers to the number of defects found in the units inspected, and hence can be more than the number of units inspected.

- Variables—measurement information

  Variables—usually measured by the mean and standard deviation. This refers to the distribution of a specific measurable characteristic of the product inspected.

Terminology of Acceptance Sampling. The terminology of acceptance sampling has evolved over the years into a precise and well-directed set of terms defining various properties of acceptance sampling plans and procedures. These are clearly described in the international standard ANSI/ISO/ASQC A3534 (1993), "Statistics—Vocabulary and Symbols."

It is important to distinguish between product which is not fit for use and product which does not meet specification requirements. This is done in ANSI/ISO/ASQC A3534 (1993), Part 2, which uses the term "defect" in relation to the former and "nonconformity" in relation to the latter. These are defined as

Defect: The nonfulfillment of an intended usage requirement.

Nonconformity: The nonfulfillment of a specified requirement.

Clearly a unit of product containing one or more defects or nonconformities is a "defective" or "nonconforming unit."

Since acceptance sampling plans usually relate to requirements imposed by the customer (internal or external), the terms "defect" and "defective" will be generally used here. Furthermore, in referring to the literature of acceptance sampling, the terms "defect" and "defective" will be commonly found. However, in appropriate instances in this section such as variables sampling plans which compare measurements to specifications, the terms "nonconformity" and "nonconforming unit" will be used.

Acceptance Sampling Procedures

Forms of Sampling. Sampling plans can be classified in two categories: attributes plans and variables plans.

Attributes Plans. In these plans, a sample is taken from the lot and each unit classified as conforming or nonconforming. The number nonconforming is then compared with the acceptance number stated in the plan and a decision is made to accept or reject the lot. Attributes plans can further be classified by one of the two basic criteria:

1. Plans which meet specified sampling risks provide protection on a lot-by-lot basis. Such risks are
   a. A specified quality level for each lot (in terms of percent defective) having a selected risk (say 0.10) of being accepted by the consumer. The specified quality level is known as the lot tolerance percent defective or limiting quality (p_2); the selected risk is known as the consumer's risk (β).
   b. A specified quality level for each lot such that the sampling plan will accept a stated percentage (say 95 percent) of submitted lots having this quality level. This specified quality level is termed the acceptable quality level (AQL). The risk of rejecting a lot of AQL quality (p_1) is known as the producer's risk (α).
2. Plans which provide a limiting average percentage of defective items for the long run. This is known as the average outgoing quality level (AOQL).

Variables Plans. In these plans, a sample is taken and a measurement of a specified quality characteristic is made on each unit. These measurements are then summarized into a simple statistic (e.g., sample mean) and the observed value compared with an allowable value defined in the plan. A decision is then made to accept or reject the lot. When applicable, variables plans provide the same degree of consumer protection as attributes plans while using considerably smaller samples.

Attributes plans are generally applied on a percent defective basis. That is, the plan is instituted to control the proportion of accepted product which is defective or out of specification. Variables plans for percent defective are also used in this way. Such plans provide a sensitivity greater than attributes but require that the shape of the distribution of individual measurements must be known and stable. The shape of the distribution is used to translate the proportion defective into specific values of process parameters (mean, standard deviation) which are then controlled.

Variables plans can also be used to control process parameters to given levels when specifications are directed toward the process average or process variability and not specifically to percent defective. These variables plans for process parameter do not necessarily require detailed knowledge of the shape of the underlying distribution of individual measurements.

Sampling plans used in reliability and in the sampling of bulk product are generally of this type. Published plans in the reliability area, however, usually require detailed knowledge of the shape of the distribution of lifetimes. Some of the important features of attributes and variables plans for percent defective are compared in Table 46.1.

The principal advantage of variables plans for percent defective over corresponding attributes plans is a reduction in the sample size needed to obtain a given degree of protection. Table 46.2 shows a comparison of variables sample sizes necessary to achieve the same protection as the attributes plan: $n = 125$, $c = 3$ (sample size of 125 units, allowable number of defectives of 3).

Types of Sampling Plans. In single-sampling plans, the decision to accept or reject a lot is based on the results of inspection of a single group of units drawn from the lot. In double-sampling plans, a smaller initial sampling is usually drawn, and a decision to accept or reject is reached on the basis of this smaller first sample if the number of defectives is either quite large or quite small. A second sample is taken if the results of the first are not decisive. Since it is necessary to draw and inspect the second sample only in borderline cases, the average number of pieces inspected per lot is generally smaller with double sampling. In multiple-sampling plans, one, or two, or several still smaller samples of n individual items are taken (usually truncated after some number of samples) until a decision to accept or reject is obtained. The term "sequential-sampling plan" is generally used when

TABLE 46.1 Comparison of Attributes and Variables Sampling Plans for Percent Defective

Feature	Attributes	Variables
Inspection	Each item classified as defective or nondefective. Go no-go gages may be employed.	Each item measured. Inspection more sophisticated. Higher inspection and clerical cost.
Distribution of individual measurements	Need not be known.	*Must* be known (normal usually assumed).
Type of defect	Any number of defect types can be assessed by one plan.	Separate plan required for each type of defect.
Sample size	Depends on protection required.	Smaller sample size for same protection as attributes (at least 30% smaller*).
Process information	Percent defective.	Percent defective plus valuable information on process average and variability for corrective action.
Severity	Weights all defectives of a given kind equally.	Weights each unit inspected by its proximity to specifications.
Evidence to supplier	Defectives available as evidence.	Possible for lot to be rejected on sample containing no defectives.
Measurement errors	Measurements not recorded.	Measurements available for review.
Screened lots	No effect on performance of plan.	Screened lots may be rejected in error even though they contain no defectives.

*Bowker and Goode (1952), pp. 32–33. Assumes single sample of one characteristic.

TABLE 46.2 Comparison of Variables and Attributes Sample Sizes*

$p_1 = 0.0109; \alpha = 0.05; p_2 = 0.0535; \beta = 0.10$

Plan	Sample size
Single-sampling attributes	125
Variables:	
σ known	19
σ unknown (s)	52
σ unknown ($\overline{R}$ of groups of 5)	75
Sequential sampling, σ known (ASN at p_1)	10.3

*Specifications assumed to be $> 6\sigma$ apart if two-sided.

the number of samples is unlimited and a decision is possible after each individual unit has been inspected.

Sampling Schemes and Systems. While simple sampling plans are often employed solely in sentencing individual lots, sampling schemes and systems are generally used in acceptance control applications involving a steady flow of product from the producer. The ANSI/ISO/ASQC A3534 (1993) standard defines a sampling plan as “a specific plan which states the sample size(s) to be used and the associated criteria for accepting the lot,” and a sampling scheme as “a combination of sampling plans with rules for changing from one plan to another.” Finally, it defines an acceptance sampling system as, “a collection of sampling schemes, each with its own rules for changing plans, together with criteria by which appropriate schemes may be chosen.” Thus, $n = 134$, $c = 3$ is a sampling plan; Code J, 1.0 percent AQL is a sampling scheme; and ANSI/ASQC Z1.4 (1993) is a sampling system.

Published Tables and Procedures. From the point of view of ease of negotiation between the producer and the consumer, it is usually best to use the published procedures and standards. This avoids problems of credibility created when one party or the other generates its own sampling plan. Also, the legal implications of using plans which appear in the literature (such as Dodge-Romig plans) or have been subjected to national consensus review (such as ANSI/ASQC Z1.4, 1993) are obvious. Unique plans, specifically generated for a given application, are probably best used internally.

Screening. In view of advances in automatic inspection equipment and computer integrated manufacturing, 100 percent inspection, or screening, has become more attractive. This is particularly true in the short run; however, dependence on screening may inhibit efforts for continual improvement as a long-term strategy. Screening procedures have been designed to address important considerations such as the selection of a screening variable, prior information on the population, cost, losses due to erroneous decisions, variation in product quality, and environmental considerations. Methods include: statistical models, such as Deming’s all-or-none rule, Taguchi’s model for tolerance design, and economic and statistical models using correlated variables. Other issues include the possibility of burn-in, group testing, allocation of inspection effort, selection of process parameters, and selective assembly. An excellent systematic literature review will be found in Tang and Tang (1994). An example of the use of a correlated variable to improve 100 percent inspection is given by Mee (1990). Jaraiedi, Kochhar, and Jaisingh (1987) discuss a model for determining the average outgoing quality for product which has been subjected to multiple 100 percent inspections. A computer program for application of their results will be found in Nelson and Jaraiedi (1987).

SAMPLING RISKS AND PARAMETERS

Risks. When acceptance sampling is conducted, the real parties of interest are the producer (supplier or Production department) and the consumer, i.e., the company buying from the supplier or the department to use the product. Since sampling carries the risk of rejecting "good" lots and of accepting "bad" lots, with associated serious consequences, producers and consumers have attempted to standardize the concepts of what constitutes good and bad lots, and to standardize also the risks associated with sampling. These risks are stated in conjunction with one or more parameters, i.e., quality indices for the plan. These indices are as follows.

Producer's Risk. The producer's risk α is the probability that a "good" lot will be rejected by the sampling plan. In some plans, this risk is fixed at 0.05; in other plans, it varies from about 0.01 to 0.10. The risk is stated in conjunction with a numerical definition of the maximum quality level that may pass through the plan, often called the acceptable quality level.

Acceptable Quality Level. Acceptable quality level (AQL) is defined by ANSI/ISO/ASQC A3534 as follows: "When a continuing series of lots is considered, a quality level which for purposes of sampling inspection is the limit of satisfactory process average." A sampling plan should have a low producer's risk for quality which is equal to or better than the AQL.

Consumer's Risk. The consumer's risk β is the probability that a "bad" lot will be accepted by the sampling plan. The risk is stated in conjunction with a numerical definition of rejectable quality such as lot tolerance percent defective.

Lot Tolerance Percent Defective. The lot tolerance percent defective (LTPD) is the level of quality that is unsatisfactory and therefore should be rejected by the sampling plan. A consumer's risk of 0.10 is common and LTPD has been defined as the lot quality for which the probability of acceptance is 0.10; i.e., only 10 percent of such lots will be accepted. (LTPD is a special case of the concept of limiting quality, LQ, or rejectable quality level, RQL. The latter terms are used in tables that provide plans for several values of the consumer's risk as contrasted to a value of 0.10 for the LTPD.) Limiting quality (LQ) plans will be found in the international standard ISO 2859-2 (1985) and in the U.S. national standard Q3 as well as the LTPD plans in Dodge and Romig (1959).

A third type of quality index, average outgoing quality limit, is used with 100 percent inspection of rejected lots and will be discussed later in this section.

The producer's and consumer's risks and associated AQL and LTPD are summarized graphically by an operating characteristic curve.

Operating Characteristic Curve. The operating characteristic (OC) curve is a graph of lot fraction defective versus the probability that the sampling plan will accept the lot.

Figure 46.2 shows an ideal OC curve for a case where it is desired to accept all lots 3 percent defective or less and reject all lots having a quality level greater than 3 percent defective. Note that all lots less than 3 percent defective have a probability of acceptance of 1.0 (certainty); all lots greater than 3 percent defective have a probability of acceptance of zero. Actually, however, no sampling plan exists that can discriminate perfectly; there always remains some risk that a "good" lot will be rejected or that a "bad" lot will be accepted. The best that can be achieved in practice is to control these risks.

Figure 46.3 shows the curve of behavior that would be obtained if an operator were instructed to take a sample of 150 pieces from a large lot and to accept the lot if no more than four defective pieces were found. Such curves can be constructed from the appropriate probability distribution. For example, using the Poisson table (Appendix II, Table E), we find that for a sample size of 150 (n = 150) and 2 percent defective (p = 0.02) we would expect np = 3.0 defectives in the sample. The table shows for r = 4 and np = 3.0, the probability of acceptance (for four or fewer defectives) is 0.815.

It is seen from this curve that a lot 3 percent defective has one chance in two of being accepted. However, a lot 3.5 percent defective, though technically a "bad" lot, has 39 chances in 100 of being accepted. In like manner, a lot 2.5 percent defective, though technically a good lot, has 34 chances in 100 of not being accepted.

The effect of parameters of the sampling plan on the shape of the OC curve is demonstrated in Figure 46.4, where the curve for perfect discrimination is given along with the curves for three particular acceptance sampling plans. The following statements summarize the effects:

1. When the sample size approaches the lot size or, in fact, approaches a large percentage of the lot size, and the acceptance number is chosen appropriately, the OC curve approaches the perfect OC curve (the rectangle at p_1).
2. When the acceptance number is zero, the OC curve is exponential in shape, concave upward (see curves 2 and 3).
3. As the acceptance number increases, the OC curve is pushed up, so to speak, for low values of p, and the probability of acceptance for these quality levels is increased, with a point of inflection at some larger value of p (see curve 1).
4. Increasing the sample size and the acceptance number together gives the closest approach to the perfect discrimination OC curve (see curve 1).

It is sometimes useful to distinguish between type A and type B OC curves (see Dodge and Romig, 1959, pp. 56–59). Type A curves give the probability of acceptance for an individual lot that comes from finite production conditions that cannot be assumed to continue in the future. Type B curves assume that each lot is one of an infinite number of lots produced under essentially the same production conditions. In practice, most OC curves are viewed as type B. With the few exceptions noted, this section assumes type B OC curves.

Average Sample Number Curve. The average sample number (ASN) is the average number of units inspected per lot in sampling inspection, ignoring any 100 percent inspection of rejected lots. In single-sampling inspection the ASN is equal to n, the sample size. However, in double- and multiple-sampling plans the probability of not reaching a decision on the initial sample, and consequently being forced to inspect a second, third, etc., sample must be considered. For a double-sampling plan with sample sizes n_1 and n_2, the average sample number, ASN is simply

$$\text{ASN} = n_1 + (1 - P_{a1})\, n_2$$

where P_{a1} is the probability of acceptance on the first sample. The ASN for multiple sampling is more complicated to calculate; see Schilling (1982).

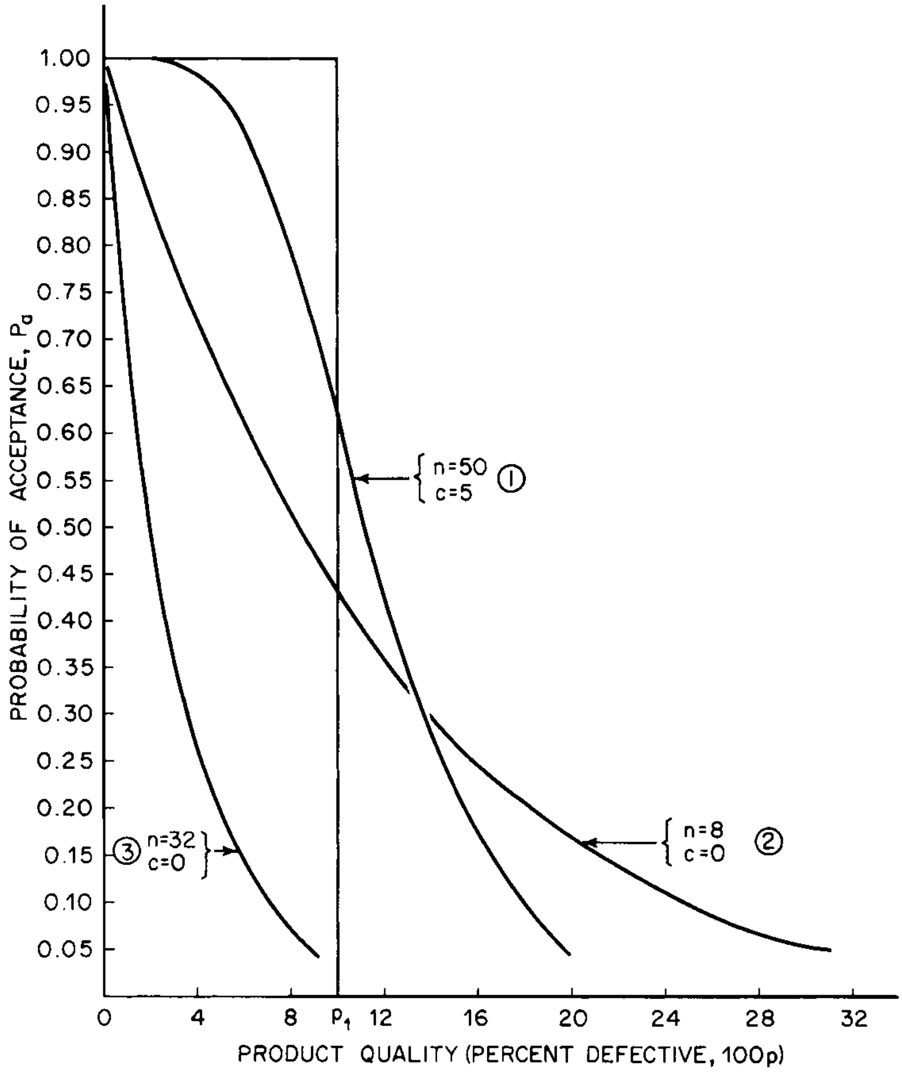

FIGURE 46.4 Shape of the OC curve.

A comparison of ASN for some specific plans, roughly matching $n = 125$, $c = 3$, is shown in Figure 46.5. (The calculations assume that all samples selected are completely inspected.) As indicated, double and multiple sampling generally lead to economies over single sampling when quality is either very good or very poor.

Average Outgoing Quality Limit. The acceptable quality level (AQL) and lot tolerance percent defective (LTPD) have been cited as two common quality indices for sampling plans. A third commonly used index is the average outgoing quality limit (AOQL). The AOQL is the upper limit of average quality of outgoing product from accepted lots and from rejected lots which have been screened.

The AOQL concept stems from the relationship between the fraction defective before inspection (incoming quality) and the fraction defective after inspection (outgoing quality) when inspection is nondestructive and rejected lots are screened. When incoming quality is perfect, outgoing quality must likewise be perfect. However, when incoming quality is very bad, outgoing quality will also be near perfect, because the sampling plan will cause most lots to be rejected and 100 percent inspected. Thus, at either extreme—incoming quality very good or very bad—the outgoing quality will tend to be very good. It follows that between these extremes is the point at which the percent defective in the outgoing material will reach its maximum. This point is known as the AOQL.

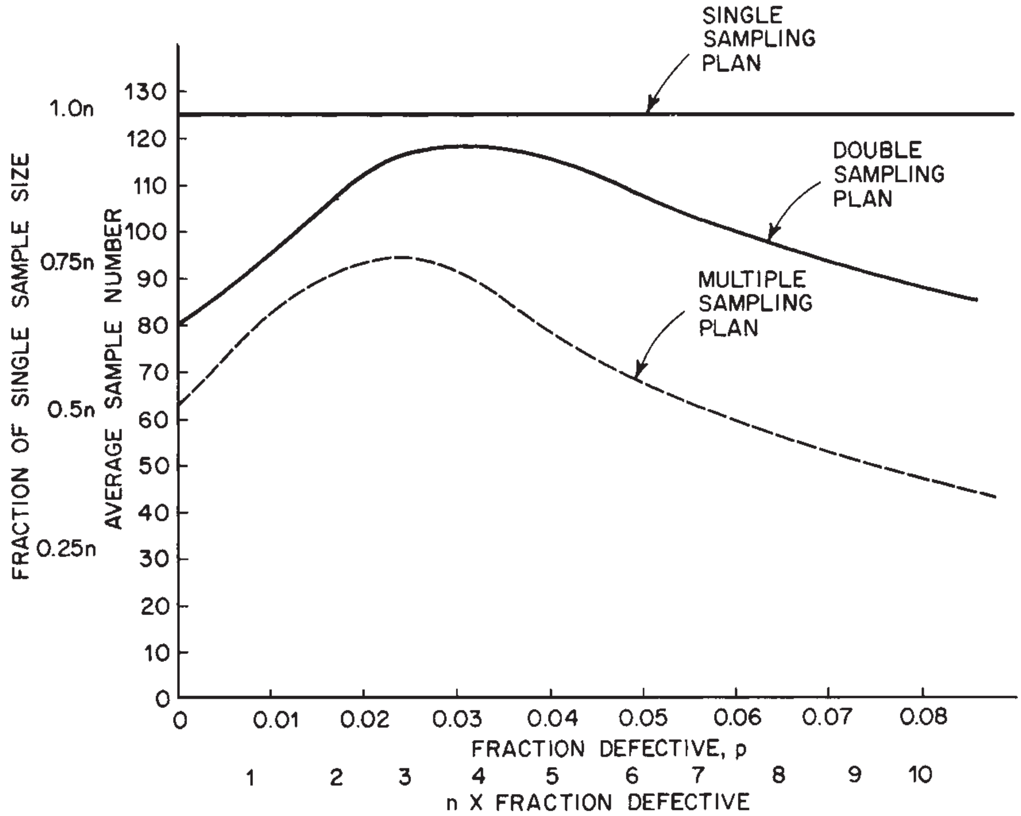

FIGURE 46.5 Average sample number versus fraction defective.

If p is incoming quality, P_r is the probability of lot rejection, and all rejected lots are screened and made free of defects (i.e., 0 percent), then

$$\text{AOQ} = (p)P_a + (0)P_r = (p)P_a$$

This calculation is approximate in that it does not account for the small effect of 100 percent inspection of the units in the samples from accepted lots. Taking this into account:

$$\text{AOQ} = pP_a\left(1 - \frac{n}{N}\right)$$

for sample size n and lot size N.

Average Outgoing Quality Curve. An example of the calculation for the AOQL and a plot of the average outgoing quality (AOQ) curve is shown in Table 46.3 and Figure 46.6. Schilling (1982) has indicated that for acceptance numbers of five or less

$$\text{AOQL} \simeq \frac{0.4}{n}(1.25c + 1)$$

Average Total Inspection. The average total inspection (ATI) takes into account the likelihood of 100 percent inspection of rejected lots when this is possible; i.e., inspection is nondestructive. The lot size N must now be taken into account. Again with single-sampling plans with sample size n, lot size N, probability of acceptance P_a, and probability of rejection P_r,

$$\text{ATI} = P_a n + P_r N$$

TABLE 46.3 Computations for Average Outgoing Quality Limit (AOQL); for This Example, $n = 78$, $c = 1$

Incoming quality fraction defective = p	np	Probability of acceptance = P_a	Average outgoing quality (AOQ) = $p \times P_a$
0.005	0.39	0.940	0.00470
0.010	0.78	0.820	0.00820
0.015	1.17	0.680	0.01020
0.020	1.56	0.550	0.01100*
0.025	1.95	0.430	0.01075
0.030	2.34	0.330	0.00990
0.035	2.73	0.250	0.00875
0.040	3.12	0.190	0.00760
0.045	3.51	0.140	0.00630
0.050	3.90	0.100	0.00500
0.055	4.29	0.075	0.00402
0.060	4.68	0.050	0.00300

*AOQL ≅ maximum AOQ ≅ 0.01100 = 1.1%.

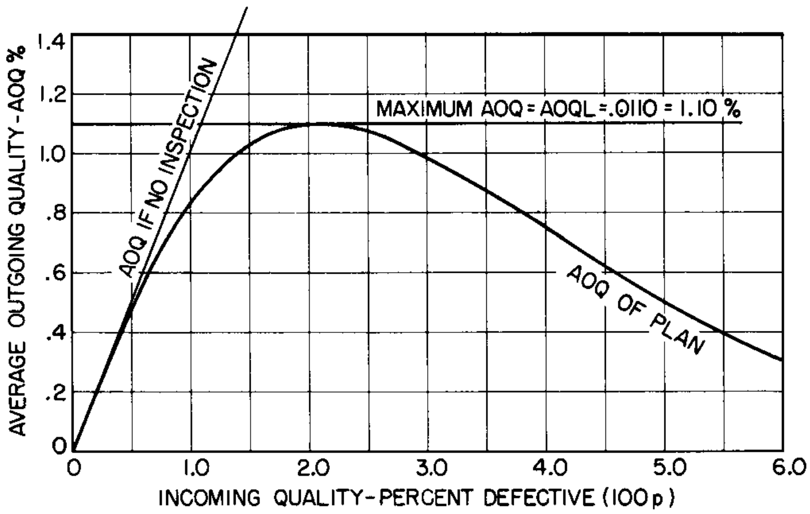

FIGURE 46.6 AOQ curve and AOQL for a typical sampling plan.

An extension to double sampling is simply

$$\text{ATI} = n_1 + (1 - P_{a1})n_2 + (N - n_1 - n_2)(1 - P_a)$$

where P_{a1}, is the probability of acceptance on the first sample. Extension to multiple-sampling plans of k levels is straightforward and will not be derived here. See Schilling (1982).

Minimum Total Inspection. Minimum inspection per lot, for a given type of protection, can be illustrated by the following example:

Assume that a consumer establishes acceptance criteria as an LTPD of 5 percent. A great many sampling plans meet this criterion. Some of these plans are

Take from each lot a sample n of	Accept the lot if the number of defectives does not exceed the maximum acceptance number c
46	0
78	1
106	2
134	3
160	4

Figure 46.7 shows the corresponding operating characteristic curves. Each plan has an LTPD of 5 percent; i.e., the probability of acceptance of a submitted inspection lot with fraction defective of 0.05 is 0.10. Which plan should be used? One logical basis for choosing among these plans is to use that one which gives the least inspection per lot.

The total number of units inspected consists of (1) the sample which is inspected for each lot and (2) the remaining units which must be inspected in those lots which are rejected by the sampling inspection. The number of lots rejected in turn depends on the normal level of defectives in the product so that minimum inspection is a function of incoming quality.

A sample computation of minimum inspection per lot is shown in Table 46.4. It is assumed that rejected lots are detail (100 percent) inspected. For small acceptance numbers the total inspection is high because many lots need to be detailed. For large acceptance numbers the total is again high, this time because of the large size of samples. The minimum sum occurs at a point between these extremes.

From the foregoing it is seen that for any specified conditions of

Lot tolerance percent defective

Consumer's risk

Lot size

Process average

it becomes possible to derive the values of

$$n = \text{sample size}$$

$$c = \text{allowable number of defectives in the sample}$$

to obtain

$$I_m = \text{minimum inspection per lot}$$

Similar reasoning may be applied in making a selection from a group of sampling plans designed to give the same average outgoing quality limit (AOQL).

Complete tables for sampling have been derived for minimum inspection per lot for a variety of $\bar{p}$, LTPD, and AOQL values (e.g., Dodge and Romig 1959). These will be described later.

Selection of a Numerical Value of the Quality Index. There are three alternatives for evaluating lots: no inspection, inspect a sample, 100 percent inspection (or more). An economic evaluation of these alternatives requires a comparison of *total* costs under each of the alternatives.

Enell (1954) has suggested that a break-even point be used in the selection of quality index:

$$P_b = \frac{I}{A}$$

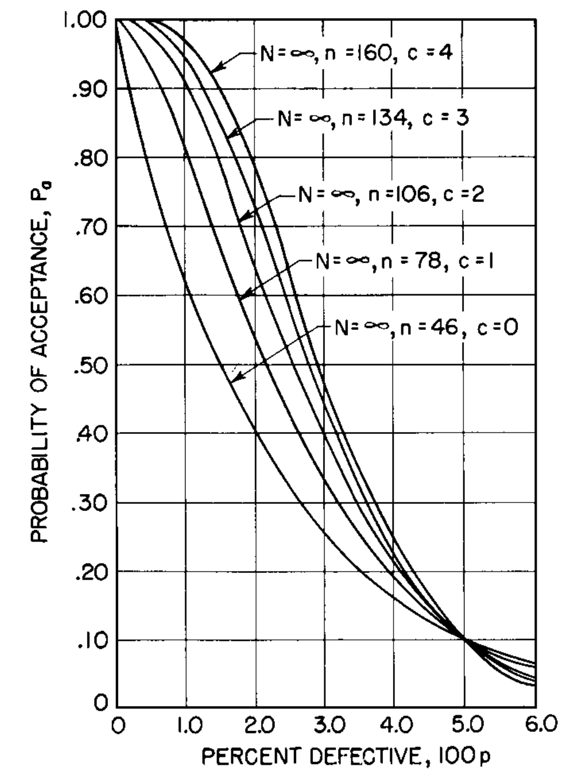

FIGURE 46.7 Family of sampling plans each having LTPD = 0.05.

TABLE 46.4 Computation of Minimum Inspection per Lot for 5 Alternative n−c Combinations Appropriate to Lots of N= 1000 Articles

All plans have the same lot tolerance percent defective. Incoming material has a process average percent defective, $\bar{p}$ = 0.5%.

Sample size n	Allowable number of defects c	Probability of acceptance by sampling P_a	Probability of inspecting residue of lot $1-P_a$	Average no. pieces inspected: In sample* n	Average no. pieces inspected: In rest of lot† $(N - n) \times (1 - P_a)$	Average no. pieces inspected: Total inspected per lot
46	0	0.795	0.205	46	196	242
78	1	0.940	0.060	78	55	133
106	2	0.983	0.017	106	15	121‡
134	3	0.995	0.005	134	4	138
160	4	0.998	0.002	160	2	162

*The sample size indicates the number inspected from each lot.

†The size of the uninspected residue of the lot, multiplied by the probability that it will have to be inspected because of rejection of the sample.

‡This is the minimum sought.

where I = inspection cost per item and A = damage cost incurred if a defective slips through inspection. If it is thought that the lot quality p is less than p_b, the total cost will be lowest with sampling inspection or no inspection. If p is greater than p_b, 100 percent inspection is best. If p is unknown, it may be best to sample using an appropriate sampling plan such as an AOQL scheme.

For example, a microcomputer device costs \$0.50 per unit to inspect. A damage cost of \$10.00 is incurred if a defective device is installed in the larger system. Therefore,

$$p_b = \frac{0.50}{10.00} = 0.05 = 5.0\%$$

If it is expected that the percent defective will be greater than 5 percent, then 100 percent inspection should be used. Otherwise, use sampling or no inspection.

The formula assumes that the sample size is small compared to the lot size, the cost to replace a defective found in inspection is borne by the producer or is small compared to the damage or inconvenience caused by a defective, and no inspection errors occur.

As a 5 percent defective quality level is the break-even point between sorting and sampling, the appropriate sampling plan should provide for a lot to have a 50 percent probability of being sorted or sampled; i.e., the probability of acceptance for the plan should be 0.50 at a 5 percent defective quality level. The operating characteristic curves in a set of sampling tables such as ANSI/ASQC Z1.4 can now be examined to determine an AQL. For example, suppose that the device is inspected in lots of 3000 pieces. The operating characteristic curves for this case (code letter K) are shown in ANSI/ASQC Z1.4. The plan closest to having a P_a of 0.50 for a 5 percent level is the plan for an AQL of 1.5 percent. Therefore, this is the plan to adopt. Other economic models have been developed. For example, see Tagaras (1994) for the case of variables sampling.

In practice, the quantification of the quality index is a matter of judgment based on the following factors:

1. Past performance on quality
2. Effect of nonconforming product on later production steps
3. Effect of nonconforming product on fitness for use
4. Urgency of delivery requirements
5. Cost to achieve the specified quality level

IMPLEMENTATION OF AN ACCEPTANCE SAMPLING PROCEDURE

Assumptions Made in Sampling Plans

Inspection Error. In implementing acceptance sampling plans, it is commonly assumed that:

1. The inspectors follow the prescribed sampling plan.
2. The inspection is made without error; i.e., no human or equipment mistakes are made in measurement or in judging conformance.

In practice, these assumptions are not fully valid. See Section 23, under Inspection Errors.

The effect of inspection error has received considerable attention in the literature of quality. For example, see Johnson, Kotz, and Rodriguez (1985, 1986, 1988, 1990) and Suich (1990).

Units of Product. These may consist of (1) discrete pieces or (2) specimens from bulk material. The criteria used for judging the conformance of a single unit of product to standard are somewhat different for these two categories. The criteria used for judging lot conformance differ even more widely.

Seriousness Classification of Defects. Many sampling plans set up their criteria for judging lot conformance in terms of an allowable number of defects in the sample. Since defects differ greatly in seriousness, the sampling plans must somehow take these differences into account.

Where there exists a formal plan for seriousness classification of defects, the sampling plans may be structured so that:

1. A separate sampling plan is used for each seriousness class, e.g., large sample sizes for critical defects, small sample sizes for minor defects.
2. A common sampling plan is used, but the allowable number of defects varies for each class; e.g., no critical defects are allowed, but some minor defects are allowed.
3. The criteria may be established in terms of defects per hundred units, the allowable number being different for each class.
4. The criteria may be based on demerits per unit; i.e., all defects found are converted to a scale of demerits based on the classification system.

In the absence of a formal system of seriousness classification, all defects are considered equally important during the sampling inspection. However, when nonconforming lots are subsequently reviewed to judge fitness for use, the review board gives consideration to the seriousness of the defects.

Lot Formation. The general approach to lot formation is discussed in Section 23, under Degree of Inspection and Test Needed. While most acceptance sampling plans can be validly applied regardless of how lots are formed (skip-lot plans are an exception), the economics of inspection and the quality of the accepted product are greatly influenced by the manner of lot formation.

The interrelation of lot formation to economics of inspection is discussed in Section 23 and will not be elaborated here. The interrelation of lot formation to quality of accepted product can be seen from a single example.

Ten machines are producing the same product. Nine of these produce perfect product. The tenth machine produces 100 percent defectives. If lots consist of product from single machines, the defective product from the tenth machine will always be detected by sampling. If, however, the lots are formed by mixing up the work from all machines, then it is inevitable that some defects will get through the sampling plan.

The fact that lot formation so strongly influences outgoing quality and inspection economics has led to some guidelines for lot formation:

1. Do not mix product from different sources (processes, production shifts, input materials, etc.) unless there is evidence that the lot-to-lot variation is small enough to be ignored.
2. Do not accumulate product over extensive periods of time (for lot formation).
3. Do make use of extraneous information (process capability, prior inspections, etc.) in lot formation. Such extraneous information is especially useful when product is submitted in isolated lots, or in very small lots. In such cases the extraneous information may provide better knowledge on which to base an acceptance decision than the sampling data.
4. Do make lots as large as possible consistent with the above to take advantage of low proportionate sampling costs. (Sample sizes do not increase greatly despite large increases in lot sizes.)

When production is continuous (e.g., the assembly line) so that the "lot" is necessarily arbitrary, the sampling plans used are themselves designed to be of a "continuous" nature. These continuous sampling plans are discussed later in this section.

Selecting the Sample. The results of sampling are greatly influenced by the method of selecting the sample. In acceptance sampling, the sample should be representative of the lot. In those cases where the inspector has knowledge of how the lot was formed, this knowledge can be used in selecting the sample by stratification (see below). Lacking this knowledge, the correct approach is to use random sampling.

Random Sampling. All published sampling tables are prepared on the assumption that samples are drawn at random; i.e., at any one time each of the remaining uninspected units of product has an equal chance of being the next unit selected for the sample. To conduct random sampling requires that (1) random numbers be generated and (2) these random numbers be applied to the product at hand.

Random numbers are available in prefabricated form in tables of random numbers (Appendix II, Table CC). One uses such a table by entering it at random (without "looking") and then proceeding in some chosen direction (up, down, right, left, etc.) to obtain random numbers for use. Numbers which cannot be applied to the product arrangement are discarded.

Random numbers may also be generated by various devices. These include:

1. *Calculators or computers:* Many calculators are available with random-number routines built in. Computers are, of course, an excellent source of random numbers. Statistical software often has random numbers built in.
2. *A bowl of numbered chips or marbles:* After mixing, one is withdrawn and its number recorded. It is then replaced and the bowl is again mixed before the next number is withdrawn.
3. *Random number dice:* One form is icosahedron (20-sided) dice. There are three of these, each of a different color, one for units, one for tens, and one for hundreds. (Each die has the numbers from 0 to 9 appearing twice.) Hence one throw of the three dice displays a random number within the interval 000 to 999.

Once the random numbers are available, they must be adapted to the form in which the product is submitted. For systematically packed material, the container system can be numbered to correspond to the system of random numbers. For example, a lot might be submitted in 8 trays, each of which has 10 rows and 7 columns. In such a case, the trays might be numbered from 0 to 7, the rows from 0 to 9, and the columns from 0 to 6. Then, using three-digit random numbers, the digits are assigned to trays, rows, and columns, respectively.

For bulk packed materials, other practical procedures may be used. In the case of small parts, they may be strewn onto a flat surface which is marked with grid lines in a 10×10 arrangement. Based on two-digit random numbers, the cell at the intersection of these digits is identified. Within the cell, further positional identity can be determined with a third digit.

For fluid or well-mixed bulk products, the fluidity obviates the need for random numbers, and the samples may be taken from "here and there."

Stratified Sampling. When the "lots" are known to come from different machines, production shifts, operators, etc., the product is actually multiple lots which have been arbitrarily combined. In such cases, the sampling is deliberately stratified; i.e., an attempt is made to draw the sample proportionately from each true lot. However, within each lot, randomness is still the appropriate basis for sampling.

A further departure from randomness may be due to the economics of opening containers, i.e., whether to open few containers and examine a few pieces from each.

Sampling Bias. Unless rigorous procedures are set up for sampling at random and/or by stratification, the sampling can deteriorate into a variety of biases which are detrimental to good decision making. The more usual biases consist of:

1. Sampling from the same location in all containers, racks, or bins
2. Previewing the product and selecting only those units which appear to be defective (or nondefective)
3. Ignoring those portions of the lot which are inconvenient to sample
4. Deciding on a pattern of stratification in the absence of knowledge of how the lot was made up

The classic example is the legendary inspector who always took samples from the four corners and center of each tray and the legendary production worker who very carefully filled these same spots with perfect product.

Because the structured sampling plans do assume randomness, and because some forms of sampling bias can significantly distort the product acceptance decisions, all concerned should be alert to plan the sampling to minimize these biases. Thereafter, supervision and auditing should be alert to assure that the actual sampling conforms to these plans.

ATTRIBUTES SAMPLING

Overview of Single, Double, Multiple, and Sequential Plans. In general, single, double, multiple, and sequential sampling plans can be planned to give lots of specified qualities nearly the same chance of being accepted; i.e., the operating-characteristic curves can be made quite similar (matched) if desired. However, the best type of plan for one producer or product is not necessarily best for another. The suitability of a plan can be judged by considering the following factors:

1. Average number of parts inspected
2. Cost of administration
3. Information obtained as to lot quality
4. Acceptability of plan to producers

The advantages and disadvantages of the four forms of sampling plans are tabulated in Table 46.5. The average number of parts that need to be inspected to arrive at a decision varies according to the plan and the quality of the material submitted. In cases where the cost of inspection of each piece is substantial, the reduction in number of pieces inspected may justify use of sequential sampling despite its greater complexity and higher administrative costs. On the other hand, where it is not practicable to hold the entire lot of parts while sampling and inspection are going on, it becomes necessary to set aside the full number of items that may need to be inspected before inspection even begins. In these circumstances single sampling may be preferable if the cost of selecting, unpacking, and handling parts is appreciable. It is of course simplest to train personnel, set up records, and administer a single-sampling plan. A crew of inspectors hastily thrown together cannot easily be taught all the intricacies of the more elaborate plans. However, double-sampling plans have been demonstrated to be simple to use in a wide variety of conditions, economical in total cost, and acceptable psychologically to both producer and consumer.

Lot-by-Lot Inspection. When product is submitted in a series of lots (termed "lot-by-lot"), the acceptance sampling plans are defined in terms of

N = lot size

n = sample size

c = acceptance number, i.e., the allowable number of defects in the sample

r = rejection number

When more than one sample per lot is specified, the successive sample sizes are designated as n_1, n_2, n_3, etc. The successive acceptance numbers are c_1, c_2, c_3, etc. The successive rejection numbers are r_1, r_2, r_3, etc.

Selecting Sampling Plans. Sampling plans are often specified by choosing two points on the OC curve: the AQL denoted by p_1 and the LTPD symbolized by p_2. Tables have been developed to facilitate the selection process by using unity (np) values from the Poisson distribution. See Cameron (1952), Duncan (1974), and Schilling and Johnson (1980). Such tables use the operating

TABLE 46.5 Comparative Advantages and Disadvantages of Single, Double, and Multiple Sampling

Feature	Single sampling	Double sampling	Multiple sampling	Sequential
Acceptability to producer	Psychologically poor to give only one chance of passing the lot	Psychologically adequate	Psychologically open to criticism as being indecisive	Psychologically open to criticism as being more indecisive than multiple
Number of pieces inspected per lot*	Generally greatest	Usually (but not always) 10 to 50% less than single sampling	Generally (but not always) less than double sampling by amounts of the order of 30%	Minimum over all attributes plans
Administration cost in training, personnel, records, drawing and identifying samples, etc.	Lowest	Greater than single sample	Greater	Greatest
Information about prevailing level of quality in each lot	Most	Less than single sample	Less than double	Least

*This is not to be confused with total cost of inspection, which includes administration cost of the plan.

ratio, $R = p_2/p_1$, in conjunction with the unity value of (np_1) or (np_2) to obtain the sample size and acceptance number(s) for the plan whose OC curve passes through the points specified. Often other unity values are provided which assess other properties of the plan. Such tables are illustrated below by the Schilling-Johnson (1980) table, reproduced in part as Table 46.6, which uses producer risk $\alpha = 0.05$ and consumer risk $\beta = 0.10$ with equal sample sizes at each stage of a double or multiple sampling plan. The table provides matched sets of single, double, and multiple sampling plans, such that their OC curves are essentially equivalent. For example, if $p_1 = 0.012$ and $p_2 = 0.053$ then $R = 4.4$. The closest value given under the column for R is 4.058, corresponding to an acceptance number of 4. The sample size is then found to be $n = 7.994 \div 0.053 = 150.8$, which would usually be taken to be 150. The operating characteristic curve for the plan can be seen in Figure 46.3.

Single Sampling. In single sampling by attributes, the decision to accept or reject the lot is based on the results of inspection of a single sample selected from the lot. The operation of a single-sampling plan by attributes is given in Figure 46.8. The characteristics of an attributes sampling plan are given in Table 46.7.

Double Sampling. In double sampling by attributes, an initial sample is taken, and a decision to accept or reject the lot is reached on the basis of this first sample if the number of nonconforming units is either quite small or quite large. A second sample is taken if the results of the first sample are not decisive. The operation of an attributes double-sampling plan is given in Figure 46.9. The characteristics of an attributes double-sampling plan are given in Table 46.8.

Multiple Sampling. In multiple sampling by attributes, more than two samples can be taken in order to reach a decision to accept or reject the lot. The chief advantage of multiple sampling plans is a reduction in sample size for the same protection. The operation of multiple sampling by attributes is given in Figure 46.10. The characteristics of a multiple sampling plan are given in Table 46.9.

Sequential Sampling. In sequential sampling, each item is treated as a sample of size 1, and a determination to accept, reject, or continue sampling is made after inspection of each item. The major advantage of sequential sampling plans is that they offer the opportunity for achieving the minimum sample size for a given protection. The operation of sequential sampling by attributes is given in Figure 46.11. The characteristics of a sequential sampling plan are given in Table 46.10. Attributes sequential plans will be found tabulated by producer's and consumer's risk in the international standard ISO 8422 (1991).

Rectification Schemes. Rectification schemes are used when it is desired to ensure that the average outgoing quality level of a series of lots will not exceed specified levels. Such schemes employ 100 percent inspection (screening), with nonconforming items replaced by conforming items.

There are two basic types of rectification schemes: LTPD schemes and AOQL schemes. LTPD schemes ensure consumer quality level protection for each lot, while AOQL schemes ensure AOQL protection for a series of lots. Both types of schemes minimize ATI at the projected process average percent nonconforming. See Dodge and Romig (1959).

Continuous Sampling. Some production processes deliver product in a continuous stream rather than on a lot-by-lot basis. Separate plans have been developed for such continuous production based on the AOQL concept. These plans generally start with 100 percent inspection until some consecutive number of units free of defects is found and then provide for inspection on a sampling basis until a specified number of defective units is found. One hundred percent inspection is then instituted again until a specified number of consecutive good pieces is found, at which time sampling is reinstituted. Continuous sampling plans have been proposed by Harold F. Dodge and modifications

TABLE 46.6 Unity Values for Construction and Evaluation of Single, Double, and Multiple Sampling Plans

($n_1 = n_2 = \cdots = n_k$; # indicates acceptance not allowed at a given stage)

				Probability of acceptance												
Plan	Acceptance numbers	$R = p_2/p_1$	np_2	.99	.95	.90	.75	.50	.25	.10	.05	.01	.005	.001	.0005	.0001

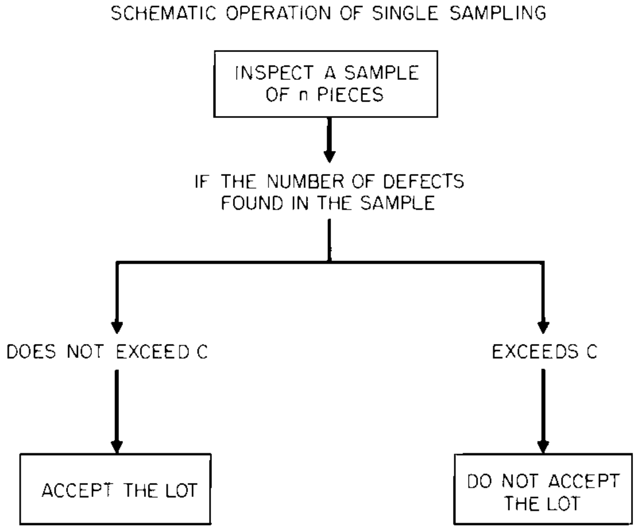

FIGURE 46.8 Schematic of single sampling. In practice, the lot not to be accepted may be repaired, junked, etc. Sampling tables usually assume that the lot is detail-inspected and the defective pieces are all repaired or replaced by good pieces.

TABLE 46.7 Single Sampling by Attributes

Example: Determine a plan which will have AQL = 100 p_1 = 1.0% with producer risk α = 0.05 and LTPD = $100p_2$ = 5.0% with consumer risk β = 0.10. Suppose a sample is taken from a lot of N = 500 and four nonconforming units are found.

Summary of plan	Calculations
I. Restrictions: Random sample of dichotomous data	
II. Necessary information *A.* Producer quality level, p_1 *B.* Producer risk, α *C.* Consumer quality level, p_2 *D.* Consumer risk, β	II. *A.* $p_1 = 0.01$ *B.* $\alpha = 0.05$ *C.* $p_2 = 0.05$ *D.* $\beta = 0.10$
III. Selection of plan (using Table 46.6) *A.* Calculate operating ratio $R = p_2 / p_1$	III. *A.* $R = 0.05/0.01$ $= 5$

IV. Elements	IV.
A. Sample size: See III above	*A.* $n = 134$
B. Statistic: d = number nonconforming in sample	*B.* $d = 4$
C. Decision criteria: 1. Accept if $d \leq c$ 2. Reject if $d > c$	*C.* $4 > 3$, reject
V. Action: Dispose of lot as indicated by decision rules.	V. Reject
VI. Measures (using Table 46.6)	VI.
A. Probability of acceptance (OC curve)	*A.*
1. Under each value of probability of acceptance listed across top of table, there corresponds a value of np shown for the plan	1. $P_a = 0.50$ corresponds to $np = 3.672$
2. Divide the values of np by sample size n to get p	2. $p = \frac{3.672}{134} = 0.027$
3. Draw OC curve from corresponding values of p and P_a	3. $p = 0.027$ $P_a = 0.50$
B. Average sample number (ASN curve)	*B.*
1. Calculate p from P_a as above	1. $P_a = 0.50$ $p = 0.027$
2. Under the value np from which p was calculated is listed a value of ASN/n_1	2. $\text{ASN}/n_1 = 1$
3. Multiply ASN/n_1 by n to obtain ASN corresponding to p	3. $(1)(134) = 134$
4. Draw ASN curve from corresponding values of p and P_a	4. $p = 0.027$ $\text{ASN} = 134$
C. Average outgoing quality	*C.*
1. Use formula $\text{AOQ} \simeq p\,P_a$	1. at $p = 0.027$, $\text{AOQ} = 0.027\,(0.50) = 0.014$
D. Average outgoing quality limit $\text{AOQL} \simeq \frac{0.4}{n}(1.25c + 1)$ for $c \leq 5$	*D.* 1. $\text{AOQL} \simeq \frac{0.4}{134}[1.25(3)+1] \simeq 0.019$
E. Average total inspection $\text{ATI} = n\,P_a + N(1 - P_a)$	*E.* at $p = 0.027$ $\text{ATI} = 134(0.5) + 500(0.5) = 317$

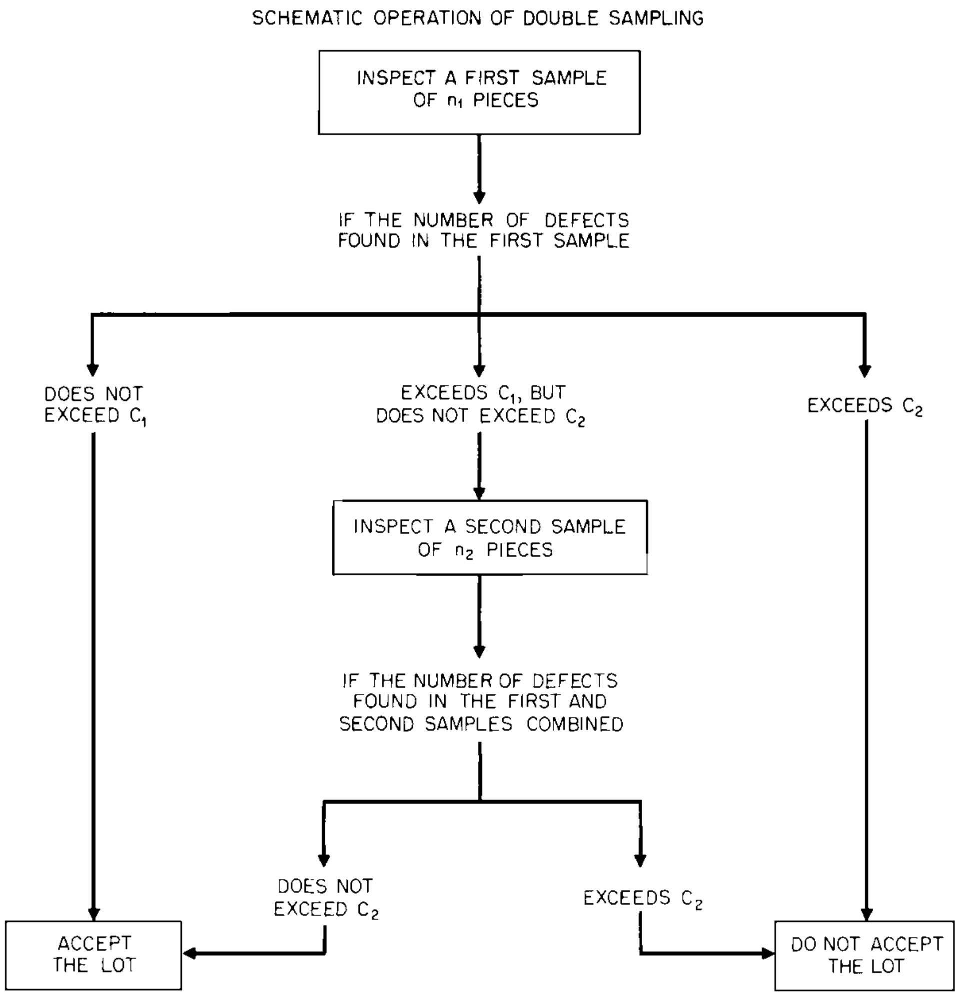

FIGURE 46.9 Schematic of double sampling. In regard to the lot not to be accepted, inspect the remainder of the pieces, replacing or repairing those defective.

TABLE 46.8 Double Sampling by Attributes

TABLE 46.8 Double Sampling by Attributes (*Continued*)

B. Choose double sampling plan (*D*) from Table 46.6 which shows $R \leq$ the value calculated	*B*. Choose plan 3D since R shown is 4.398
C. Acceptance (a) and rejection (r) numbers are given as Ac and Re	*C*. First sample $a_1 = 1, r_1 = 4$ Second sample $a_2 = 4, r_2 = 5$
D. Determine first sample size from value of (np_2) shown for the plan as $n = (np_2)/p_2$ This is sample size of each of the double samples	*D*. $n = 4.398/0.05 = 88$
IV. Elements	IV.
A. Sample size: See III above	*A*. $n_1 = 88$ $n_2 = 88$
B. Statistic: d_1 = number nonconforming in first sample; d_2 = number nonconforming in second sample	*B*. $d_1 = 4$
C. Decision criteria	*C*.
1. Accept on first sample if $d_1 \leq a_1$	1. 4 not ≤ 1
2. Reject on first sample if $d_1 \geq r_1$	2. 4 $\geq$4, reject
3. Take a second sample if $a_1 < d_1 < r_1$	
4. Accept on second sample if $d_1 + d_2 \leq a_2$	
5. Reject on second sample if $d_1 + d_2 \geq r_2$	
V. Action: Dispose of lot as indicated by decision rules.	V. Reject
VI. Measures (using Table 46.6)	VI.
A. Probability of acceptance (OC curve)	*A*.
1. Under each value of probability of acceptance listed across the top of table, there corresponds a value of np shown for the plan	1. $P_a = 0.50$ corresponds to $np = 2.465$
2. Divide the values of np by sample size n to get p	2. $p = \frac{2.465}{88} = 0.028$

TABLE 46.8 Double Sampling by Attributes (*Continued*)

D. Average outgoing quality limit 1. AOQL = maximum of AOQ curve	*D.* 1. AOQL = 0.0151 at p = 0.022
E. Average total inspection ATI $= n_1 + n_2(1 - P_{a_1}) + (N - n_1 - n_2)(1 - P_a)$ where P_{a_1} is probability of acceptance on first sample	*E.* P_{a_1} = probability no more than one defective in 88 $P_{a_1} = 0.29$ ATI = 88 + 88 (0.71) + 324(0.50) = 312.5

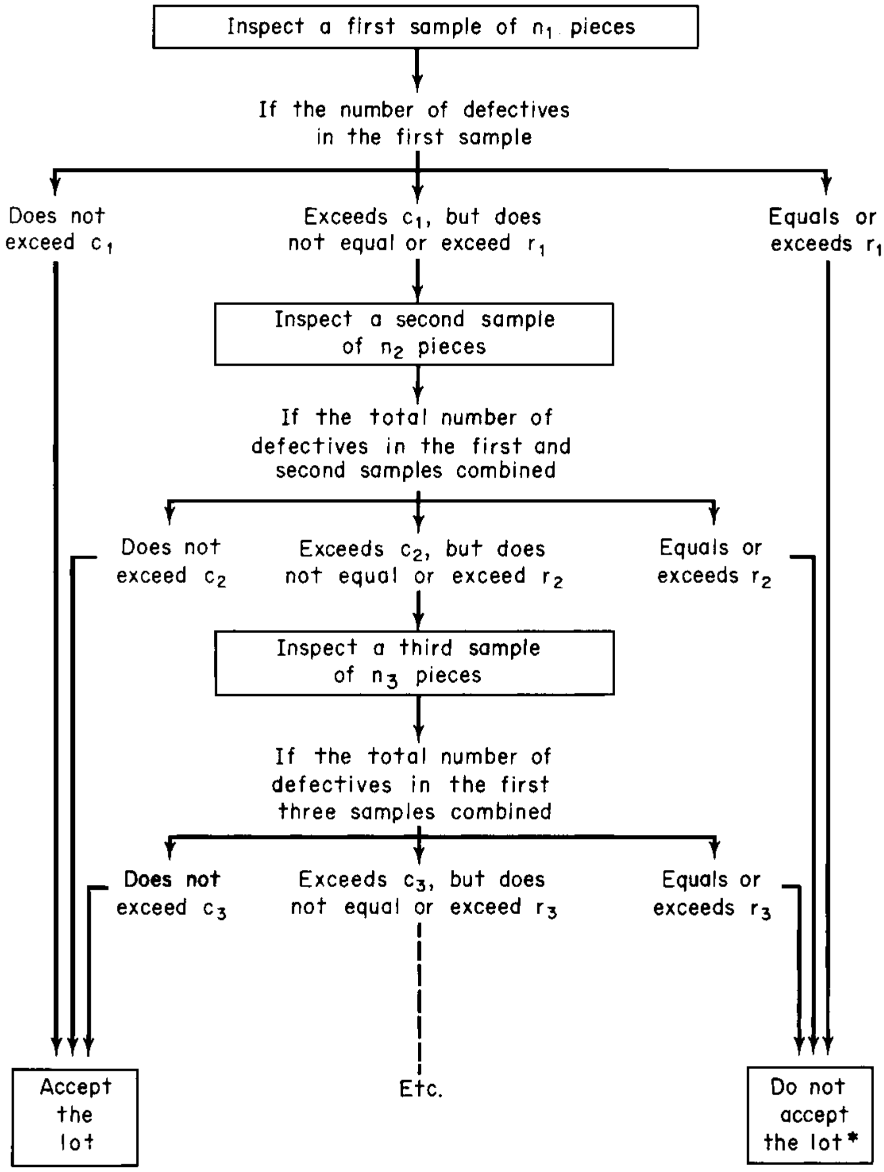

FIGURE 46.10 Schematic operation of multiple sampling. The asterisk means some of these plans continue to the "bitter end"; i.e., the taking of samples continues if necessary until the lot is fully inspected, unless the plan has meanwhile "made up its mind." Other plans, described as "truncated," are designed to force a decision after a certain number of inconclusive samples have been examined.

TABLE 46.9 Multiple Sampling by Attributes

TABLE 46.9 Multiple Sampling by Attributes (*Continued*)

Summary of plan	Calculations
2. Divide the values of np by sample size n to get p	2. $p = \frac{0.910}{33} = 0.028$
3. Draw OC curve from corresponding values of p and P_a	3. $p = 0.028$ $P_a = 0.50$
B. Average sample number (ASN curve)	*B*.
1. Calculate p from P_a as above	1. $P_a = 0.50$ $p = 0.028$
2. Under the value np from which p was calculated is listed a value of ASN/n_1	2. $\text{ASN}/n_1 = 3.288$
3. Multiply ASN/n_1 by n to obtain ASN corresponding to p	3. $\text{ASN} = 3.288(33)$ $= 108.5$
4. Draw ASN curve from corresponding values of p and P_a	4. $p = 0.028$ $\text{ASN} = 108.5$
C. Average outgoing quality	*C*.
1. Use formula $\text{AOQ} \simeq p\,P_a$	1. at $p = 0.028$ $\text{AOQ} \simeq 0.028(.5)$ $= 0.014$
D. Average outgoing quality limit AOQL = maximum of AOQ curve	*D*. $\text{AOQL} = 0.0148$ at $p = 0.022$
E. Average total inspection (approximate) $\text{ATI} \approx \text{ASN}\,(P_a) + N(1 - P_a)$	*E*. $\text{ATI} \simeq 108.5(0.5) + 500(0.5)$ $\simeq 304$

TABLE 46.10 Sequential Sampling by Attributes (*Continued*)

Summary of plan	Calculations
B. Average sample number (ASN curve) 1. Calculate p and P_a as above 2. Then $$\text{ASN} = \frac{\left\{\begin{matrix} P_a \log [(\beta/(1-\alpha)] + \\ (1-P_a) \log [(1-\beta)/\alpha] \end{matrix}\right\}}{\left\{\begin{matrix} p \log (p_2/p_1) + (1-p) \\ \log [(1-p_2)/(1-p_1)] \end{matrix}\right\}}$$	*B.* 1. $p = 0.01$ $P_a = 0.95$ 2. ASN = 81
C. Average outgoing quality (AOQ curve) 1. Use approximate formula $\text{AOQ} \simeq p\,P_a$	*C.* 1. At $p = 0.01$ $\text{AOQ} \simeq 0.01(0.95)$ $= 0.0095$
D. Average outgoing quality limit AOQL = maximum of AOQ curve	*D.* AOQL = 0.0145 at $p = 0.021$
E. Average total inspection (approximate) $\text{ATI} \approx \text{ASN}\,(P_a) + N(1 - P_a)$	*E.* At $p = 0.01$ $\text{ATI} \approx 81(.95) + 500\,(0.05)$ $\simeq 102$

developed by Dodge and Torrey (1951). Dodge (1970) recounts these and other developments in the evolution of continuous sampling plans. MIL-STD-1235B (1981) provides a tabulation of continuous sampling plans.

The following are prerequisites for application of the single-level continuous sampling plans:

1. The inspection must involve "moving product," i.e., product which is flowing past the inspection station, e.g., on a conveyer belt.
2. Rapid 100 percent inspection must be feasible.
3. The inspection must be relatively easy.
4. The product must be homogeneous.

CSP-1 Plans. CSP-1 plans use 100 percent inspection at the start. When i successive units are found to be acceptable and when there is assurance that the process is producing homogeneous product, 100 percent inspection is discontinued and sampling is instituted to the extent of a fraction f of the units. The sampling is continued until a defective unit is found. One hundred percent inspection is then reinstated and the procedure is repeated. Dodge (1943) has provided graphs which can be used to select such plans based on the AOQL desired. See also Schilling (1982).

For example, consider the plan $i = 35, f = 1/10$. One hundred percent inspection is used until 35 successive units are found nondefective. Sampling, at the rate of 1 unit in 10, is then instituted. If a defective is found, 100 percent inspection is reinstituted and continued until 35 successive units are found nondefective, at which time sampling is reinstituted.

CSP-2 Plans. CSP-2 plans were developed to permit sampling to continue even if a single defective is found. Again, 100 percent inspection is used at the start until i successive units are found free of defects. When sampling is in effect and a defective is found, 100 percent inspection is instituted only if a second defective occurs in the next i or fewer sample units inspected. Details on the selection of such plans will be found in Dodge and Torrey (1951) or in Schilling (1982).

Stopping Rules. Murphy (1959) has investigated four different "stopping rules" for use with CSP-1 plans. He concludes that a useful rule is to stop when a specified number of defectives is found during any one sequence of 100 percent inspection.

Multilevel Continuous Sampling. Continuous sampling plans have been developed by Lieberman and Solomon (1955) and others which reduce the sampling rate beyond that of other continuous plans when the quality level is better than a defined AOQL. In addition, sudden changes in the amount of inspection are avoided by providing for several "levels" of inspection. Figure 46.12 outlines the procedure for using the plan. The procedure provides for reducing the sampling rate each time i successive units are found to be free of defects. The first sampling rate after leaving 100 percent inspection is f, and each succeeding sampling rate is f raised to one larger power. The number of sampling levels is k. Thus, if $f = 1/2$ and $k = 3$, the successive sampling rates are $1/2$, $1/4$, and $1/8$.

Notice that all the continuous plans discussed so far are based on the AOQL concept, i.e., requiring periods of 100 percent inspection during which only nondefective product is accepted. This action controls the average defectiveness of accepted product at some predetermined level. In all these cases, inspection must be nondestructive to employ the AOQL principle.

Additional detail on continuous sampling plans will be found in Stephens (1979). A sequential approach to continuous sampling is given in Connolly (1991).

Skip-Lot Schemes. Skip-lot sampling plans are used when there is a strong desire to reduce the total amount of inspection. The approach is to require some initial criterion be satisfied, such as 10 or so consecutively accepted lots, and then determine what fraction of lots will be inspected. Given this fraction, the lots to be inspected are chosen using some random selection procedure. The assumptions on which the use of skip-lot plans are based are much like those for chain-sampling plans (see below). OC curves for the entire skip-lot sampling scheme can be derived, and an AOQL concept does apply. Strong dependence is made on the constancy of production and consistency of process quality as well as faith in the producer and inspector. The skip-lot sampling scheme was devised by Dodge (1955*a*) and is, in essence, an extension of continuous sampling acceptance plans with parameters i and f. He designated the initial scheme as SkSP-1. Here, a single determination of acceptability was made from a sample of size $n = 1$ from each lot. Lots found defective under sampling were 100 percent inspected or replaced with good lots. The AOQL then was in terms of the long-term average proportion of defective *lots* that would reach the customer. This is called AOQL-2. Later, Dodge and Perry (1971) incorporated the use of a "reference" sampling plan to determine whether a lot is acceptable or rejectable. Such skip-lot plans are designated SkSP-2 and require 100 percent inspection of rejected lots. The resulting AOQL is in terms of the long-term average proportion of individual product *units* that would reach the customer. This is called AOQL-1.

The application of skip-lot sampling should be confined to those instances where a continuous supply of product is obtained from a reasonably stable and continuous process.

The parameters of the SkSP-2 plan (in terms of continuous sampling) are:

i = number of successive lots to be found conforming to qualify for skipping lots either at the start or after detecting a nonconforming lot

f = fraction of lots to be inspected after the initial criteria have been satisfied

n = sample size per inspected lot

c = acceptance number for each inspected lot

For example, if $i = 15$, $f = 1/3$, $n = 10$, $c = 0$, the plan is to inspect 10 units from each lot submitted until 15 consecutive lots are found conforming, i.e., have no defective in the sample. Then one-third of the submitted lots are chosen at random for inspection and inspected to the same n and c. As long as all lots are found to be conforming, the f-rate applies. When a nonconforming lot is identified, reversion to inspection of all lots occurs until again 15 consecutive lots are found to be conforming, and then skipping is permissible again.

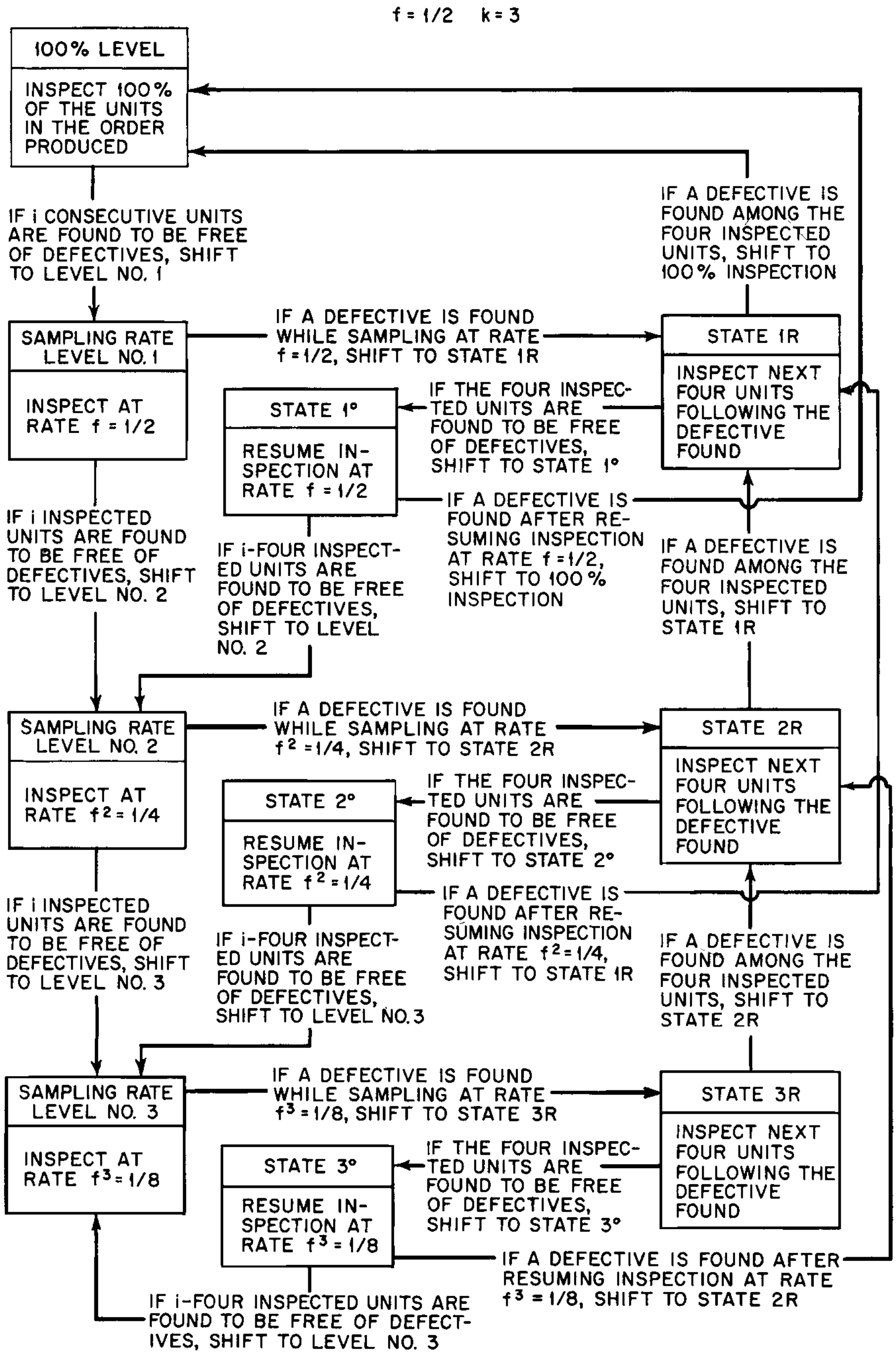

FIGURE 46.12 Procedure for multilevel continuous sampling.

Many values of f and i are possible. For SkSP-1 these may be obtained from the Basic Curves for Plan CSP-1. See Dodge (1955*a*). In using SkSP-2, unity values for selection of a plan having a designated operating ratio will be found in Dodge and Perry (1971). See also Schilling (1982).

One further consideration is whether to apply the skip-lot procedure to only one characteristic of a product, to all characteristics simultaneously, or just to some. The plan is obvious and straightforward when applied only to one product characteristic, perhaps a particularly expensive one in time or dollars. If applied to several characteristics, it might be best to inspect the f fraction for these characteristics on each lot. For example, if 6 characteristics are candidates for skip-lot sampling and $f = 1/3$, perhaps 2 of these would be examined on one lot, two on another, and two on the third, again in some random fashion, so that all lots receive some inspection. Of course, if the most expensive part of the sampling scheme were forming the lot, sampling from it, and keeping records, the choice might be to inspect all characteristics on the same sample units. This argument may be extended to the case where all inspection characteristics are on a skip-lot basis; but if inspection is very complex, this is difficult to visualize.

Perry (1973) has extended skip-lot plans in many ways including the incorporation of multi-level plans. See Perry (1973*a*). Skip-lot plans are described in detail by Stephens (1982).

Chain Sampling. Chain-sampling plans utilize information over a series of lots. The original plans by Dodge (1955), called ChSP-1, utilized single sampling on an attributes basis with n small and $c = 0$. The distinguishing feature is that the current lot under inspection can also be accepted if one defective unit is observed in the sample provided that no other defective units were found in the samples from the immediately preceding i lots, i.e., the chain. Dodge and Stephens (1965, 1966) derived some two-stage chain-sampling plans which make use of cumulative inspection results from several samples and are generalizations and extensions of ChSP-1. Conversely, ChSP-1 is a subset of the new plans, and the discussion here will be in terms of the original plan.

Before discussing the details of the plans, the general characteristics of chain sampling are described. Chain-sampling plans, in comparison with single-sampling plans, have the characteristic of "bowing up" the OC curve for small fractions defective while having little effect on the end of the curve associated with higher fractions defective. The effectiveness of chain-sampling plans strongly depends upon the assumptions on which the plans are based, namely:

1. Production is steady, as a continuing process.
2. Lot submittal is in the order of production.
3. Attributes sampling is done where the fraction defective p is binomially distributed.
4. A fixed sample size from each lot is assumed.
5. There is confidence in the supplier to the extent that lots are expected to be of essentially the same quality.

Chain-sampling plans are particularly useful where inspection is costly and sample sizes are relatively small. However, they may also be found useful with large sample sizes. The advantage over double-sampling plans is the fixed sample size. The disadvantage is that moderate changes in quality are not easily detected. However, major changes in quality are detected as easily with chain-sampling plans having much smaller sample sizes as with single-sampling with the same AQL.

The original ChSP-1 plan is to inspect a sample of n units from a lot, accept the lot if zero defectives were found in the n units, or accept the lot if one defective were found in the sample and no defectives were found in the samples from the immediately preceding i lots. See Figure 46.13.

Formulas for the OC curves for ChSP-1 are given by Dodge (1955) and for the two-stage procedure will be found in Dodge and Stephens (1965, 1966) while Raju (1991) has developed three-stage chain plans. All are based on the type B sampling situation. Soundararajan (1978) has prepared tables of unity values for selection of chain-sampling plans from various criteria. Stephens (1982) provides additional detail on implementing chain plans. Govindaraju (1990) gives tables for determining ChSP-1 plans given acceptable quality level and limiting quality level.

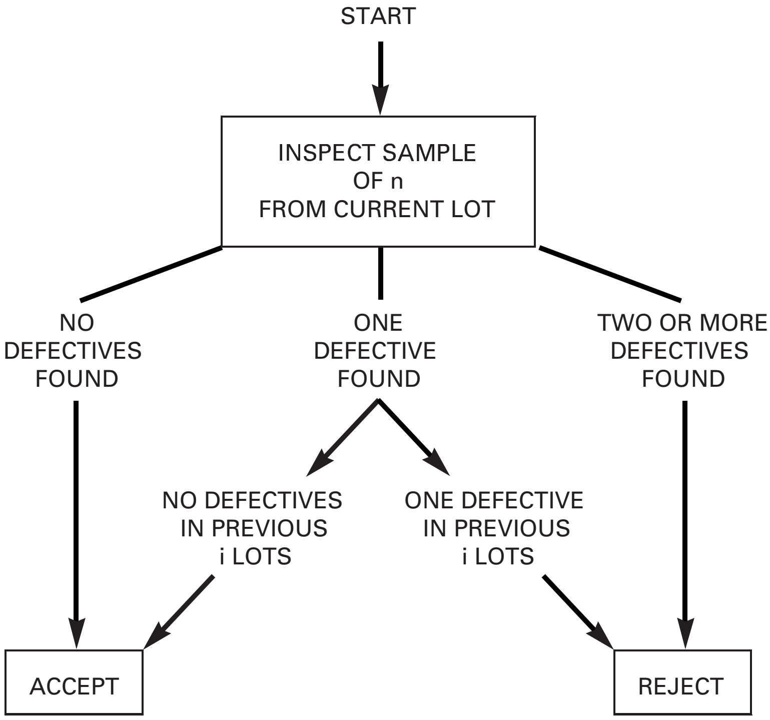

FIGURE 46.13 Flowchart of ChSP-1 chain sampling scheme. (*From Schilling, 1982, p. 452.*)

Cumulative Sum Sampling Plans. A scheme has been devised by Beattie (1962) whereby the continuous inspection approach may be employed when inspection is destructive. Acceptance or rejection of the continuously produced product is based on the cumulation of the observed number of defectives. In addition, it is possible to discriminate between two levels of quality, an acceptable quality level (AQL) and rejectable quality level (RQL).

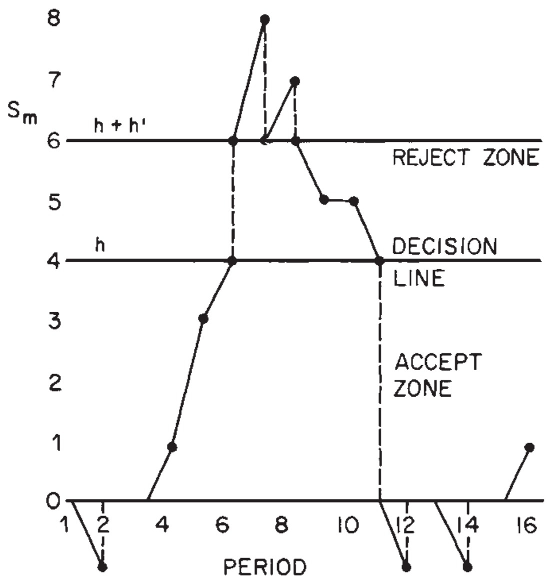

FIGURE 46.14 Form of chart for acceptance under cumulative sum sampling plans.

Prior to the development of continuous and cumulative sum plans, the following situation frequently occurred when inspection was destructive. Since production was continuous, usually there was no technically rational procedure for defining lots. A common procedure was to accumulate product until a lot was formed, but then the time delay in making a decision on the product was often too long, and a few short periods of bad production could cause a large quantity of good product to be rejected. Various approaches to lessening this jeopardy were used, all with some logical or practical drawback.

The procedure of Beattie (1962) establishes two zones, an accept zone and a reject zone, and product is accepted or rejected according to the cumulative sum of defectives observed. Figure 46.14 illustrates the plan. To implement the procedure, a sample of size n is chosen at regular intervals from production. The units are inspected and the number of defective units d_i in the ith sample recorded. Then,

$$S_m = \sum_{i=1}^{m} (d_i - k)$$

is accumulated, where k is a parameter of the scheme and S_m is computed and plotted according to the following rules:

1. Start the cumulation, S_m, at zero.
2. Accept product as long as $S_m < h$, where h is a second parameter of the scheme. When $S_m < 0$, return the cumulation to zero.
3. When h is crossed or reached from below, reject product and restart the cumulation at $h + h'$ (h' is a third parameter of the plan).
4. Continue rejecting product as long as $S_m > h$. When $S_m > h + h'$, return calculation to $h + h'$ and continue rejecting product until $S_m < h$.
5. When h is crossed or reached from above, accept product and restart cumulation at zero.

The discriminatory capability of the plans is controlled through the choice of k, h, and h' and their combinations. In familiar terms, k is akin to an acceptance number and h defines how far one is willing to deviate from this number over a long run and still accept product. The quantity h' is essentially how much evidence is required to be assured the process has been corrected.

The protection offered by this type of plan, which is similar to a type B acceptance situation, is defined by an OC curve which is determined by the ratio of the average run length (ARL) in the accept zone to the sum of the ARL in the accept and reject zones.

Published Tables and Procedures. The published plans cover primarily three quality indices: AQL, LTPD, and AOQL.

AQL Plans. Sampling schemes indexed by AQL are designed to give high assurance for type B situations (that is, sampling from a process) of lot acceptance when the process fraction defective is less than or equal to that AQL. These plans are devised for producer's protection or to keep the producer's risk small. Consideration is given to what might be called the other end of the plan, the consumer's risk, through the switching rules. If there is interest in controlling both producer's risk and consumer's risk with the use of schemes indexed by AQL values, the best approach is to study the scheme OC curves and choose the plan which provides adequate protection against both types of risks. See Schilling and Sheesley (1978); also Schilling and Johnson (1980).

MIL-STD-105 is the best-known sampling scheme indexed by AQL. The AQLs presented in this standard form a sequence which is evident in the standard. The sequence chosen, while arbitrary, does provide a sensible grouping and indexing of plans for use. The steps to be applied in AQL sampling acceptance procedures are summarized in Table 46.11.

LTPD Plans. Sampling schemes indexed by LTPD or other values of limiting quality (LQ) are designed for type A situations that are essentially the mirror image of plans indexed by AQL. That is, the plans are chosen such that the consumer's risk of accepting a submitted lot of product with quality equal to or worse than the LQ is equal to or less than some specified value. Specifically, LTPD plans provide a 10 percent probability of accepting lots of the listed LTPD quality. The Dodge-Romig sampling scheme is the best-known set of plans indexed by LTPD. The derivative standards of MIL-STD-105 also provide some LQ plans. Schilling (1978) has developed a set of type A lot sensitive plans with $c = 0$ which are selected directly from the lot size and the LTPD using a simple table. Properties of these plans under curtailment have been developed by Govindaraju (1990).

AOQL Plans. Sampling schemes indexed by AOQL are derived to provide assurance that the long-run average of accepted quality, given screening of rejected lots, will be no worse than the indexed AOQL value. The Dodge-Romig (1959) plans provide the best available tables of sampling plans indexed by AOQL.

TABLE 46.11 Summary of Lot-by-Lot Sampling Procedure

Establishment of standards (by inspection executive):
Decide what shall be a unit of product.
Classify the quality characteristics for seriousness.
Fix an acceptable quality level for each class.
Fix an inspection level for the product.
Installation of procedure (by inspection supervisor):
Arrange for formation of inspection lots.
Decide what type of sampling shall be used (single, double, multiple).
Choose sampling plan from tables.
Operation of procedure (by line inspector):
Draw sample units from each inspection lot.
Inspect each sample unit.
Determine whether to accept or reject the inspection lot (if sampling for acceptance) or whether to urge action on the process (if sampling for control).
Review of past results (by inspection supervisor):
Maintain a record of lot acceptance experience and cumulative defects by successive lots.
Determine whether to tighten or reduce inspection on future lots.
Feedback
Insure that the results of inspection are fed back to all components and especially the producer to motivate action for quality improvement.

AQL Attributes Sampling System. Most attributes AQL sampling systems are derivatives of the 105 series of acceptance sampling procedures, issued by the United States Department of Defense. This series was initiated with MIL-STD-105A in 1950 and concluded with MIL-STD-105E issued 10 May 1989. This series was canceled 27 February 1995. The best known 105 derivatives are the U.S. commercial version ANSI/ASQC Z1.4 (1993) and the international standard ISO-2859-1 (1989). With the cancellation of MIL-STD-105E, ANSI/ASQC Z1.4 was adopted by the U.S. military for future acquisitions.

ISO 2859-1 (1989) is part of the international ISO 2859 series of integrated standards to be used with the ISO 2859-1 AQL system. ISO 2859-0 (1995) provides an introduction to acceptance sampling as well as a guide to the use of ISO 2859-1, which is the central AQL sampling system in the series. ISO 2859-2 (1985) gives Limiting Quality (LQ) sampling plans for isolated lots to be used individually or in cases where the assumptions implicit in AQL schemes do not apply. ISO 2859-3 (1991) presents skip-lot sampling procedures which may be used as a substitute for Reduced Sampling in ISO 2859-1. Related U.S. national standards are ANSI/ASQC S2 (1995) which is an introduction to the ANSI/ASQC Z1.4 (1993) AQL scheme; ANSI/ASQC Q3 (1988) for Limiting Quality; and ANSI/ASQC S1 (1996) for skip-lot inspection. The international standard ISO 8422 (1991) also provides attribute sequential plans which may be used with ISO 2859-1.

The quality index in ANSI/ASQC Z1.4 (1993) and its international counterpart ISO 2859-1 (1989) is the acceptable quality level (AQL):

- A choice of 26 AQL values is available ranging from 0.010 to 1000.0. (Values of 10.0 or less may be interpreted as percent defective or defects per hundred units. Values above 10.0 must be interpreted as defects per hundred units.)
- The probability of accepting at AQL quality varies from 89 to 99.5 percent.
- Nonconformances are classified as A, B, C according to degree of seriousness.
- The purchaser may, at its option, specify separate AQLs for each class or specify an AQL for each kind of defect which a product may show.

The user also specifies the relative amount of inspection or inspection level to be used. For general applications there are three levels, involving inspection in amounts roughly in the ratio of 1 to 2.5 to 4. Level II is generally used unless factors such as the simplicity and cost of the item, inspection cost, destructiveness of inspection, quality, consistency between lots, or other factors make it appropriate to use another level. The standard also contains special procedures for "small-sample inspection" where small sample sizes are either desirable or necessitated by some aspects of inspection. Four additional inspection levels (S1 through S4) are provided in these special procedures.

The procedure for choice of plan from the tables is outlined below.

1. The following information must be known:
   a. Acceptable quality level.
   b. Lot size.
   c. Type of sampling (single, double, or multiple).
   d. Inspection level (usually level II).
2. Knowing the lot size and inspection level, obtain a code letter from Table 46.12.
3. Knowing the code letter, AQL, and type of sampling, read the sampling plan from one of the nine master tables (Table 46.13 is for single-sampling normal inspection; the standard also provides tables for double and multiple sampling).

For example, suppose that a purchasing agency has need for a 1 percent AQL for a certain characteristic. Suppose also that the parts are bought in lots of 1500 pieces. From the table of sample size code letters (Table 46.12), it is found that letter K plans are required for inspection level II, the one generally used. Then the plan to be used initially with normal inspection would be found in Table 46.13 in row K. The sample size is 125. For AQL = 1.0, the acceptance number is given as 3 and the rejection number is given as 4. This means that the entire lot of 1500 units may be accepted if 3 or fewer defective units are found in the sample of 125, but must be rejected if 4 or more are found. Where an AQL is expressed in terms of "nonconformities per hundred units," this term may be substituted for "nonconforming units" throughout. Corresponding tables are provided in the standard for tightened and reduced inspections.

Inspection Severity—Definitions and General Rules for Changing Levels. The commonly used AQL attributes acceptance sampling plans make provisions for shifting the amount of inspection

TABLE 46.12 Sample Size Code Letters*

Lot or batch size	General inspection levels		
	I	II	III
2 to 8	A	A	B
9 to 15	A	B	C
16 to 25	B	C	D
26 to 50	C	D	E
51 to 90	C	E	F
91 to 150	D	F	G
151 to 280	E	G	H
281 to 500	F	H	J
501 to 1,200	G	J	K
1,201 to 3,200	H	K	L
3,201 to 10,000	J	L	M
10,001 to 35,000	K	M	N
35,001 to 150,000	L	N	P
150,001 to 500,000	M	P	Q
500,001 and over	N	Q	R

*Sample size code letters given in body of table are applicable when the indicated inspection levels are to be used. The Standard includes an added table of code letters for small-sample inspection.

and/or the acceptance number as experience indicates. If many consecutive lots of submitted product are accepted by an existing sampling plan, the quality of submitted product must exceed that specified as necessary for acceptance. This makes it desirable to reduce the amount and cost of inspection (with a subsequent higher risk of accepting an occasional lot of lesser quality) simply because the quality level is good. On the other hand, if more than an occasional lot is rejected by the existing sampling plan, the quality level is either consistently lower than desired or the quality level fluctuates excessively among submitted lots. In either case it is desirable to increase the sampling rate and/or reduce the acceptance number to provide greater discrimination between lots of adequate and inadequate quality.

Three severities of inspection are provided: normal, tightened, and reduced. All changes between severities are governed by rules associated with the sampling scheme. Normal inspection is adopted at the beginning of a sampling procedure and continued until evidence of either lower or higher quality than that specified exists. Schematically, the rules for switching severities specified in MIL-STD-105D and E, ANSI/ASQC Z1.4 (1993) and ISO 2859-1 (1989) are given in Figure 46.15.

From the schematic it can be seen that the criteria for change from normal to tightened and back to normal are simple and straightforward. (This is a vast improvement over early issues of MIL-STD-105.) Again, the change from reduced to normal is straightforward and occurs with the first indication that quality has slipped. Considerably more evidence is required to change from normal to reduced.

In using schemes like MIL-STD-105E, ANSI/ASQC Z1.4 (1993), or ISO 2859-1 (1989), the customer assumes that the changes from the normal to tightened and reduced to normal are a necessary part of the scheme. (Recall that such AQL plans provide primarily producer's assurance that quality at the AQL level will be accepted.) The change from normal to tightened or reduced to normal occurs when evidence exists that the quality level has deteriorated. In this way, the consumer's protection is maintained.

On the other hand, the supplier is interested in keeping inspection costs as low as possible consistent with the demands placed upon it. Certainly it is desirable to change from tightened to normal inspection when conditions warrant. Generally, it would be economic to change from normal to reduced except in those cases where record-keeping costs or bother exceed the saving in reduced sampling effort. The initiative for these types of change generally rests with the supplier; only in those instances where an economic advantage exists would a customer insist on reduced inspection. This might occur, e.g., when the customer is using these sampling plans, where inspection is destructive or degrading, or when the psychological impact on the supplier makes reduced inspection a motivational force.

The case could arise where a complex item is being inspected for many characteristics, some of which are classified as critical (A), some major (B), and some minor (C). A different sampling plan could be in effect for each class. It is also possible on this same product to have reached the situation where, for example, reduced inspection could be in effect for A defects, tightened inspection for B defects, and normal inspection for C defects. The bookkeeping involved with complete flexibility of sampling plan choice by classification of characteristics and inspection severity can be enormous. On the other hand, the savings involved with lower inspection rates, especially with destructive inspection, could be sizable. Inspection cost and convenience, along with the consequences of the error of applying an improper sampling plan, are the main determinants in any decision to use anything but the highest sampling rate associated with all defect classifications and whether to take advantage of reduced inspection for one or more defect classes when allowed. Of course, when tightened inspection is dictated by sampling experience, there is no recourse but to adopt it for at least that classification or characteristic, other than to discontinue acceptance inspection of submitted product.

The standard also provides limiting quality (LQ) single-sampling plans with a consumer's risk of 10 percent (LTPD) and 5 percent for use in isolated lot acceptance inspection. If other levels of LQ are desired, the individual OC curves can be examined to adopt an appropriate plan.

Finally, a table of AOQL values is included for each of the single-sampling plans for normal and tightened inspection. These may also be used as rough guides for corresponding double- and multiple-sampling plans.

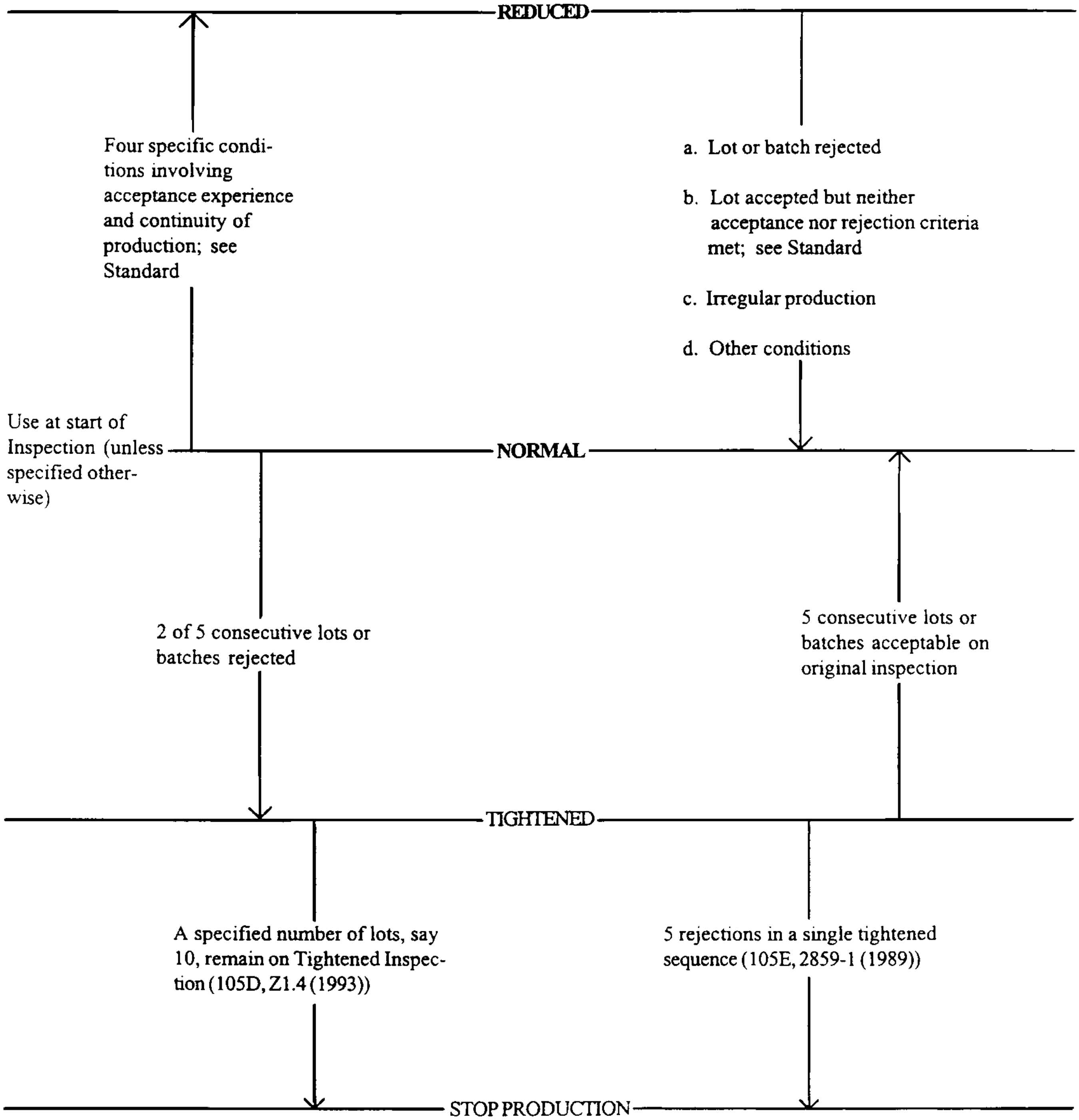

FIGURE 46.15 Rules for switching inspection severity, MIL-STD-105D and E, ANSI/ASQC Z1.4 (1993) and ISO 2859-1 (1989).

The sampling plans in MIL-STD-105E, ANSI/ASQC Z1.4, and ISO 2859-1 are sufficiently varied in type (single, double, multiple), amount of inspection, etc., to be useful in a great number of situations. The inclusion of operating characteristic curves and average sample size curves for most of the plans is a noteworthy advantage of the standards.

Dodge-Romig Sampling Tables. Dodge and Romig (1959) provide four different sets of tables:

1. Single-sampling lot tolerance tables (SL)
2. Double-sampling lot tolerance tables (DL)

3. Single-sampling AOQL tables (SA)
4. Double-sampling AOQL tables (DA)

All four types of plans were constructed to give minimum total inspection for product of a given process average. All lots rejected are assumed to be screened, and both the sampling and the expected amount of 100 percent inspection were considered in deriving the plan which would give minimum inspection per lot. This is a particularly appropriate approach for a manufacturer's inspection of its own product as when the product of one department is examined prior to use in another, especially when processes are out of control. Practically, it may be reasonable to use the same theory even when the sampling is done by the purchasing company and the detailing of rejected material is done by the supplying company, since in the long run all the supplier's costs to provide material of a specified quality are reflected in the price.

The first and second sets of tables are classified according to lot tolerance percent defective at a constant consumer's risk of 0.10. Available lot tolerance plans range from 0.5 to 10.0 percent defective. In contrast, the third and fourth sets of tables are classified according to the average outgoing quality limit (AOQL) which they assure. Available AOQL values range from 0.1 to 10.0 percent. Lot tolerance plans emphasize a constant low consumer's risk (with varying AOQLs). In other words, they are intended to give considerable assurance that individual lots of poor material will seldom be accepted. The AOQL plans emphasize the limit on poor quality in the long run, but do not attempt to offer uniform assurance that individual lots of low quality will not get through. The relative importance of these two objectives will guide the choice of types of plan.

Table 46.14 shows a representative Dodge-Romig table for single sampling on the lot tolerance basis. All the plans listed in this table have the same risk (0.10) of accepting submitted lots that contain exactly 5 percent of defective units. The table has six columns. Each of these lists a set of plans appropriate to a specified average value of incoming quality. For example, if the estimated process average percent defective is between 2.01 and 2.50 percent, the last column at the right gives the plans that will provide the minimum inspection per lot. However, the assurance that a lot of quality 5 percent defective will be rejected is the same for all columns, so an initial incorrect estimate of the process average would have little effect except to increase somewhat the total number of pieces inspected per lot. The selection of a plan from this table thus requires only two items of information: the size of the lot to be sampled and the prevailing average quality of the supplier for the product in question. If the process average is unknown, the table is entered at the highest value of process average shown.

Zero Acceptance Number Plans. In an increasingly litigious society, zero acceptance number plans have enjoyed increasing popularity. This is due, in part, from the notion that no defectives in the sample implies there are no defectives in the rest of the lot; and that defectives found in the sample indicate that the rest of the lot is equally contaminated. This is in line with a zero defects approach to quality. Continual improvement, however, implies lack of perfection and a constant effort to reveal and appropriately deal with quality levels that are not zero. If the state-of-the-art quality levels are indeed not zero, plans with nonzero acceptance numbers and correspondingly larger sample sizes will better discriminate between quality levels, thus facilitating corrective action when needed. Sample size is important as well as the acceptance number! A lot 13 percent defective has 50:50 odds of rejection by the plan $n = 5$, $c = 0$ or by the plan $n = 13$, $c = 1$. Thus, $c = 0$ and $c = 1$ can give equal protection at this point. However, the $c = 1$ plan can discriminate between AQL and LTPD levels that are closer together.

The zero acceptance number plans are useful for emphasizing zero defects and in product liability prevention. The $c = 0$ requirement, however, forces the consumer quality level to be 41 times higher than the producer quality level, since this is a mathematically unavoidable characteristic of $c = 0$ plans. If the consumer quality level is to be closer to the producer quality level, other acceptance numbers must be used. This is why lawyers like and statisticians dislike exclusive use of $c = 0$ plans. A simple estimator of the number of nonconformances outgoing from a $c = 0$ plan with rectification has been given by Greenberg and Stokes (1992).

Several plans have been developed to incorporate the $c = 0$ requirement. These are

- *Lot sensitive plan:* Presents a simple table for Type A sampling that can be used to determine the sample size given the lot size for a desired LTPD. See Schilling (1978).
- *Parts per million AOQL sampling plans:* Give Type B plans to achieve stated AOQL in parts per million given lot size. Also shows LTPD of plans presented. See Cross (1984).
- *AQL sampling plans:* Codified Type B plans indexed by AQL and lot size compatible with MIL-STD-105E Normal Inspection (only). See Squeglia (1994).
- *AQL sampling scheme:* Switches sample sizes between two $c = 0$ plans to increase discrimination over single sampling plans. Called TNT plans, they utilize switching rules in the manner of MIL-STD-105E. See Calvin (1977).

The tightened-normal-tightened (TNT) plan is of special interest in that it improves upon the operating characteristics of a single sample $c = 0$ plan. The switching rules of 105 or its derivatives may be used to switch between normal and tightened inspection. Two plans are used, each with $c = 0$: a normal plan with a given sample size n_N and a tightened plan with a larger sample size n_T. Switching between them will build a shoulder on the OC curve of the normal plan, thus increasing discrimination.

The normal and tightened plan sample sizes n_N and n_T can be found using a method developed by Schilling (1982) as follows:

$$n_T = \frac{230.3}{\text{LTPD}(\%)} \qquad n_N = \frac{5.13}{\text{AQL}(\%)}$$

Thus, if it is desired to achieve AQL = 1 percent and LTPD = 5 percent, $n_T = 230.3/5 = 46.1 \simeq 46$ and $n_N = 5.1/1 \simeq 5$. Then, using Z1.4 switching rules to switch between the plans $n = 46$, $c = 0$ for tightened inspection and $n = 5$, $c = 0$ for normal inspection, the desired values of AQL and LTPD can be obtained. Procedures and tables for the selection of TNT plans are given in Soundararajan and Vijayaraghavan (1990).

VARIABLES SAMPLING

Overview. In using variables plans, a sample is taken and a measurement of a specified quality characteristic is made on each unit. These measurements are then summarized into a simple statistic (e.g., sample mean) and the observed value is compared with an allowable value defined in the plan. A decision is then made to accept or reject the lot. When applicable, variables plans provide the same degree of consumer protection as attributes plans while using considerably smaller samples.

Table 46.15 summarizes various types of variables plans showing, for each, the assumed distribution, the criteria specified, and special features.

Sampling for Percent Nonconforming. When interest is centered on the proportion of product outside measurement specifications, and when the underlying distribution of individual measurements is known, variables plans for percent nonconforming may be used. These plans relate the proportion of individual units outside specification limits to the population mean through appropriate probability theory. The sample mean, usually converted to a test statistic, is then used to test for the position of the population mean.

Assume the distribution of individual measurements is known to be normal and a plan is desired such that the OC curve will pass through the two points $(p_1, 1 - \alpha)$ and (p_2, β) where

p_1 = acceptable quality level

$1-\alpha$ = probability of acceptance at p_1

TABLE 46.15 Summary of Variables Sampling Plans

Type of plan	Plan	Assumed distribution	Criteria specified	Features
Percent nonconforming	Single-sampling variables plan	Normal	Acceptable and rejectable percent nonconforming	Formulas for determining sample size and acceptance criteria to meet defined risks.
	Double-sampling variables plan	Normal	Acceptable and rejectable percent nonconforming	Tables for determining sample size and acceptance criteria to meet defined risks.
	Narrow-limit gauging	Normal	Acceptable and rejectable percent nonconforming	Tables for determining sample size and acceptance criteria to meet defined risks.
	Lot plot	None	Allowable percent nonconforming	Requires 50 measurements. Simple calculations and graphical procedure used to evaluate lot.

TABLE 46.15 Summary of Variables Sampling Plans

Type of plan	Plan	Assumed distribution	Criteria specified	Features

p_2 = rejectable quality level
β = probability of acceptance at p_2

The OC curve should appear as indicated in Figure 46.16. Some important plans of this type are described below.

Single Sampling. The rationale for variables sampling plans for percent non-conforming is illustrated in Figure 46.17, which assumes the underlying distribution of measurements to be normal with standard deviation σ known.

Suppose the following sampling plan is used to test against an upper specification limit *U:*

1. Sample *n* items from the lot and determine the sample mean $\bar{X}$.
2. Test against an acceptance limit $(U - k\sigma)$, *k* standard deviation units inside the specification.
3. If $\bar{X} \leq (U - k\sigma)$, accept the product; otherwise reject the product.

The situation is analogous, but reversed, for a lower specification limit. If the distribution of individual measurements is normal, as shown, a proportion *p* of the product above the specification limit *U* implies the mean of the distribution must be fixed at the position indicated by μ. Means of samples of size *n* are, then, distributed about μ, as shown; so the probability of obtaining an $\bar{X}$ not greater than $(U - k\sigma)$ is indicated by the shaded area of the distribution of sample means. This shaded area is the probability of acceptance when the fraction nonconforming in the process is *p.* Note that the normal shape supplies the necessary connection between the distribution of the sample means and the proportion of product nonconforming. While, for reasonable sample sizes, the distribution of sample means will be normal, regardless of the shape of the underlying distribution of individual measurements, it is the underlying distribution of measurements itself that determines the relationship of μ and *p.* Hence, the plan will be quite sensitive to departures from normality.

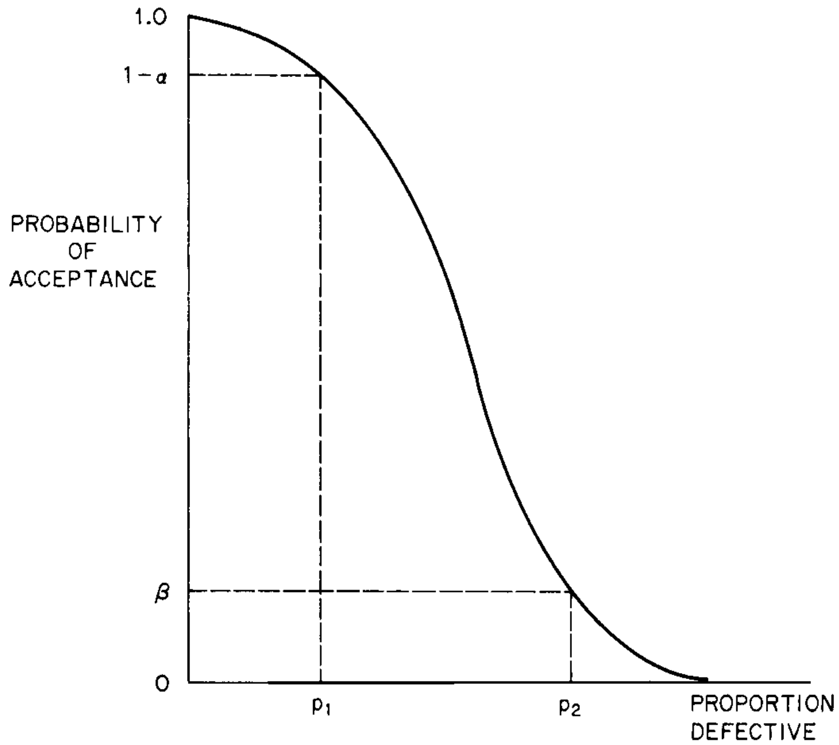

FIGURE 46.16 Operating characteristic curve.

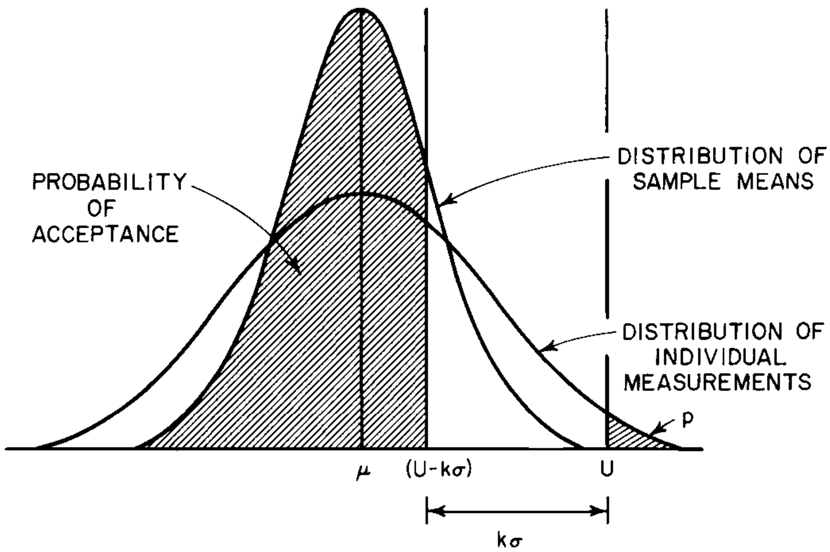

FIGURE 46.17 ($U-k\sigma$) method.

Since $\overline{X} \le U - k\sigma$ is equivalent to $(U - \overline{X})/\sigma \ge k$, the above sampling plan may be expressed as follows:

1. Sample n items from the lot and determine the sample mean $\overline{X}$.
2. If $(U - \overline{X})/\sigma \ge k$, accept the product; otherwise reject the product. Note that for a lower specification limit L, the inequality becomes $(\overline{X} - L)/\sigma \ge k$.

This is the method used to specify variables sampling plans in MIL-STD-414 (1957) and its civilian versions, ANSI/ASQC Z1.9 (1993) and ISO 3951 (1989).

Using Figure 46.17 and normal probability theory, it is possible to calculate the probability of acceptance P_a for various possible values of p, the proportion defective. For example, from Figure 46.17 we see that using an upper tail z value, $z_{1-P_a} = (z_p - k)\sqrt{n}$. So the probability of exceeding z_{1-P_a} is $1 - P_a$ from which we obtain P_a. By symmetry, this relationship can be used with a lower specification limit as well.

A graph of P_a versus p traces the operating characteristic curve of the acceptance sampling plan. Figure 46.18 shows the operating characteristic curve of the variables plan $n = 19$, $k = 1.908$, testing against a single-sided specification limit with known standard deviation. For comparative purposes, the OC curve of the attributes plan $n = 125$, $c = 3$ is also given. Note that the OC curves intersect at about $p = 0.01$ and $p = 0.05$, indicating roughly equivalent protection at these fractions defective.

Probability theory appropriate to other methods of specifying variables plans (e.g., standard deviation unknown, double specification limits) can be used to give the OC curves and other properties of these procedures. Formulas for determining plans to meet specific prescribed conditions can also be derived. Note that the OC curves of variables plans are generally considered to be type B. That is, they are regarded as sampling from the process producing the items inspected, rather than the immediate lot of material involved.

Two-Point Variables Plans for Percent Nonconforming. Two-point single-sampling plans for variables inspection can be readily obtained using an approximation derived by Wallis. For example, a two-point plan testing against a single specification limit, incorporating producer's quality level p_1 and consumer's quality level p_2 with producer's and consumer's risks fixed at $\alpha = 0.05$ and $\beta = 0.10$ may be found using

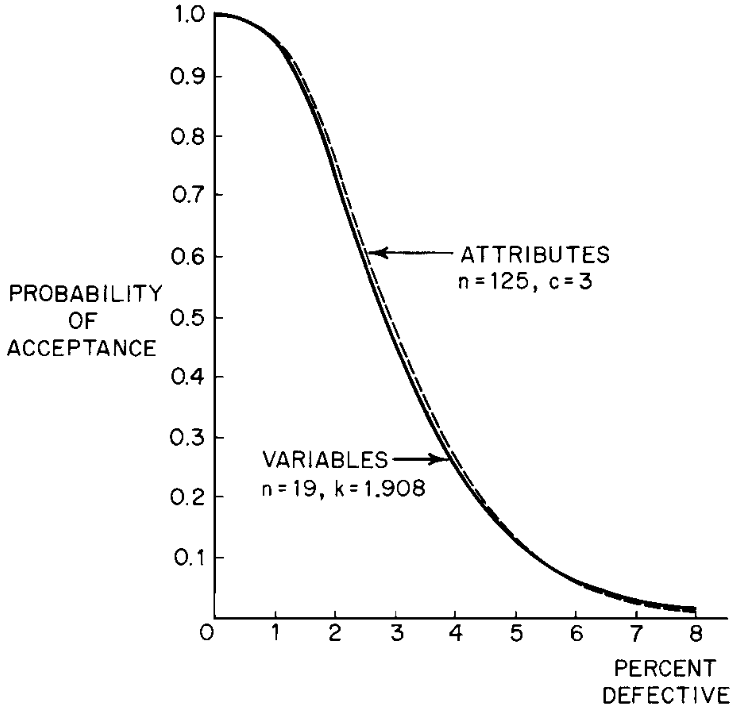

FIGURE 46.18 Operating characteristic curve.

$$k = 0.438Z_1 + 0.562Z_2$$

$$n_\sigma = \frac{8.567}{(Z_1 - Z_2)^2}$$

or

$$n_s = \left(\frac{1 + k^2}{2}\right) n_\sigma$$

Here Z_1 is the standard normal deviate corresponding to an upper tail area of p_1, and Z_2 is the standard of normal deviate corresponding to an upper tail area of p_2. Use n_σ or n_s depending on whether the standard deviation is known or unknown. For other risks or double specification limits more detailed procedures are necessary. See Duncan (1974).

As an example of the use of the procedure, consider $p_1 = 0.01$, $p_2 = 0.05$, $\alpha = 0.05$, $\beta = 0.10$. Application of the formulas with $Z_1 = 2.326$ and $Z_2 = 1.645$ gives $k = 1.94$, $n_\sigma = 18.5 \simeq 19$ and $n_s = 53.3 \simeq 54$. This plan closely matches the attributes plan $n = 125$, $c = 3$. Using the formula given above we see that for $p = p_1 = 0.01$, $Z_{1-P_a} = (2.326 - 2.94)\sqrt{19} = 1.68$ so $1 - P_a = 0.046$ and $P_a = 0.954$.

A procedure for applying double-sampling variables plans was first presented by Bowker and Goode (1952). Tables to expedite the selection and application of these plans are given by Sommers (1981). Double sampling by variables can account for about a 20 percent reduction in average sample number from single sampling. For example, the plan matching the single-sampling plan derived above would have average sample size of 14.9 for the known standard deviation plan and 41.4 when the standard deviation is unknown. The application of double-sampling by variables is analogous to double sampling by attributes.

The relative efficiency of two-point variables plans over the matched attributes plans varies with k. Hamaker (1979) has shown that

$$\frac{n_a}{n_\sigma} = 2\pi p_k(1 - p_k)e^{k^2}$$

where p_k is the upper tail normal area corresponding to the standard normal deviate $Z = k$, and n_a is the attributes sample size. For the example above, $k = 1.94$ so $p_k = .0262$ and $n_a/n_\sigma = 6.9$ and we observe that $131/19 = 6.9$.

Narrow-Limit Gaging for Percent Nonconforming. Narrow- (or compressed-) limit gaging plans bridge the gap between variables and attributes inspection by combining the ease of attributes with the power of variables inspection to reduce sample size. An artificial specification limit is set inside the specification limit. Samples are selected and gaged against this artificial "narrow limit." The narrow limit is set using the properties of the normal distribution, to which the product is assumed to conform. A standard attributes sampling plan is applied to the results of gaging to the narrow limit and the lot is sentenced accordingly.

The criteria for the narrow-limit gaging plan are, then, as follows:

n = sample size

c = acceptance number

t = number of standard deviation units the narrow-limit gage is set inside the specification limit.

The standard deviation σ is assumed to be known.

A set of two-point plans having a minimum sample size for selected values of producer's and consumer's quality levels with $\alpha = 0.05$ and $\beta = 0.10$ has been given by Schilling and Sommers (1981). They have also compiled a complete set of narrow-limit plans matching MIL-STD-105E, ANSI/ASQC Z1.4, and ISO 2859-1.

Consider the plan having producer's and consumer's quality levels $p_1 = 0.01$ and $p_2 = 0.05$ with $\alpha = 0.05$ and $\beta = 0.10$. The appropriate plan given in the Schilling-Sommers tables is $n = 28$, $c = 14$, $t = 1.99$. A narrow-limit gage is set 1.99σ inside the specification limit. A sample of 28 is taken and gaged against each unit. If 15 or more units fail the narrow limit, the lot is rejected. Otherwise, it is accepted.

A good approximation to the optimum narrow limit plan can be constructed from the corresponding known standard deviation variables plan (n_σ, k) as follows:

$$n = \frac{3n_\sigma}{2} \qquad t = k \qquad c = \frac{3n_\sigma}{4} - \frac{2}{3}$$

For the example of the two-point variables plan given above, we have seen that $n_\sigma = 18.5$ and $k = 1.94$

$$n = 27.8 \simeq 28 \qquad t = 1.94 \qquad c = 13.2 \simeq 14$$

Narrow-limit plans have had many successful applications and are readily accepted by inspectors because of the ease with which they can be applied.

Lot Plot. Probably no variables acceptance sampling plan matches the natural inclination of the inspector better than the lot plot method developed by Dorian Shainin (1950) at the Hamilton Standard Division of United Aircraft Co. The procedure employs a histogram and rough estimates of the extremes of the distribution of product to determine lot acceptance or rejection. A standard sample size of 50 observations is maintained.

The method is useful as a tool for acceptance sampling in situations where more sophisticated methods may be inappropriate or not well received by the parties involved. The lot plot plan is especially useful in introducing statistical methods. The subjective aspects of the plan (classification of frequency distributions, their construction, etc.) and its fixed sample size suggest the use of more objective procedures in critical applications. The wide initial acceptance of Shainin's (1952) work attests to its appeal to inspection personnel. The lot plot method is outlined in Table 46.16. See Grant and Leavenworth (1979) for details. For a critical review, see Moses (1956).

TABLE 46.16 Variables Plans for Percent Defective Lot Plot

Example: A lot plot is to be used in inspecting the width of caps. A sample of 50 is taken in 10 subgroups of 5 with the following results:

1	2	3	4	5	6	7	8	9	10
0.2538	0.2581	0.2556	0.2531	0.2501	0.2521	0.2541	0.2555	0.2489	0.2529
0.2519	0.2571	0.2542	0.2566	0.2506	0.2557	0.2499	0.2569	0.2557	0.2579
0.2508	0.2521	0.2521	0.2534	0.2534	0.2569	0.2514	0.2553	0.2542	0.2565
0.2537	0.2545	0.2521	0.2557	0.2516	0.2541	0.2536	0.2496	0.2529	0.2577
0.2529	0.2563	0.2518	0.2519	0.2559	0.2524	0.2492	0.2512	0.2546	0.2541

These data are shown analyzed in Fig. 46.19. Should the lot be accepted?

Summary of plan	Calculations
I. Restrictions: None.	
II. Necessary information: Specification limits.	
III. Selection of plan. *A.* Plan is constant lot to lot. *B.* A special form (Fig. 46.19) is used to apply plan.	
IV. Elements *A.* Sample size: A random sample of 50 pieces is taken from the lot in 10 subsamples of 5 each. Subsample identification is maintained.	
B. Statistic (for symmetric distribution).	*B.*
1. Statistic is $\bar{\bar{X}} \pm 3\hat{\sigma}$ calculated as follows: *a.* Construct cells for frequency distribution on chart. (1) Determine mean $\bar{X}_1$ and range R_1 for the first subgroup (2) Position $\bar{X}_1$ at line number 0 on lot plot form (3) Set cell width w so that $w \simeq R_1/4$ (4) Fill in lot plot form with cell midpoints	1*a.* $\bar{X}_1 = 0.2526$, $R_1 = 0.003$ $\bar{X}_1 \sim 0.253$ at 0 $w \simeq \frac{0.003}{4} = 0.00075$ take $w = 0.001$
b. Tally measurements for frequency distribution using subsample number as tally mark. Tally marks form a histogram.	1*b.* See Fig. 46.19
c. Record range of each subsample on form in terms of number of cells between lowest and highest tally mark for each subgroup.	1*c.* See Fig. 46.19 under "range" on right side
d. Calculate grand mean $\bar{\bar{X}}$ from frequency distribution in terms of line numbers above (+) and below (−) arbitrary origin taken as the zero cell.	1*d.* Zero cell shown as arrow in Fig. 46.19 $\bar{\bar{X}} = +0.14$

TABLE 46.16 Variables Plans for Percent Defective Lot Plot (*Continued*)

Summary of plan	Calculations
e. Draw $\overline{\overline{X}}$ on chart in terms of line numbers.	1*e.* $\overline{\overline{X}}$ drawn 0.14 cell widths above middle of zero cell
f. Estimate 3σ of line numbers from average of subsample ranges $3\hat{\sigma} = 3\overline{R}/d_2 = 1.29\,\overline{R}$	1*f.* $3\hat{\sigma} = 1.29\left(\frac{51}{10}\right) = 6.6$
g. Label $\overline{\overline{X}} \pm 3\sigma$ in terms of line numbers as (1) ULL = Upper Lot Limit ($\overline{\overline{X}} + 3\hat{\sigma}$) (2) LLL = Lower Lot Limit ($\overline{\overline{X}} - 3\hat{\sigma}$)	1*g.* See Fig. 46.19 marked ULL and LLL
h. Draw specification limits on chart.	1*h.* See Fig. 46.19 marked SPEC
C. Decision criteria. 1. Acceptance criterion. *a.* Symmetric distribution well within specification limits—accept automatically. *b.* Symmetric distribution other than above: (1) Lot limits within specification—accept. (2) Lot limits outside specification—estimate proportion of product outside specification with special technique using code strip. If less than allowable value—accept. See reference below. *c.* Nonsymmetric, bimodal distributions, etc. Special technique provided for estimating proportion out of specification using code strip. See reference below. If less than allowable value—accept. 2. Rejection criterion: Reject otherwise.	
V. Action: Dispose of lot as indicated. The Lot plot form provides a useful communication device with vendor.	V. Reject the lot
VI. Characteristics: See source reference below.	
VII. Reference: Shainin, 1950.	

A special lot plot card is helpful in simplifying some of the calculations. Figure 46.19 shows the form filled out for the example given in Table 46.16.

Grand Lot Schemes. Acceptance inspection and compliance testing often require levels of protection for both the consumer and the producer that make for large sample size relative to lot size. A given sample size can, however, be made to apply to several lots jointly if the lots can be shown to be homogeneous. This reduces the economic impact of a necessarily large sample size. Grand lot schemes, as introduced by L. E. Simon (1941), can be used to effect such a reduction. Application of the grand lot scheme has been greatly simplified by incorporating graphical analysis of means procedures in verifying the homogeneity of a grand lot. In this way individual sublots are subjected to control chart analysis for uniformity in level and variation before they are combined into a grand lot. The resulting approach can be applied to attributes or variables data, is easy to use, provides high levels of protection economically, and can reduce sample size by as much as 80 percent. It may be applied to unique "one-off" lots, isolated lots from a continuing series, an isolated sequence of lots, or to a continuing series of lots. The procedure has been described in depth by Schilling (1979).

Sampling for Process Parameter. When process parameters are specified, sampling plans can be developed from analogous tests of significance with corresponding OC curves. These plans do not require percent nonconforming to be related to the process mean, since the specifications to which they are applied are not in terms of percent nonconforming. This means that assumptions of process distribution may not need to be as rigorously held as in variables plans for percent nonconforming.

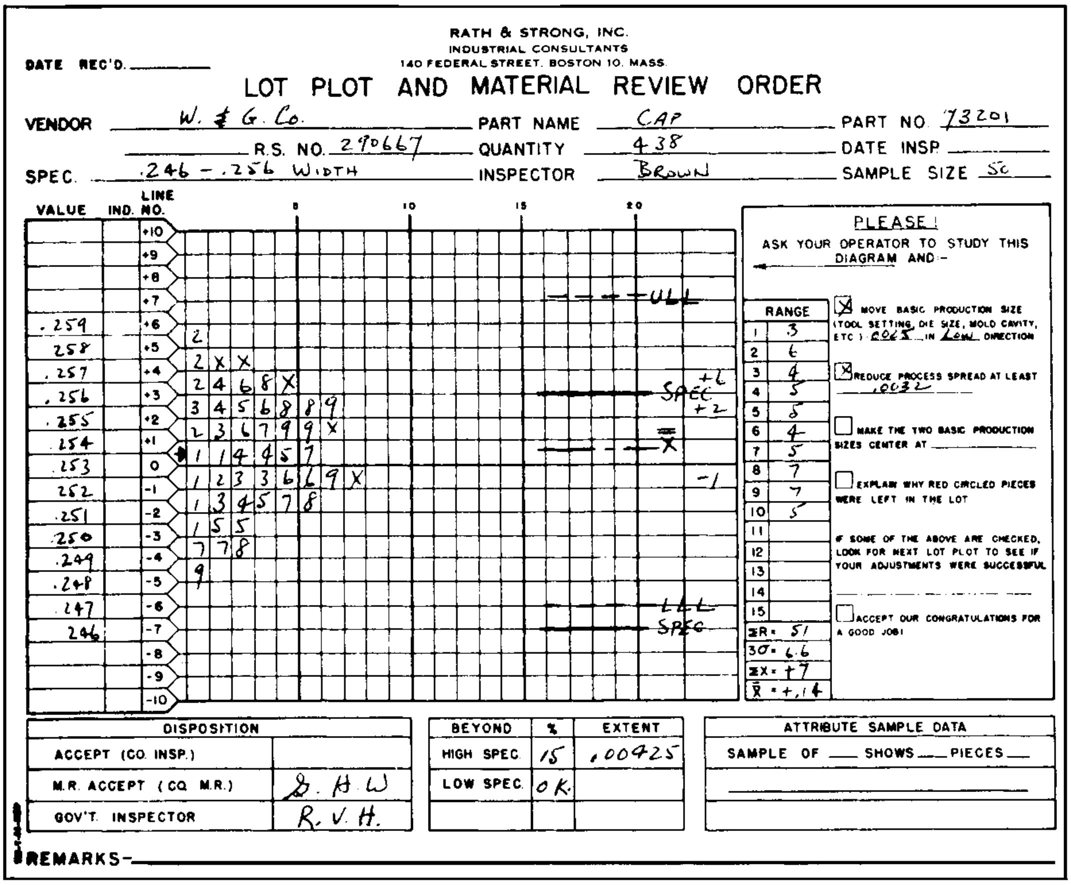

FIGURE 46.19 Illustration of lot plot method.

With specifications stated in terms of process location or variability, as measured by specific values of the mean μ or the process standard deviation σ, interest is centered not on fraction defective, but rather on controlling the parameters of the distribution of product to specified levels. From specifications of this type, it is usually possible to distinguish two process levels which may be used as bench marks as conceived by Freund (1957):

1. APL = acceptable process level—a process level which is acceptable and should be accepted most of the time by the plan.
2. RPL = rejectable process level—a process level which is rejectable and should be rejected most of the time by the plan.

The probability of acceptance for each of these process levels is usually specified as:

$$1-\alpha = \text{probability of acceptance at the APL}$$
$$\beta = \text{probability of acceptance at the RPL}$$

where α = producer's risk, β = consumer's risk.

Variables plans appropriate for this type of specification can be derived from the operating characteristic curves of appropriate tests of hypotheses. This is the case for single-sampling plans for process parameter which are, simply, appropriately constructed tests of hypotheses, e.g., testing the hypothesis that μ equals a specific value, against a one- or two-sided alternative. Thus, the statistical tests presented in Section 44 under Statistical Tests of Hypotheses can be used for this type of acceptance sampling plan.

Sequential sampling procedures have been developed which are particularly useful when levels of process parameter are specified. They usually offer a substantial decrease in sample size over competing procedures, although they may be difficult to administer. To use sequential sampling:

1. Take a sample of one measurement at a time.
2. Plot the cumulative sum T of an appropriate statistic against the sample number n.
3. Draw two lines

$$T_2 = h_2 + sn$$
$$T_1 = -h_1 + sn$$

   where the intercepts h_1 and h_2 are values associated with the plan used and the symbol s is not a standard deviation but is a constant computed from the values of the acceptable process level (APL) and the rejectable process level (RPL). The use of s here corresponds to its use in the literature of sequential sampling plans.
4. Continue to sample if the cumulative sum lies between these lines, and take the appropriate action indicated if the plot moves outside the lines.

Procedures for constructing such plans and determining appropriate values of h_1, h_2, and s are given in detail in Duncan (1974) and Schilling (1982).

Acceptance control charts offer a unique answer to the problem of sampling for process parameter and can be used to implement such plans when an acceptable process level and rejectable process level are defined in terms of the mean value. They satisfy the natural desire of inspection personnel to observe quality trends and to look upon sampling as a continuing process.

These charts incorporate predetermined values of consumer and producer risk in the limits and so provide the balanced protection for the interested parties that is often lacking in the use of a conventional control chart for product acceptance.

It is not necessary that the population of individual measurements be normally distributed. The distribution must be known so that acceptable and rejectable values of the mean can be calculated. The procedure then uses the normal distribution in the analysis of the sample mean because the distribution of sample means of samples of reasonable size may be regarded as normal for any distribution of individual measurements.

The procedure for implementing this technique is shown in Table 46.17. The acceptance control chart concept is shown in Figure 46.20, and an acceptance control chart example is shown in Figure 46.21. See Freund (1957) for additional details.

TABLE 46.17 Variables Plans for Process Parameter—Acceptance Control Charts

Example: The specification limits for electrical resistance are 620 and 680, the AQL 2.5%, and the standard deviation 13. Assuming a normal distribution of individual measurements, the mean may be as low as 620 + 1.96 (13), or 646, or as high as 680 − 1.96 (13), or 654. This pair of values represents the range of the acceptable process level (APL). It was decided that the rejectable process level would occur when 14% was beyond a specification limit. Thus, the range of RPL was 620 + 1.08 (13) and 680 − 1.08 (13), or 634 and 666. Should the lot be accepted if $\overline{X} = 647$?

Summary of plan	Calculations
I. Restrictions: None	
II. Necessary information (single-sided specification)	II.
A. σ = known standard deviation	*A.* $\sigma = 13$
B. μ_1 = APL (acceptable process level) with $P_a = 1 - a$	*B.* $\mu_1 = 654$, $P_a = 0.95$
C. μ_2 = RPL (rejectable process level) with $P_a = \beta$	*C.* $\mu_2 = 666$, $P_a = 0.10$
III. Selection of plan: See below	
IV. Elements	IV.
A. Sample size $n = \left[\frac{(z_\alpha + z_\beta)\,\sigma}{\mu_2 - \mu_1}\right]^2$ where z_p cuts off upper tail area of p in standard normal curve	*A.* $n = \left[\frac{(1.645 + 1.282)(13)}{12}\right]^2$ $n = 10.06 \sim 10$
B. Statistic: $\overline{X}$ = mean of sample of n	*B.* $\overline{X} = 647$
C. Decision criteria	*C.* $d = \left(\frac{1.645}{1.645 + 1.282}\right)(12) = 6.74$
1. Compute: $d = \frac{z_\alpha}{z_\alpha + z_\beta} \mid \mu_2 - \mu_1 \mid$	Upper ACL = 654 + 6.74 = 660.74
and set the acceptance control limit, ACL, a distance d from APL in the direction of the RPL. Sign of $\mid \mu_2 - \mu_1 \mid$ ignored.	By symmetry Lower ACL = 646 − 6.74 = 639.26
2. Construct an acceptance control chart (Fig. 46.20) and accept if $\overline{X}$ falls within acceptance control limits; reject otherwise. Double-sided specification chart shown (see remarks below). Use appropriate half of chart for single-sided specification.	Plot as in Fig. 46.21

TABLE 46.17 Variables Plans for Process Parameter—Acceptance Control Charts (*Continued*)

V. Action: Single lot disposed of as indicated by chart	V. Accept the lot
VI. Characteristics: Two points originally specified give indication of OC curve	
VII. Reference:	Freund, 1957.
VIII. Remarks	VIII.
A. Above formulas are for single upper or single lower process limits, or for both if (Upper ACL − Lower ACL) $\geq k\sigma / \sqrt{n}$ where: α = 0.05, k = 5; α = 0.01, k = 6; α = 0.001, k = 7 If (Upper ACL − Lower ACL) $< k\sigma / \sqrt{n}$, see above reference for appropriate factors	*A.* Can use both upper and lower limits since $(660.74 - 639.26) > \frac{(5)(13)}{\sqrt{10}}$ $21.48 > 20.56$
B. If standard deviation is estimated from control chart, see reference above for appropriate limits	
C. Advisable to run range chart with acceptance control chart to ensure stability of variation	

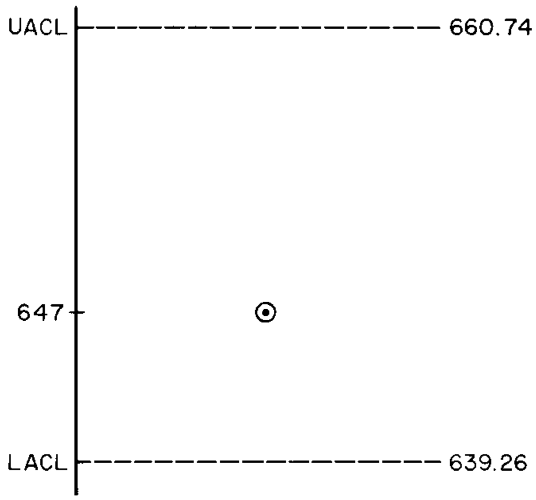

FIGURE 46.21 Acceptance control chart example.

Published Tables and Procedures. There is often much more involved in acceptance sampling than simple tests of hypotheses. Sampling plans applied individually to guard against an occasional discrepant lot can be reduced to hypothesis tests. Sampling plans, however, may be combined into sampling schemes, intended to achieve a predetermined objective. Sampling schemes, as overall strategies using one or more sampling plans, have their own measures such as AOQ (average outgoing quality) or ATI (average total inspection), not to be found in hypothesis testing. Thus, MIL-STD-105E (1989) and its civilian versions ANSI/ASQC Z1.4 (1993) and ISO 2859-1 (1989) together with their variables counterpart MIL- STD-414 (1957) and its modified civilian versions ANSI/ASQC Z1.9 (1993) and ISO 3951 (1989) are sampling schemes which specify the use of various sampling plans under well-defined rules. The latter assume the individual measurements to which they are applied to be normally distributed.

The variables standards allow for the use of three alternative measures of variability: known standard deviation σ, estimated standard deviation s, or average range $\bar{R}$. If the variability of the process producing the product is known and stable as verified by a control chart, it is profitable to use σ. The choice between s and $\bar{R}$, when σ is unknown, is an economic one. The range requires larger sample sizes but is easier to understand and compute. The operating characteristic curves given in the standards are based on the use of s, the σ and $\bar{R}$ plans having been matched, as closely as possible, to those using s.

MIL-STD-414 and ANSI/ASQC Z1.9 (1993) offer two alternative procedures. In addition to the method using an acceptance constant k, each standard also presents a procedure for estimating the proportion of defective in the lot from the variables evidence. The former method is called Form 1; the latter is called Form 2. Form 2 is the preferred procedure in MIL-STD-414 since the switching rules for reduced and tightened inspection cannot be applied unless the fraction defective of each lot is estimated from the sample. The switching rules of ANSI/ASQC Z1.9 were patterned after MIL-STD-105D (1964) and can be used with Form 1 or Form 2. ISO 3951 presents a graphical alternative to the Form 2 procedure to be used with switching rules also patterned after MIL-STD-105D.

Application of these variables schemes follows the pattern of MIL-STD-105E, which was also an AQL sampling scheme. Sample sizes are determined from lot size, and after choosing the measure of variability to be used and the Form of the acceptance procedure, appropriate acceptance limits are obtained from the standard. As in MIL-STD-105E, operating characteristic curves are included in the variables standards. These should be consulted before a specific plan is instituted. Note that the plans contained in MIL-STD-414 and MIL-STD-105E did not match; however the plans in ANSI Z1.9 are matched to MIL-STD-105E, ANSI/ASQC Z1.4 and ISO 2859-1 as are those in ISO 3951.

Since they are AQL schemes, these standards are based on an overall strategy which incorporates switching rules to move from normal to tightened or reduced inspection and return depending on the quality observed. These switching rules are indicated in Figure 46.22 for ANSI/ASQC Z1.9 (1993) and should be used if the standard is to be properly applied. The switching rules for ANSI/ASQC Z1.9 (1993) are the same as those of ANSI/ASQC Z1.4 (1993) except that the limit numbers for reduced inspection have been eliminated. This allows these standards to be readily interchanged. ISO 3951 switching rules differ slightly but are analogous to 105, while MIL-STD-414 rules are unique.

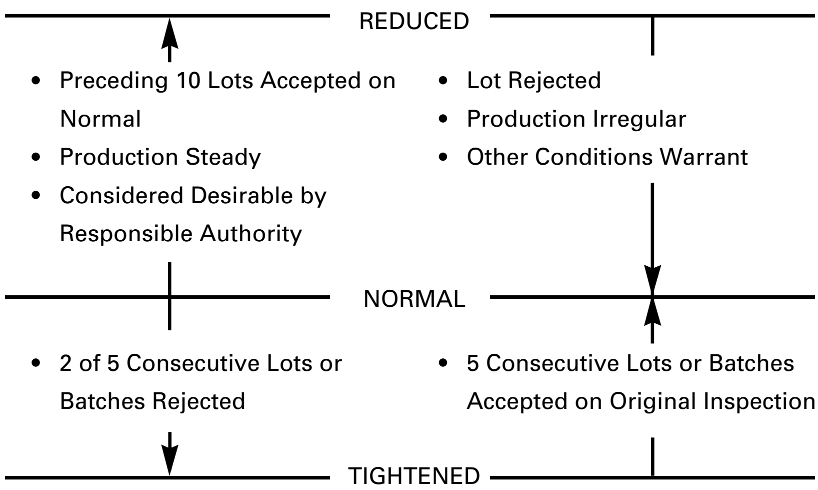

FIGURE 46.22 Switching rules for ANSI/ASQC Z1.9 (1993).

A check sequence for application of MIL-STD-414, ANSI/ASQC Z1.9 and ISO 3951 is given in Figure 46.23. Table 46.18 and Table 46.21 show the specific steps involved in application of the two forms using the sample standard deviation as a measure of variability. The graphical method of ISO 3951 is shown in Table 46.24 and Figure 46.24. Although the basic procedures of ANSI/ASQC Z1.9 (1993) and the graphical methods of ISO 3951 are analogous to MIL-STD-414, it should be noted that ranges, inspection levels, and tables of plans are different in these standards, since Z1.9 and ISO 3951 were revised to match MIL-STD-105D while 414 remained matched to MIL-STD-105A (1950). Procedures for upper, lower, and double specification limits are indicated in these tables together with appropriate references to the standard and an illustrative example. Modifications to the procedure, necessary when variability is measured by average range or a known standard deviation, are described. Table 46.25 shows the relationship of the statistics and procedures used under the various measures of variability allowed, for each of the forms.

These standards have a liberal supply of excellent examples. The reader is referred to the standards for more detailed examples of their applications.

Variables sequential plans for use with ISO 3951 (1989), ISO 2859-1 (1989), and their counterparts are provided in ISO 8423 (1991).

Mixed Plans. One disadvantage of variables is the fact that screened lots may at times be rejected by a sample $\bar{X}$ or s indicating percent defective to be high when, actually, the discrepant material has been eliminated. To prevent rejection of screened lots, double-sampling plans have been developed which use a variables criterion on the first sample and attributes on the second sample. Lots are accepted if they pass variables inspection; however, if they do not pass, a second sample is taken and the results are judged by an attributes criterion. In this way screened lots will not be rejected, since rejections are made only under the attributes part of the plan. These plans provide a more discriminating alternative when it is necessary to use $c = 0$. This type of plan was proposed by Dodge (1932). The plan has been discussed in some detail by Bowker and Goode (1952), Gregory and Resnikoff (1955), and Schilling and Dodge (1969). MIL-STD-414 and ANSI/ASQC Z1.9 (1993) allow for the use of such procedures, although only ANSI/ASQC Z1.9 (1993) is properly matched to MIL-STD-105E or ANSI/ASQC Z1.4 (1993) to allow for proper use of mixed plans. The steps involved are shown in Table 46.26.

RELIABILITY SAMPLING

Overview. Sampling plans for life and reliability testing are similar in concept and operation to the variables plans previously described. They differ to the extent that, when units are not all

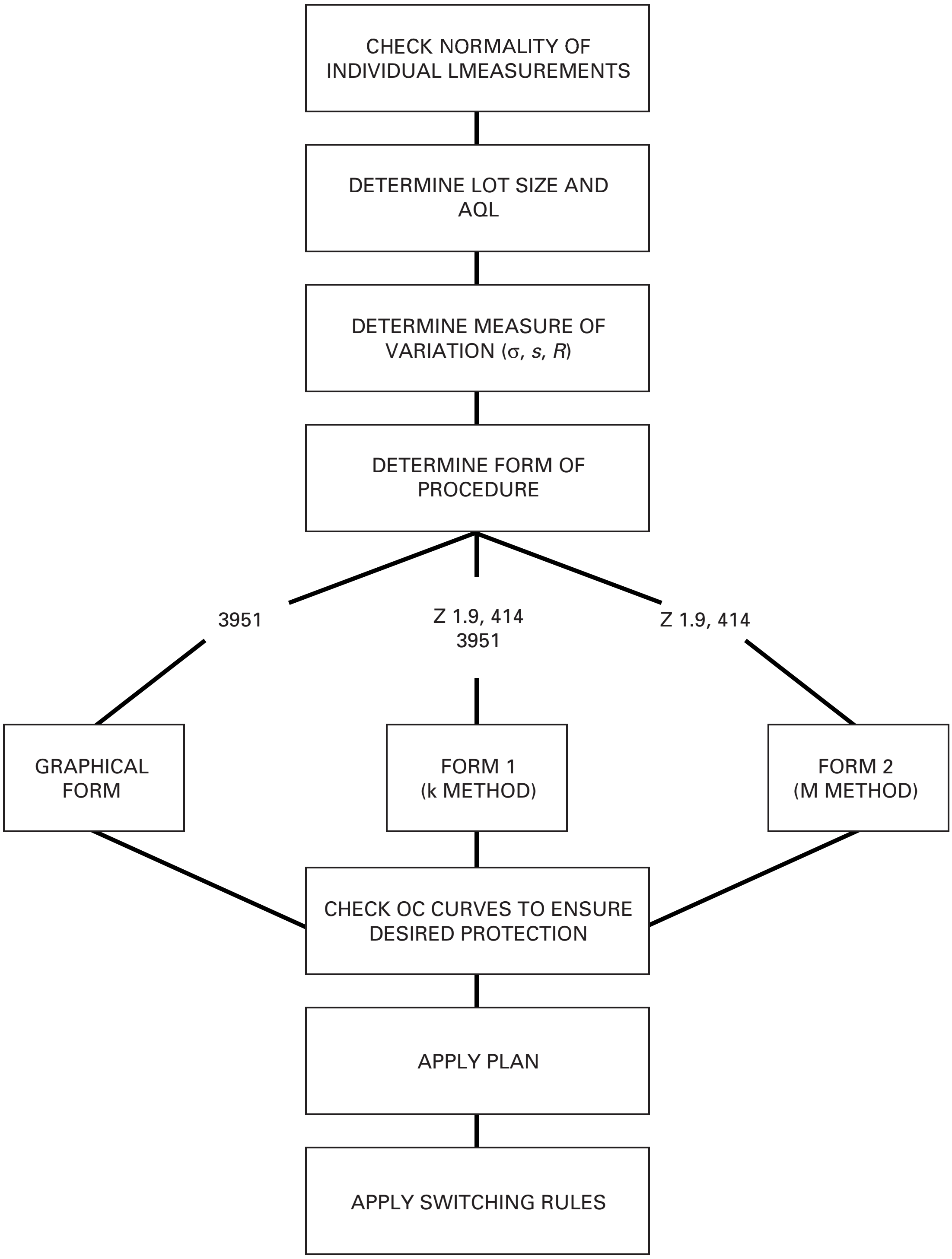

FIGURE 46.23 Check sequence for MIL-STD-414, ANSI/ASQC Z1.9, and ISO 3951.

TABLE 46.18 MIL-STD-414 (1957), ANSI/ASQC Z1.9 (1993) and ISO 3951 (1989), Variability Unknown, Standard Deviation Method, Form 1

Example: The specification for electrical resistance of a certain component is 620 to 680 ohms. A lot of 100 is submitted for inspection, normal inspection, with AQL = 2.5 percent. Should the lot be accepted if $\overline{X} = 647$ and $s = 17.22$?

Summary of plan	Calculations
I. Restrictions: Individual measurements normally distributed	MIL-STD-414 Example
II. Necessary information	II.
A. Lot size	*A.* Lot size=100
B. AQL	*B.* AQL=2.5%
C. Severity of inspection: normal, tightened, reduced	*C.* Normal inspection
III. Selection of plan	III.
A. Determine code letter (Table 46.19 for MIL-STD-414 only) from lot size and inspection level [normally, inspection level IV is used in MIL-STD-414 and inspection level II in ANSI/ASQC Z1.9 (1993) and ISO 3951 unless otherwise specified]	*A.* Code F
B. From code letter and AQL, determine (Table 46.20 for MIL-STD-414 only)	*B.*
1. Sample size = n	n=10
2. Acceptance constant = k	k=1.41
C. Double specification limits: obtain MSD = $F(U-\mathrm{L})$, where F is obtained from appropriate table in the Standard	*C.* MSD = 0.298(680 − 620) = 17.88
IV. Elements	IV.
A. Sample size: See above	*A.* $n = 10$
B. Statistic	*B.* $T_U = (680 - 647)/17.22 = 1.92$
1. Upper specification: $T_U = (U - \overline{X})/s$	$T_L = (647 - 620)/17.22 = 1.57$
2. Lower specification: $T_L = (\overline{X} - L)/s$	
3. Double specification: T_U and T_L	
C. Decision Criteria	*C.*
1. Acceptance criterion	
a. Upper specification: $T_U \geq k$	1.92 > 1.41
b. Lower specification: $T_L \geq k$	1.57 > 1.41
c. Double specification: $T_U \geq k$, $T_L \geq k$, and $s \leq$ MSD	17.22 < 17.88
2. Rejection criterion: Reject otherwise	
V. Action: Dispose of lot as indicated and refer to switching rules for next lot	V. Accept the lot
VI. Characteristics: OC curves given.	
VII. Reference: 1957 version of MIL-STD-414, ANSI/ ASQC Z1.9 (1992) and ISO 3951 (1989)	
VIII. Remarks (use appropriate tables from Standard)	
A. Range method	
1. Use $\overline{R}$ of subsamples of 5 if $n \geq 10$; use R if $n < 10$	
2. Substitute $\overline{R}$ for s in statistics	
3. Double specifications—use values of f (for MAR) in place of F (for MSD), where MAR is the maximum allowable range	
B. Variability known: Substitute σ for s in statistics	
C. ISO 3951 uses a special procedure for separate AQLs	

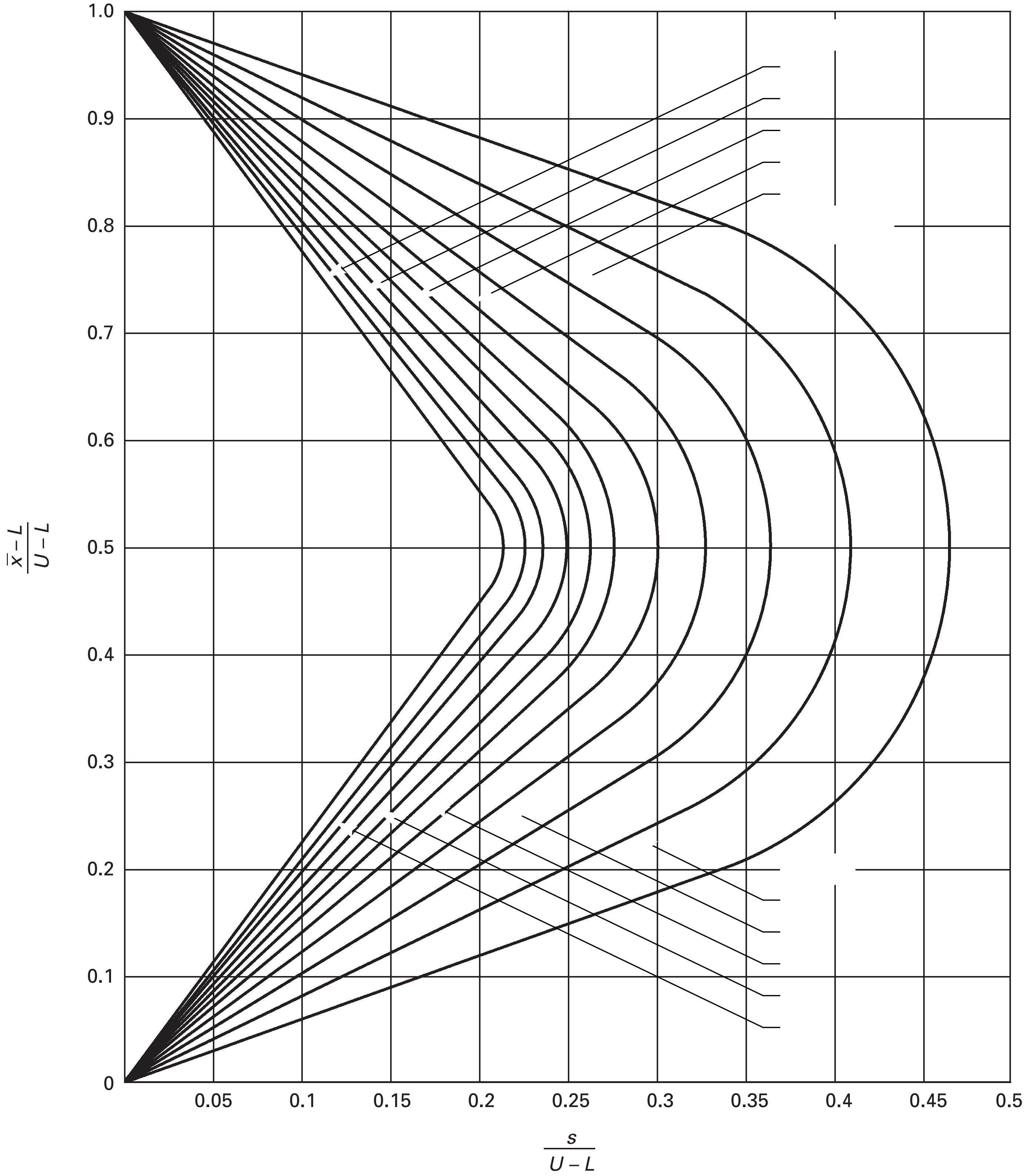
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
$\frac{\bar{x} - L}{U - L}$
$\frac{s}{U - L}$

TABLE 46.19 MIL-STD-414 Sample Size Code Letters

Lot size	Inspection levels I	II	III	IV	V
3–8	B	B	B	B	C
9–15	B	B	B	B	D
16–25	B	B	B	C	E
26–40	B	B	B	D	F
41–65	B	B	C	E	G
66–110	B	B	D	F	H
111–180	B	C	E	G	I
181–300	B	D	F	H	J
301–500	C	E	G	I	K
501–800	D	F	H	J	L
801–1,300	E	G	I	K	L
1,301–3,200	F	H	J	L	M
3,201–8,000	G	I	L	M	N
8,001–22,000	H	J	M	N	O
22,001–110,000	I	K	N	O	P
110,001–550,000	I	K	O	P	Q
550,001 and over	I	K	P	Q	Q

*Sample size code letters given in body of table are applicable when the indicated inspection levels are to be used.

run to failure, the length of the test becomes an important parameter determining the characteristics of the procedure. Further, time to failure tends to conform naturally to skewed distributions such as the exponential or as approximated by the Weibull. Accordingly, many life test plans are based on these distributions. When time to failure is normally distributed and all units tested are run to failure, the variables plans assuming normality, discussed above, apply; attributes plans such as ANSI/ASQC Z1.4 or ISO 2859 may also be used. Life tests, terminated before all units have failed, may be

1. Failure terminated—a given sample size n is tested until the rth failure occurs. The test is then terminated.
2. Time terminated—a given sample size n is tested until a preassigned termination time T is reached. The test is then terminated.

Furthermore, these tests may be based upon specifications written in terms of one of the following characteristics:

1. *Mean life:* The mean life of the product
2. *Hazard rate:* Instantaneous failure rate at some specified time t
3. *Reliable life:* The life beyond which some specified proportion of items in the lot or population will survive
4. *Failure rate* (FR or λ): The percentage of failures per unit time (say 1000 hours of test)

Relation of Life Characteristics. Specification and test of various life characteristics are intimately related. Tables 46.27 and 46.28 will be found useful in converting life test characteristics. Formulas for various characteristics are shown in terms of mean life μ. Thus, using the tables, it will be found that a specification of mean life $\mu = 1000$ hours for a Weibull distribution with $\beta = 2$ is

TABLE 46.21 MIL-STD-414 and ANSI/ASQC Z1.9 (1993), Variability Unknown, Standard Deviation Method, Form 2 (*Continued*)

equivalent to a hazard rate of 0.000157 at 100 hours or to a reliable life of 99.22 percent surviving at 100 hours.

Exponential Distribution: H108. *Quality Control and Reliability Handbook* MIL-HDBK-108 (1960) is widely used and presents a set of life test and reliability plans based on the exponential model for time to failure. The plans contained therein are intended for use when mean time to failure β is specified[2] in terms of acceptable mean life β_0 and unacceptable mean life β_1. Testing may be conducted:

With replacement: Units replaced when failure occurs. Test time continues to be accumulated on replacement unit.

Without replacement: Units not replaced upon failure.

The handbook contains three types of plans:

1. *Life tests terminated upon occurrence of a preassigned number of failures:* Here, n units are tested until r failures occur. The average life is calculated and compared with an acceptable value defined by the plan, and a decision is made.
2. *Life tests terminated at a preassigned time:* Here, n units are tested for a specified length of time T. If T is reached before r failures occur, the test is stopped and the lot accepted. If r failures occur before T is reached, the test is stopped and the lot rejected.
3. *Sequential life testing plans:* Here n units are placed on test, and time and failures are recorded until sufficient data are accumulated to reach a decision at specified risk levels. Periodically

[2]Note that elsewhere in this handbook the mean value is described by the symbol μ.

TABLE 46.22 MIL-STD-414, Master Table for Normal and Tightened Inspection for Plans Based on Variability Unknown, Standard Deviation Method

(Double specification limit and form 2, single specification limit)

Sample size code letter	Sample size	Acceptable quality levels (normal inspection)													
		0.04	0.065	0.10	0.15	0.25	0.40	0.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00
		M	*M*	*M*	*M*	*M*	*M*	*M*	*M*	*M*	*M*	*M*	*M*	*M*	*M*
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	7.59	18.86	26.94	33.69	40.47
C	4	↓	↓	↓	↓	↓	↓	↓	1.53	5.50	10.92	16.45	22.86	29.45	36.90
D	5	↓	↓	↓	↓	↓	↓	1.33	3.32	5.83	9.80	14.39	20.19	26.56	33.99
E	7	↓	↓	↓	↓	0.422	1.06	2.14	3.55	5.35	8.40	12.20	17.35	23.29	30.50
F	10	↓	↓	↓	0.349	0.716	1.30	2.17	3.26	4.77	7.29	10.54	15.17	20.74	27.57
G	15	0.099	0.186	0.312	0.503	0.818	1.31	2.11	3.05	4.31	6.56	9.46	13.71	18.94	25.61
H	20	0.135	0.228	0.365	0.544	0.846	1.29	2.05	2.95	4.09	6.17	8.92	12.99	18.03	24.53
I	25	0.155	0.250	0.380	0.551	0.877	1.29	2.00	2.86	3.97	5.97	8.63	12.57	17.51	23.97
J	30	0.179	0.280	0.413	0.581	0.879	1.29	1.98	2.83	3.91	5.86	8.47	12.36	17.24	23.58
K	35	0.170	0.264	0.388	0.535	0.847	1.23	1.87	2.68	3.70	5.57	8.10	11.87	16.65	22.91
L	40	0.179	0.275	0.401	0.566	0.873	1.26	1.88	2.71	3.72	5.58	8.09	11.85	16.61	22.86
M	50	0.163	0.250	0.363	0.503	0.789	1.17	1.71	2.49	3.45	5.20	7.61	11.23	15.87	22.00
N	75	0.147	0.228	0.330	0.467	0.720	1.07	1.60	2.29	3.20	4.87	7.15	10.63	15.13	21.11
O	100	0.145	0.220	0.317	0.447	0.689	1.02	1.53	2.20	3.07	4.69	6.91	10.32	14.75	20.66
P	150	0.134	0.203	0.293	0.413	0.638	0.949	1.43	2.05	2.89	4.43	6.57	9.88	14.20	20.02
Q	200	0.135	0.204	0.294	0.414	0.637	0.945	1.42	2.04	2.87	4.40	6.53	9.81	14.12	19.92
		0.065	0.10	0.15	0.25	0.40	0.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00	
		Acceptable quality levels (tightened inspection)													

Note: All AQL and table values are in percent defective.

↓ Use first sampling plan below arrow, that is, both sample size as well as *M* value. When sample size equals or exceeds lot size, every item in the lot must be inspected.

Source: 1957 version of MIL-STD-414.

TABLE 46.24 ISO 3951 (1989) Variability Unknown, Standard Deviation Method, Double Specifications with Combined AQL, Graphical Procedures

Example: The specification for electrical resistance of a certain component is 620 to 680 ohms. A lot of 100 is submitted for inspection, inspection level II, normal inspection, with AQL = 2.5 percent. Should the lot be accepted if $\overline{X} = 647$ and $s = 17.227$?

Summary of plan	Calculations
I. Restrictions: Individual measurements, normally distributed	
II. Necessary information	II.
A. Lot size	A. Lot size = 100
B. AQL (combined for both specification limits)	B. AQL = 2.5 percent
C. Severity of inspection	C. Normal inspection
III. Selection of plan	III.
A. Determine code letter from lot size and inspection level; usually inspection level II is used	A. Code F
B. From code letter and AQL, determine sample size	B. $n = 10$
IV. Elements	IV.
A. Sample size: See above	A. $n = 10$
B. Statistic: $\dfrac{s}{U - \mathrm{L}}$ and $\dfrac{\overline{X} - \mathrm{L}}{\mathrm{U} - \mathrm{L}}$	B. 0.287 and 0.450
C. Decision criteria	C.
1. If $s >$ MSSD $= f_s(U - L)$ reject outright	1. $17.22 < .298(60) = 17.88$
2. Select acceptance chart corresponding to code letter and AQL and plot above statistics on chart	2. See Figure 46.24
3. Acceptance criterion: Point plots inside accept zone	3. Point plots inside accept zone. Accept the lot
4. Rejection criterion: Point plots outside accept zone	
V. Action: Dispose of lot as indicated and refer to switching rules for next lot; see above	
VI. Characteristics: OC curves given	
VII. Reference: ISO 3951 (1989)	
VIII. Remarks: Analogous methods are available for average range $\overline{R}$ or known standard deviation σ, by substituting $\overline{R}$ or σ in the above.	

throughout the test, the time accumulated on all units is calculated and compared with the acceptable amount of time for the total number of failures accumulated up to the time of observation. If the total time exceeds the limit for acceptance, the lot is accepted; if the total time is less than the limit for rejection, the lot is rejected. If the total time falls between the two limits, the test is continued.

Plans are given for various values of the consumer and producer risks, and operating characteristic curves are provided for life tests terminated at a preassigned number of failures or preassigned time. Special tables are also included showing the expected saving in test time by increasing the sample size or by testing with replacement of failed units.

TABLE 46.25 Application of MIL-STD-414, ANSI/ASQC Z1.9 (1993), and ISO 3951

Step	Section	Form 1	Form 2	Graphical
Preparatory		Obtain k and n from appropriate tables	Obtain M and n from appropriate tables	Select appropriate acceptance chart
Determine criteria	Section B (s)	$T_U = \frac{U - \overline{X}}{s}$ $T_L = \frac{\overline{X} - L}{s}$	$Q_U = \frac{U - \overline{X}}{s}$ $Q_L = \frac{\overline{X} - L}{s}$	$A = \frac{\overline{X} - L}{U - L}$ $V = \frac{s}{U - L}$
	Section C ($\overline{R}$)	$T_U = \frac{U - \overline{X}}{\overline{R}}$ $T_L = \frac{\overline{X} - L}{\overline{R}}$	$Q_U = \frac{(U - \overline{X})c}{\overline{R}}$ $Q_L = \frac{(\overline{X} - L)c}{\overline{R}}$	$A = \frac{\overline{X} - L}{U - L}$ $V = \frac{\overline{R}}{U - L}$
	Section D (σ)	$T_U = \frac{U - \overline{X}}{\sigma}$ $T_L = \frac{\overline{X} - L}{\sigma}$	$Q_U = \frac{(U - \overline{X})v}{\sigma}$ $Q_L = \frac{(\overline{X} - L)v}{\sigma}$	$A = \frac{\overline{X} - L}{U - L}$ $V = \frac{\sigma}{U - L}$
Estimation			Enter table with n and Q_U or Q_L to get p_U or p_L	
Action	Single specification	Accept if $T_U \geq k$ or $T_L \geq k$	Accept if $p_U \leq M$ or $p_L \leq M$	
	Double specification	Accept if* $T_U \geq k$, $T_L \geq k$ and s < MSD or $\overline{R}$ < MAR	Accept if $p_U + p_L \leq M$	Accept if (A, V) plots inside acceptance curve
Standard/ Specification		414, 1.9/single, double* 3951/single, double with separate AQL's	414, 1.9/single, double	3951 double with combined AQL

Note: c = scale factor; $v = \sqrt{\dfrac{n}{n-1}}$

* Not official procedure.

Source: ANSI/ASQC Z1.9 (1993), ISO 3951 (1989).

Weibull Distribution: TR-3, TR-4, TR-6, TR-7. United States Defense Department quality control and reliability technical reports TR-3 (1961), TR-4 (1962), TR-6 (1963), and TR-7 (1965) have presented sampling plans based on an underlying Weibull distribution of individual measurement t.

Plots on probability paper or goodness-of-fit tests must be used to assure that individual measurements are distributed according to the Weibull model. See Wadsworth, Stephens, and Godfrey (1982). When this distribution is found to be an appropriate approximation to the failure distribution, methods are available to characterize a product or a process in terms of the three parameters of the Weibull distribution (see Sections 44 and 48 under the Weibull Distribution). These include probability plots and also point and interval estimates. Sampling plans are available for use with the Weibull approximation, which assumes shape and location parameters to be known. The plans are given in the technical reports mentioned above and are based in the following criteria:

TABLE 46.26 Variables Plans for Percent Defective, Mixed Variables—Attributes, ANSI/ASQC Z1.9 (1993)

Example: The specification for electrical resistance of a certain component is 620 to 680 ohms. A lot of 100 is submitted for inspection, inspection level II, normal inspection, with AQL = 2.5 percent. Should the lot be accepted if $\overline{X}$ = 647 and s = 17.22?

Summary of plan	Calculations
I. Restrictions: Measurements normally distributed	
II. Necessary information *A.* Lot size *B.* AQL	II. *A.* Lot size = 100 *B.* AQL = 2.5%
III. Selection of plan *A.* Using AQL and Lot Size, select appropriate variables plan from ANSI/ASQC Z1.9 (1993), Normal Inspection *B.* Using AQL and Lot Size, select single-sampling attributes plan from ANSI/ASQC Z1.4 (1993), using Tightened Inspection	III. *A.* n =10 M=7.26 *B.* ANSI/ASQC Z1.4 gives Code F n=32 c=1
IV. Elements *A.* Sample size: See above; use items drawn in first sample as part of second sample *B.* Statistic: Use appropriate statistics from ANSI/ASQC Z1.9 (1993) and ANSI/ASQC Z1.4 (1993), as indicated in each standard	IV. *A.* First sample: n=10 Second sample: n=(32−10) n=22
C. Decision criteria 1. Apply ANSI/ASQC Z1.9 (1993) plan *a.* Accept lot if plan accepts *b.* Otherwise, apply ANSI/ASQC Z1.4 (1993) plan, taking additional samples to satisfy sample size requirements 2. Apply ANSI/ASQC Z1.4 (1993) plan if necessary *a.* Accept lot if plan accepts *b.* Otherwise reject lot	*C.* Table 46.21 indicates ANSI/ASQC Z1.9 (1993) plan accepts; if the plan rejected the lot, an additional 22 samples would be drawn and the ANSI/ASQC Z1.4 (1993) plan applied to the number of defectives in the combined sample of 32
V. Action *A.* Dispose of lot as indicated	V. Accept the lot
VI. Characteristics *A.* Since the procedure outlined is a “dependent” mixed plan, see the following references: 1. σ known: Schilling and Dodge (1969) 2. σ unknown: Gregory and Resnikoff (1955) 3. Approximation for σ unknown: Bowker and Goode (1952)	

Source: ANSI/ASQC Z1.9 (1993).

TABLE 46.27 Life Characteristics for Two Failure Distributions

$$\text{Exponential } f(t) = \frac{1}{\mu} e^{-t/\mu}$$

$$\text{Weibull* } f(t) = \frac{\beta t^{\beta-1}}{\eta^{\beta}} e^{-(t/\eta)^{\beta}} \text{ where } \mu = \eta\Gamma\left(1 + \frac{1}{\beta}\right)$$

Life characteristic	Exponential	Weibull
Proportion $F(t)$ failing before time t	$F(t) = 1 - e^{-t/\mu}$	$F(t) = 1 - e^{-g(t/\mu)^{\beta}}$
Proportion $R(t)$ of population surviving to time t	$R(t) = e^{-t/\mu}$	$R(t) = e^{-g(t/\mu)^{\beta}}$
Mean life, ML or mean time between failures	μ	μ
Hazard rate, $Z(t)$, instantaneous failure rate at time t	$Z(t) = \frac{1}{\mu}$	$Z(t) = \frac{\beta g t^{\beta-1}}{\mu^{\beta}}$
Cumulative hazard rate $M(t)$ for period 0 to t	$M(t) = \frac{t}{\mu}$	$M(t) = \frac{g t^{\beta}}{\mu^{\beta}}$
Failure rate λ or average hazard rate period 0 to t, $m(t)$	$\lambda = \frac{1}{\mu}$	$m(t) = \frac{g t^{\beta-1}}{\mu^{\beta}}$

*Weibull parameters explained in Section 48. The formulas given here are those of H108 (exponential) and TR-3 (Weiball).

TABLE 46.28 Values of $g=[\Gamma(1+1/\beta)]^{\beta}$ for Weibull Distribution*

β	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	· · ·	4.5287	2.6052	1.9498	1.6167	1.4142	1.2778	1.1794	1.1051	1.0468
1.0	1.0000	0.9615	0.9292	0.9018	0.8782	0.8577	0.8397	0.8238	0.8096	0.7969
2.0	0.7854	0.7750	0.7655	0.7568	0.7489	0.7415	0.7348	0.7285	0.7226	0.7172
3.0	0.7121	0.7073	0.7028	0.6986	0.6947	0.6909	0.6874	0.6840	0.6809	0.6778

β	0.33	0.67	1.33	1.67	3.33	4.00	5.00
g	1.8171	1.2090	0.8936	0.8289	0.6973	0.6750	0.6525

*The columns of this table are subdivisions of the rows. Thus when β = 1.2, the value of g is 0.9292.

1. Mean life criterion (TR-3)
2. Hazard rate criterion (TR-4)
3. Reliable life criterion (TR-6)
4. All three (TR-7)

The tables cover a wide range of the family of Weibull distributions by providing plans for shape parameter β from 1/3 to 5. The technical reports abound in excellent examples and detailed descriptions of the methods involved.

Technical report TR-7 (1965) provides factors and procedures for adapting MIL-STD-105D plans to life and reliability testing when a Weibull distribution of failure times can be assumed. Tables of the appropriate conversion factors are provided for the following criteria:

Table	Criterion	Conversion factor
1	Mean life	$(t/\mu) \times 100$
2	Hazard rate	$tZ(t) \times 100$
3	Reliable life ($r = 0.90$)	$(t/p) \times 100$
4	Reliable life ($r = 0.99$)	$(t/p) \times 100$

Each table is presented in three parts, each of which is indexed by 10 values of $\beta(\beta = 1/3, 1/2, 2/3, 1, 1\frac{1}{3}, 1\frac{2}{3}, 2, 2\frac{1}{2}, 3\frac{1}{2}, 4)$. TR-7 is used in a manner analogous to the other three technical reports.

BULK SAMPLING[3]

Bulk material may be of gaseous, liquid, or solid form. Usually it is sampled by taking increments of the material, blending these increments into a single composite sample, and then, if necessary, reducing this gross sample to a size suitable for laboratory testing.

If bulk material is packaged or comes in clearly demarked segments, if it is for all practical purposes uniform within the packages, but varying between packages, and if the quality of each package in the sample is measured, then the sampling theory developed for discrete units may be employed.

A special theoretical discussion is necessary for the sampling of bulk material:

1. If the packages are uniform but the increments from individual packages are not tested separately; instead they are physically composited, in part at least, to form one or more composite samples that are tested separately.
2. If the contents of the packages are not uniform so that the question of sampling error arises with respect to the increments taken from the packages.
3. If the bulk material is not packaged and sample increments have to be taken from a pile, a truck, a railroad car, or a conveyer belt.

In the above circumstances, the special aspects that make bulk sampling different from the sampling of discrete indivisible units are:

1. The possibility of physical compositing and the subsequent physical reduction (or subsampling) that is generally necessary.
2. The need in many cases to use a mechanical sampling device to attain the increments that are taken into the sample. In this case the increments are likely to be "created" by the use of the sampling device and cannot be viewed as preexisting.

Objectives of Bulk Sampling. In most cases the objective of sampling bulk material is to determine its mean quality. This may be for the purpose of pricing the material or for levying custom duties or other taxes, or for controlling a manufacturing process in which the bulk material may be used. It is conceivable that interest in bulk material may also at times center on the variability of the material; or, if it is packaged, on the percent defective; or on the extreme value attained by a segment or package, as described in ASTM (1968). In view of the limited space that is available, the discussion will be restricted to estimation of the mean quality of a material.

[3]This section is condensed from Section 25A of the third edition of this handbook, prepared by Acheson J. Duncan.

Special Terms and Concepts. A number of special terms and concepts are used in the sampling of bulk material. These are

1. *Lot:* The mass of bulk material the quality of which is under study—not to be confused with a statistical population.
2. *Segment:* Any specifically demarked portion of the lot, actual or hypothetical.
3. *Strata:* Segments of the lot that are likely to be differentiated with respect to the quality characteristic under study.
4. *Increment:* Any portion of the lot, generally smaller than a segment.
5. *Sample increments:* Those portions of the lot initially taken into the sample.
6. *Gross sample:* The totality of sample increments taken from the lot.
7. *Composite sample:* A mixture of two or more sample increments.
8. *Laboratory sample:* That part of a larger sample which is sent to the laboratory for test.
9. *Reduction:* The process by which the laboratory sample is obtained from a composite sample. It is a method of sampling the composite sample. It may take the form of hand-quartering or riffling or the like.
10. *Test-unit:* That quantity of the material which is of just sufficient size to make a measurement of the given quality characteristic.
11. *Quality of a test-unit:* The expected value of the hypothetically infinite number of given measurements that might be made on the test-unit. Any single measurement is a random sample of one from this infinite set. The analytical variance is the variance of such measurements on the infinite set.
12. *Mean of a lot:* If a lot is exhaustively divided into a set of M test-units, the mean of the qualities of these M test-units is designated the mean of the lot. It is postulated that this mean will be the same no matter how the M test-units are obtained. This assumes that there is no physical interaction between the quality of test-units and the method of division. See item 16 below.
13. *Mean of a segment (stratum, increment, composite sample, or laboratory sample):* Defined in a manner similar to that used to define the mean of a lot. It is assumed that the segment is so large relative to the size of a test- unit that any excess over the integral number of test-units contained in the segment can be theoretically ignored. If this is not true, then the quality of the fraction of a test-unit remaining is arbitrarily taken to be the quality of the mean of the segment minus this fraction.
14. *Uniformity:* A segment of bulk material will be said to be uniform if there is no variation in the segment. If, for example, every cubic centimeter of a material contains exactly the same number of "foreign particles," the density of these particles would be said to be uniform throughout the segment. See the note under item 15, however.
15. *Homogeneous:* A segment of bulk material will be said to be homogeneous with respect to a given quality characteristic if that characteristic is randomly distributed throughout the segment.

    Note: The character of being uniform or homogeneous is not independent of the size of the units considered. The number of foreign particles may be the same for every cubic meter of a material, and with respect to this size unit the material will be said to be uniform. For units of size 1 cubic centimeter, however, there may be considerable variation in the number of foreign particles, and for this size of unit the material would not be judged to be uniform. The same considerations are involved in the definition of homogeneity. The number of foreign particles per cubic meter could vary randomly from one cubic meter to the next, but within each cubic meter there might be considerable (intraclass) correlation between the number of foreign particles in the cubic centimeters that make up the cubic meter.
16. *Systematic physical bias:* If the property of the material is physically affected by the sampling device or method of sampling employed, the results will have a systematic bias. A boring or cutting device, for example, might generate sufficient heat to cause loss of moisture.

17. *Physical selection bias:* If a bulk material is a mixture of particles of different size, the sampling device may tend to select more of one size particle than another. This means that if a segment was exhaustively sampled by such a device, early samples would tend to have relatively more of certain size particles than later samples.

18. *Statistical bias:* A function of the observations that is used to estimate a characteristic of a lot, e.g., its mean, is termed a "statistic." A statistic is statistically biased if in many samples its mean value is not equal to the lot characteristic it is used to estimate.

Determination of the Amount of Sampling. Since the variance of a sample mean $\sigma_{\overline{X}}^2$ is a function of the amount of sampling, say the number of increments taken, then once a model has been adopted and a formula obtained for $\sigma_{\overline{X}}^2$, it becomes possible to determine the amount of sampling required to attain a confidence interval of a given width or to attain a specified probability of making a correct decision.

A similar approach is involved in determining the amount of sampling to get a desired set of risks for lot acceptance and rejection.

Models and Their Use. Sampling plans for discrete product have been cataloged in a number of tables. This has not yet been possible for bulk sampling, and instead, a "sampling model" must be created for each type of bulk material and the model used to determine the sample size and acceptance criteria for specific applications.

A bulk sampling model consists of a set of assumptions regarding the statistical properties of the material to be sampled plus a prescribed procedure for carrying out the sampling. A very simple model, for example, would be one in which it is assumed that the quality characteristics of the test-units in a lot are normally distributed and simple random sampling is used.

With the establishment of a model, a formula can generally be derived for the sampling variance of an estimate of the mean of a given lot. This must be uniquely derived based on the type of product, lot formation, and other factors. The reader is urged to consult the references for specific formulas (see especially Duncan, 1962). From an estimate of this sampling variance, confidence limits can be established for the lot mean, and/or a decision with given risk can be made about the acceptability of the lot.

Let the variance of a sample mean be denoted as $\sigma_{\overline{X}}^2$ and its estimate as $s_{\overline{X}}^2$. Then "0.95 confidence limits" for the mean of the lot will be given by

$$0.95 \text{ confidence limits for } \mu = \overline{X} \pm t_{0.025} S_{\overline{X}}$$

where μ is the mean of the lot, $\overline{X}$ is the sample mean, and $t_{0.025}$ is the 0.025 point of a t-distribution for the degrees of freedom involved in the determination of $s_{\overline{X}}^2$.

If a decision is to be made on the acceptability of a lot, a criterion for acceptability will take some such form as

$$\text{Accept if } t = \frac{\overline{X} - L}{s_{\overline{X}}} \text{ is positive or if it has a negative value numerically less than } t_{0.025}$$

where L is the lower specification limit on the product and $t_{0.025}$ is the 0.025 point of a t-distribution for the degrees of freedom involved in the determination of $s_{\overline{X}}^2$. See Grubbs and Coon (1954).

Models for Distinctly Segmented Bulk Material ("Within and Between" Models). Much bulk material comes in distinctly segmented form. It may be packaged in bags, bales, or cans, for example, or may come in carloads or truckloads.

For distinctly segmented material it can be established that the overall variance of individual test-units is, for a large number of segments each with a large number of test-units, approximately equal to the sum of the variance between segments and the average variance within segments. In what follows, the variance within segments is assumed the same for all segments.

Model 1A. Isolated Lots, Nonstratified Segments. For an isolated lot of distinctly segmented material, the sampling procedure will be to take an increment of m test-units from each of n segments, reduce each increment to a laboratory sample, and measure its quality X. The mean of the n test-units is taken as an estimate of the mean of the lot.

Model 1B. One of a Series of Lots. Suppose the current lot is one of a series of lots of distinctly segmented material and that estimates of the between and within variances have been made in a pilot study, together with estimates of the reduction variance and test variance. It will be assumed that the reduction variance yielded by the pilot study is valid for larger amounts than that used in the study.

With the given prior information, an estimate of the sampling variance for the current sample estimate of mean lot quality can be based on the pilot study, and there is need only for a current check on the continued validity of this study. Consequently, composite samples can be used requiring only a few measurements, and the cost of inspection of the current lot may be considerably curtailed.

The "Within and Between" Models: Stratified Segments. In some situations the quality characteristic of the bulk material may be stratified in that in each segment it may vary from layer to layer and is not randomly distributed throughout the segment. Difficulties in formulating a model for stratified segments of this kind can be overcome if the strata are reasonably parallel and if the increments taken from the sample segments are taken perpendicular to the strata and penetrate all strata. In taking a sample from a bale of wool, for example, a thief could cut a sample running vertically from top to bottom of the bale.

Models for Bulk Material Moving in a Stream. In many instances the bulk material to be sampled is moving in a stream, say on a conveyer belt. In such instances it is the common practice to take increments systematically from the stream, the increment being taken across the full width of the stream.

Isolated Lots. If increments are taken at random from the stream, we would have a simple random sample from the lot, and we could proceed much as indicated for isolated lots of distinctly segmented material.

Model 2A. A Stream of Lots: A Segregation Model. When a stream of bulk material persists for some time, with possible interruptions, determinations of quality and/or action decisions may have to be made for a number of lots. Here, a pilot study might be profitable.

The kind of pilot study needed will depend on the assumptions about the statistical properties of the material in the stream. If the quality of the material varies randomly in the stream, then a pilot study based on a number of randomly or systematically taken increments would be sufficient to determine the variance of the material, and this could be used to set up confidence limits for the means of subsequent lots or make decisions as to their acceptability, even though in each of these lots a single composite sample was taken.

Obtaining the Test-Units. Three factors are important in obtaining the test-units.

1. *The models discussed above assume random sampling:* Either the increments are picked at random (using, preferably, random numbers if the units can be identified) or the increments are selected systematically from material that is itself random. Random sampling may have to be undertaken while the material is in motion or being moved. If random sampling cannot be used, special efforts should be made to get a representative picture of a lot, noting strata and the like. Some element of randomness must be present in a sampling procedure to yield a formula for sampling variance.

2. *Grinding and mixing:* In the processing of bulk material and in the formation of composite samples, grinding and/or mixing may be employed in an attempt to attain homogeneity or at least to reduce the variability of the material.

If a material can be made homogeneous for the size increment that is to be used in subsequent sampling, then an increment of that size can be viewed as a random sample of the material. The attainment of homogeneity of bulk material in bags, cartons, barrels, etc., is thus a worthy objective when the contents of these containers are to be sampled by a thief or other sampling instruments. However, random sampling does not guarantee minimum sampling variation, and grinding and mixing may lead to a reduction in overall sampling variation without attaining homogeneity. See Cochran (1977).

Although grinding and mixing are aimed at reducing variability, these operations may in some circumstances cause segregation and thus increase variability.

3. *Reduction of a sample to test-units:* Measurements often may be made directly on the sample itself. However, sometimes a portion of the sample must be carefully reduced in either particle size or physical quantity to facilitate laboratory testing. The unreduced portion of the sample may be retained for subsequent reference for legal purposes or verification of results. An example of a technique is coning and quartering. The material is first crushed and placed in a conical pile, which is then flattened. The material is then separated into quarters and opposite quarters selected for further quartering or as the final test-units.

Bicking (1967) and Bicking et al. (1967) provide unique details on obtaining test-units for a large variety of specific bulk products.

Tests of Homogeneity. The homogeneity of bulk material may be tested by $\overline{X}$-charts and c-charts. For mixtures of particles that can be identified, local homogeneity may be tested by running a χ^2 short-distance test. See Shinner and Naor (1961).

BAYESIAN PROCEDURES

The concept that experience or analytical studies can yield prior frequency distributions of the quality of submitted lots and that these "prior" distributions can in turn be used to derive lot-by-lot sampling plans has gained some popularity in recent years. This is generally termed the "Bayesian approach." Bayes' theorem and the Bayesian decision theory concept are discussed in Section 44, Basic Statistical Methods. The general concept of the use of prior information is discussed in Section 23, under Degree of Inspection and Testing Needed.

Considerable literature exists on Bayesian sampling. However, a very limited number of sampling tables based on particular prior distributions of lot quality are available. Calvin (1984) provides procedures and tables for implementing Bayesian plans. Another example is the set of tables by Oliver and Springer (1972), which are based on the assumption of a Beta prior distribution with specific posterior risk to achieve minimum sample size. This avoids the problem of estimating cost parameters. It is generally true that a Bayesian plan requires a smaller sample size than does a conventional sampling plan with the same consumer's and producer's risk. Among others, Schaefer (1967) discusses single sampling plans by attributes using three prior distributions of lot quality. Given specified risks α and β, sampling plans which satisfied these risks and which minimized sample size were determined. For example, with a particular prior distribution, $n = 6$ and $c = 0$ gave protection equivalent to a conventional sampling plan with $n = 34$, $c = 0$.

Advantages similar to those quoted above are usually cited. However, one prime factor is frequently ignored: How good is the assumption on the prior distribution? Hald (1960) gives an extensive account of sampling plans based on discrete prior distributions of product quality. Hald also employs a simple economic model along with the discrete prior distribution and the hypergeometric sampling distribution to answer a number of basic questions on costs, relationship between sample size and lot size, etc. Schaefer (1964) also considers the Bayesian operating characteristic curve. Hald (1981) has also provided an excellent comparison of classical and Bayesian theory and methodology for attributes acceptance sampling. The Bayesian approach to sequential and nonsequential acceptance sampling has been described by Grimlin and Breipohl (1972). Most such plans incorporate cost data reflecting losses involved in the decision making process to which the plan is to be applied. An excellent review of Bayesian and non-Bayesian plans has been given by Wetherill and Chin (1975).

While the explicit specification of a prior distribution is characteristic of the classical Bayes approach, procedures have been developed to incorporate much of the philosophy and approach of Bayes without explicit specification of a prior distribution. These methods are based on the incorporation of past data into an empirical estimate of the prior, and hence the approach is called "empirical Bayes." An excellent description of this method of estimation has been presented by Krutchkoff (1972). Application of the empirical Bayes methodology to attributes sampling has been given by Martz (1975). Craig and Bland (1981) show how it can be used in variables sampling.

These ideas have been further extended by the application of shrinkage estimators to the empirical Bayes problem as described by Morris (1983). One outstanding application of such procedures is in the so-called "universal sampling plan" which has been utilized in the quality measurement plan (QMP) of American Telephone and Telegraph. Audit sample sizes are based on historical process control, economics, quality standards, and the heterogeneity of audit clusters as described by Hoadley (1981). Strictly speaking, the weights for the Stein shrinkage estimators utilized in the procedure are developed through a classical Bayes approach, and so this application is more properly viewed as Bayes empirical Bayes methodology. A method for estimating quality using data from both accepted and rejected lots based on QMP is given in Brush and Hoadley (1990).

Application of Bayesian methods has been hindered by the difficulty involved in correct assessment of the necessary prior distributions and in collecting and keeping current the required cost information. Clearly, greater reliance on prior empirical information and the potential of the computer for generating cost information should be helpful in this regard. Pitfalls in the selection of a prior distribution have been pointed out by Case and Keats (1982).

While studies have indicated that Bayesian schemes may be quite robust to errors in the prior distributions and loss functions, they nevertheless assume the prior to be stationary in the long-term sense (i.e., a process in control). Classical methods do not make this assumption. Bayesian plans are, in fact, quite application-specific, requiring extensive information and update for proper application, and like variables sampling, they are applied one characteristic at a time. Nevertheless, where appropriate, they provide yet another tool for economic sampling.

HOW TO SELECT THE PROPER SAMPLING PROCEDURES

The methods of acceptance sampling are many and varied. It is essential to select a sampling procedure appropriate to the acceptance sampling situation to which it is to be applied. This will depend upon the nature of the application itself, on quality history, and the extent of knowledge of the process and the producer. Indeed, according to Dodge (1950, p. 8):

> A product with a history of consistently good quality requires less inspection than one with no history or a history of erratic quality. Accordingly, it is good practice to include in inspection procedures provisions for reducing or increasing the amount of inspection, depending on the character and quantity of evidence at hand regarding the level of quality and the degree of control shown.

This will be discussed further in the Conclusion, below.

The steps involved in the selection and application of a sampling procedure are shown in Figure 46.25 taken from Schilling (1982). Emphasis here is on the feedback of information necessary for the proper application, modification, and evolution of sampling. As Ott and Schilling (1990) have pointed out:

> There are two standard procedures that, though often good in themselves, can serve to postpone careful analysis of the production process:

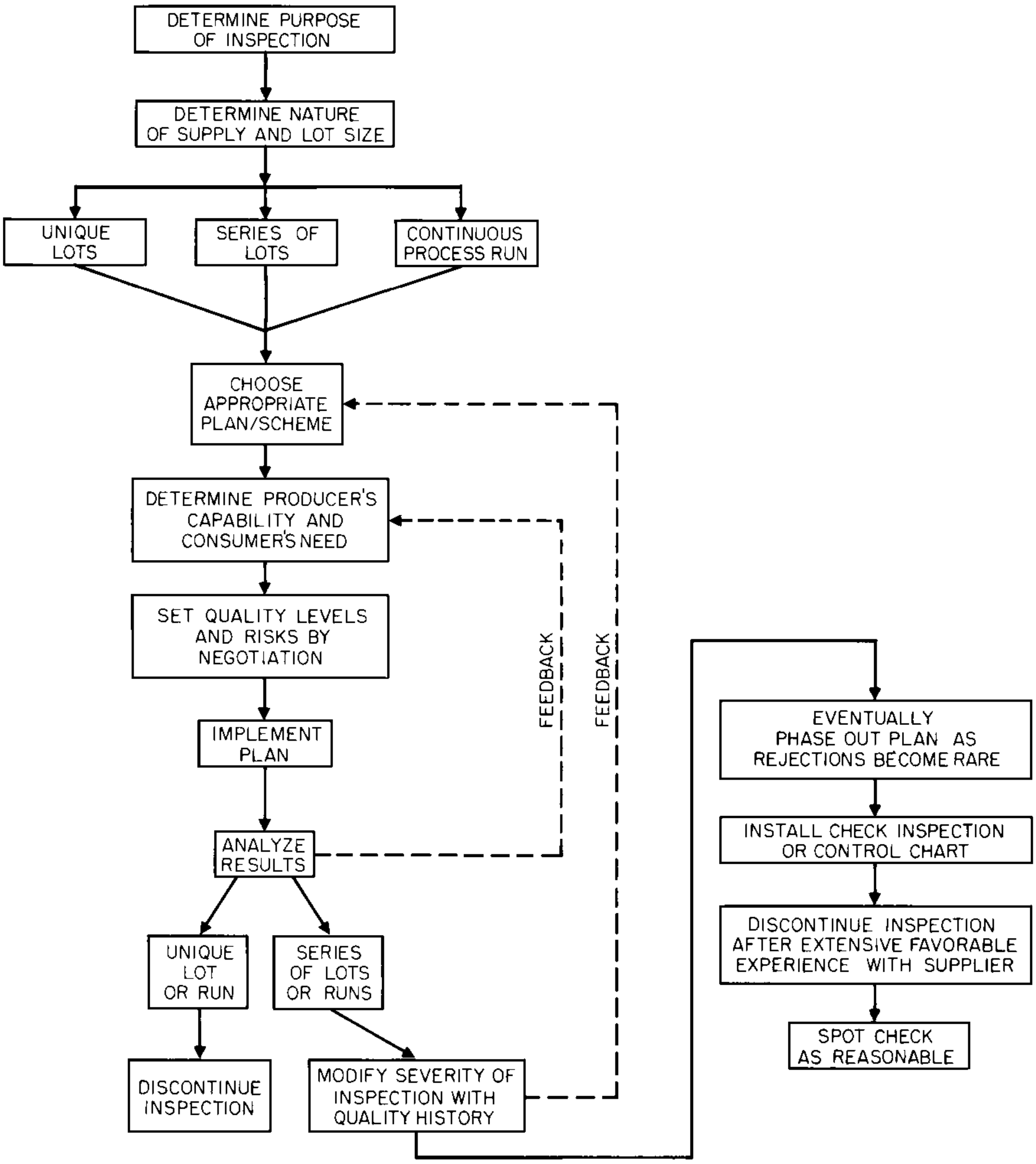

FIGURE 46.25 Check sequence for implementation of sampling procedure.

> 1. On-line inspection stations (100% screening). These can become a way of life.
> 2. On-line acceptance sampling plans which prevent excessively defective lots from proceeding on down the production line, but have no feedback procedure included.
>
> These procedures become bad when they allow or encourage carelessness in production. It gets easy for production to shrug off responsibility for quality and criticize inspection for letting bad quality proceed.

It is therefore necessary for sampling to be constantly subject to modification in a system of acceptance control. Each plan has a specific purpose for which it is to be applied. Table 46.29 shows the relation of plan and purpose for some of the plans which have been discussed. Those not discussed here will be found described in detail in Schilling (1982). These plans then are the elements involved in a system of acceptance control. Their effective use in a continuing effort to provide protection while reducing inspection and moving to process control is a function of the ingenuity, integrity, and industry of the user.

TABLE 46.29 Selection of Plan

Purpose	Supply	Attributes	Variables
Simple guarantee of producer's and consumer's quality levels at stated risks	Unique lot	Two-point plan (Type A)	Two-point plan (type B)
	Series of lots	Dodge-Roming LTPD Two-point plan (Type B)	Two-point plan (type B)
Maintain level of submitted quality at AQL or better	Series of lots	MIL-STD-105E ANSI/ASQC 1.4 ISO 2859	MIL-STD-414 ANSI/ASQC Z1.9 ISO 3951
Rectification guaranteeing AOQL to consumer	Series of lots	Dodge-Roming AOQL	Use measurements as go-no go
	Flow of individual units	CSP-1, 2, 3 Multilevel plan MIL-STD-1235B	
Reduced inspection after good history	Series of lots	Skip-lot Chain	Lot plot Mixed variable-attributes Narrow-limit gaging
Check inspection	Series of lots	Demerit rating	Acceptance control chart
Compliance to mandatory standards	Unique lot	Lot-sensitive plan	Mixed variables-attributes with $c = 0$
	Series of lots	TNT plan	
Reliability sampling	Unique lot	Two-point plan (type B)	MIL-HDBK-108 TR-7
	Series of lots	LTPD plan	TR-7 using MIL-STD-105D switching rules

Source: Schilling, (1982) p.569.

COMPUTER PROGRAMS FOR ACCEPTANCE SAMPLING

A variety of computer programs are available for application of specific sampling plans. They have become increasingly important, particularly in the application of selected plans and in the development of plans for specific applications. Some early fundamental programs are listed in Table 46.30. It should be noted, however, that it is questionable whether these programs will ever replace hard copy procedures and standards which play a vital role in the negotiations between producer and consumer and which are immediately available for in-plant meetings and discussions at any location.

A number of commercial packages intended for quality control applications contain acceptance sampling options. In addition, various programs have been presented in the literature. For example, Guenther (1984) presents a program for determining rectifying single sampling plans while Garrison and Hickey (1984) give sequential plans, both by attributes. A program for determining the ASN of curtailed attributes plans is given by Rutemiller and Schaefer (1985). McWilliams (1990) has provided a program which deals with various aspects of hypergeometric, Type A, plans including double sampling. McShane and Tumbull (1992) present a program for analysis of CSP-1 plans with finite run length. Goldberg (1992) gives a PC program for rapid application of the sequential approach to continuous sampling. A program for selecting quick switching systems will be found in Taylor (1996). These programs supply code. Implementation of plans on spread sheets is presented in Cox (1992–1993). Expert systems are discussed in Fard and Sabuncuoglu (1990) and Seongin (1993). Thus, the computer can be expected to enhance and diversify application of sampling plans.

TABLE 46.30 Elementary Computer Programs for Acceptance Sampling (Optional Input/Output in Brackets)

Purpose	Input	Output	Reference
Single sampling—derivation	p_1, p_2, α, β, (lot size, p)	Plan (ATI at p, AOQL)	Snyder and Storer, 1972
Double sampling—derivation	AQL, LTPD (lot size) (assumes $\alpha = 0.05$, $\beta = 0.10$)	Plans for n_2, $= n_1$ and $n_2 = 2n_1$ with AOQL (p, P_a, AOQ, ATI, ASN)	Chow, Dickinson, and Hughes, 1973 and 1975
Multiple sampling—derivation	AQL, LTPD (lot size) (assumes $\alpha = 0.05$, $\beta = 0.10$)	Plans for equal n with AOQL (p, P_a, AOQ, ATI, ASN)	Hughes, Dickinson, and Chow, 1973
Single, double, multiple sampling—evaluation (P_a given p or p given P_a) (hypergeometric, binomial, Poisson, normal)	Distribution, sample sizes, acceptance/rejection numbers, fractions defective to be evaluated, probabilities of acceptance to be determined (lot size)	Plan control table, p, P_a, ASM, AOQ, ATI	Schilling, Sheesley, and Nelson, 1978

TABLE 46.30 Elementary Computer Programs for Acceptance Sampling (Optional Input/Output in Brackets) (*Continued*)

CONCLUSION—MOVING FROM ACCEPTANCE SAMPLING TO ACCEPTANCE CONTROL

There is little control of quality in the application of a sampling plan to an individual lot. Such an occurrence is static, whereas control implies movement and direction. When used in a system of *acceptance control,* over the life of the product, however, the plan can provide:

Protection for the consumer

Protection for the producer

Accumulation of quality history

Feedback for process control

Pressure on the producer to improve the process

Acceptance control involves adjusting the acceptance sampling procedure to match existing conditions with the objective of eventually phasing out the inspection altogether. This is much as the old-time inspector varied the inspection depending on quality history. Thus, acceptance control is "a continuing strategy of selection, application and modification of acceptance sampling procedures to a changing inspection environment" [see Schilling (1982, p. 546)]. This involves a progression of sampling procedures applied as shown in Table 46.31.

These procedures are applied as appropriate over the lifetime of the product in a manner consistent with the improvement of quality and the reduction and elimination of inspection. This can be seen in Table 46.32. For a detailed discussion and example of moving to audit sampling, see Doganaksoy and Hahn (1994).

Modern manufacturing is not static. It involves the development, manufacture, and marketing of new products as well as an atmosphere of continuing cost reduction, process modification, and other forms of quality improvement. Changes are continually introduced into the production process which are deliberate, or unexpected from an unknown source. When this happens, acceptance sampling is necessary to provide protection for the producer and the consumer until the process can be brought into control. This may take days, months, or years. Under these circumstances, the inspection involved should be systematically varied in a system of acceptance control which will reduce inspection reciprocally with the increase of the learning curve in the specific application. This can provide the protection necessary for the implementation of proper process control in a manner portrayed roughly in Figure 46.26. Acceptance control and process control are synergistic in the sense that one supports proper use of the other. As Dodge (1969, p. 156) has pointed out:

> The "acceptance quality control system"…encompassed the concept of protecting the consumer from getting unacceptable defective material and encouraging the producer in the use of process quality control

TABLE 46.31 Progression of Attribute Sampling Procedures

	Quality history			
Past results	Little	Moderate	Extensive	Criterion
Excellent	AQL plan	Chain	Demerit rating or remove inspection	Almost no (<1%) lots rejected
Average	Rectification or LTPD plan	AQL plan	Chain	Few (<10%) lots rejected
Poor	100% inspection	Rectification or LTPD plan	Discontinue acceptance	Many (≥10%) lots rejected
Amount	Fewer than 10 lots	10–50 lots	More than 50 lots	

TABLE 46.32 Life Cycle of Acceptance Control Application

Stage	Step	Method
Preparatory	Choose plan appropriate to purpose	Analysis of quality system to define the exact need for the procedure
	Determine producer capability	Process performance evaluation using control charts
	Determine consumer needs	Process capability study using control charts
	Set quality levels and risks	Economic analysis and negotiation
	Determine plan	Standard procedures if possible
Initiation	Train inspector	Include plan, procedure, records, and action
	Apply plan properly	Ensure random sampling
	Analyze results	Keep records and control charts
Operational	Assess protection	Periodically check quality history and OC curves
	Adjust plan	When possible change severity to reflect quality history and cost
	Decrease sample size if warranted	Modify to use appropriate sampling plans taking advantage of credibility of supplier with cumulative results
Phase out	Eliminate inspection effort where possible	Use demerit rating or check inspection procedures when quality is consistently good Keep control charts
Elimination	Spot check only	Remove all inspection when warranted by extensive favorable history

Source: Schilling, 1982, p. 566.

by: varying the quantity and severity of acceptance inspections in direct relation to the importance of the characteristics inspected, and in inverse relation to the goodness of the quality level as indicated by these inspections.

It is in this sense that acceptance control is an essential part of the quality system.

REFERENCES

ANSI/ASQC Q3 (1988). *Sampling Procedures and Tables for Inspection of Isolated Lots by Attributes,* American Society for Quality, Milwaukee.

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