Integrating Experimental Design and Statistical Control for Quality Improvement

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As a result of a successful case study, we advance a methodology for combining design of experiments (DOE) and cumulative score (Cuscore) charts to control and monitor an industrial process subject to a feedback control scheme. We use DOE to screen out the most important variable from a larger set of variables. We use the critical variable found in the DOE phase as a compensating factor in a feedback controller. We derive the proper Cuscore statistic that will identify spike signals in a dynamic system that is subject to feedback control. We then apply this Cuscore statistic to the output error from the control scheme. We illustrate the effectiveness of this process improvement methodology using an industrial case study where we identify and eliminate a recurring output quality problem in a pleating and gluing manufacturing operation.

Introduction

THE manufacturing environment in which quality engineering is practiced is changing rapidly, with many companies facing higher demands with the introduction of new systems and new products. System transitions are becoming a more significant part of overall operations. Furthermore, there is increased pressure for quality engineering as well as other manufacturing activities to support the economic objectives and profitability of the firm. Quality engineers need more tools to cope with these changes and to meet the intense international competition.

In this paper, we discuss an industrial case where system transitions occur frequently. HomeWindows manufactures several types of window blinds including horizontal and vertical blinds, and wood and fabric blinds. (This case is based on an actual company; however, for purposes of confidentiality we have changed identifying information.) It is a vertically integrated company that also produces many of the components, such as metal railings and plastic levers, needed for assembly.

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There have been several approaches to problem solving and improving quality in industry. Taguchi's methods included the use of a loss function to relate the quality of a product with profit. He also used orthogonal arrays for experimentation with the aim of making products more robust to environmental changes and minimizing product sensitivity to transmitted variation (Hicks (1993) and Box (1988)). Deming's philosophy led him to introduce new management principles that revolutionized quality and productivity ideas based on his famous fourteen points for management (Deming (1986)). Wheeler and Chambers (1992) supported the application of control charts for process improvement. More recently, Six Sigma has evolved in the quality context as an approach that combines quality engineering and systems engineering. This approach uses the steps of define, measure, analyze, improve, and control (DMAIC) to increase quality in a way that drives business profitability (Harry and Schroeder (2000)).

From this industry perspective, we developed an initial framework that incorporated an industry-based problem-solving approach with a joint monitoring and adjustment process, as shown in Figure 1. The problem definition step closely parallels the define step in DMAIC. Designed experiments (DOE) help us to measure and analyze the process, the second two DMAIC steps. From DOE we develop an understanding of the factors to be controlled, which we can then control and monitor in keeping with the

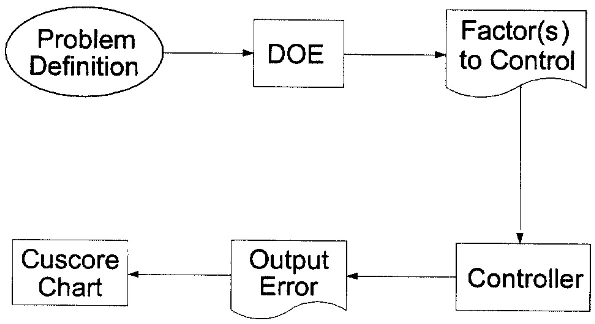


FIGURE 1. Conceptual Framework for Using DOE with Monitoring and Control.

last two DMAIC steps. The monitoring in this case is accomplished using a cumulative score (Cuscore) chart.

Although the statistical foundation for Cuscore statistics can be traced back to Fisher's (1925) efficient score statistic, it still needs further development to realize its true potential as a quality engineering tool. We explore the technical details concerning Cuscore statistics in the next section. However, at this point we would like to demonstrate why they are useful. The traditional process control view is that a process should be monitored to detect any aberrant behavior. Many situations occur, however, where certain process signals are *anticipated* because they are characteristic of a system or operation. For example, a tool may suddenly exhibit accelerated wear due to a heat transfer problem during an annealing process. We may be able to describe this problem in terms of a ramp signal with a change in slope. In the case of HomeWindows, blinds are made out of pleated and glued fabric with alternating colored and white stripes. We learned from discussions with engineers and operators that the stripe-printing process could introduce some problems in the spacing of the stripes that could be described by a spike signal. In general, we can characterize the anticipated problem in terms of a mathematical model of the signal.

Another hurdle comes from the fact that the signal may be contained not in simply white noise, but rather in noise that is serially correlated or nonstationary. In these cases, we can develop Cuscore charts that address the correlation or nonstationarity. The distinction is that the Cuscore chart allows us to investigate situations that are not possible with standard charts because we can devise them for almost any kind of signal hidden in almost any kind of noise (Box and Luceño (1997)).

Our experience with the HomeWindows case led us to refine our framework and reflect on how it could be more broadly applied in industry. One component of the approach is DOE, which many companies are already familiar with as a tool for process improvement. Another component is process control, which is quite prevalent, particularly in continuous processing industries. Still another component is process monitoring, which is also common in industry (albeit at various levels of sophistication). Since our framework rests on these well-established ideas, it can be argued that it is not new. However, we believe that there is much work to be done (and advantage to be gained) by using DOE, process control, and process monitoring in a *coordinated* way, as our framework suggests. At HomeWindows, there were engineers in various groups and pockets throughout the facility who were to some degree familiar with the separate components. However, they had not made much headway on the stripe-printing problem. Only through our coordinated framework were we able to identify, and later eliminate, an expensive process defect. Certainly we introduced a higher level of technical detail, particularly in the control and monitoring phase, than had previously existed in their operation. This is one aspect of making the framework *work* that we explore in this paper.

In the following sections, we first discuss the technical details of the Cuscore statistic, and then illustrate our working model for joint adjustment and monitoring. Next, we explain the application and refinement of the DOE-Cuscore framework for our industrial case. We finish by summarizing the results and identifying future work.

Background and Evolution of the Cuscore Method

Statistical process control (SPC) has developed into a rich collection of tools for monitoring a system. The first control chart, proposed by Shewhart (1926), is still the one most used in industrial systems today (Stoumbos et al. (2000)). The general assumption under which this chart is designed is that the observational data used to construct the chart are independent and identically normally distributed, although this need not be the case (Woodall (2000)). The process is declared *in-control* as long as the points on the chart stay within the control limits. If a point falls outside those limits, an *out-of-control* situation is declared, and a search for an assignable cause is initiated.

Soon, practitioners realized that the ability of the Shewhart chart to detect small changes was not as good as its ability to detect large changes. One approach to improving the sensitivity of the chart was the use of several additional rules (see, for example, Western Electric (1956)). Another approach was to design complementary charts, which could be used in conjunction with the Shewhart chart, but which were better at detecting small changes. Page (1954) and Barnard (1959) developed the cumulative sum (CUSUM) chart, where past and present data are accumulated to detect small shifts in the mean. Roberts (1959) proposed the exponentially weighted moving average (EWMA) chart as another way to detect small changes. This ability comes from the fact that the EWMA statistic can be written as a moving average of the current and past observations, where the weights of the past observations fall off exponentially, as in a geometric series.

When the assumption of independence of the data is violated because of systematic non-random patterns, the effectiveness of the Shewhart, CUSUM, and EWMA charts is highly degraded. Alwan and Roberts (1988) proposed a solution to this problem by modeling the non-random patterns using autoregressive integrated moving average (ARIMA) time series models. They proposed the construction of two charts: 1) a common-cause chart to monitor the process; and 2) a special-cause chart on the residuals of the ARIMA model.

Extensions of these charts to the handling of correlated data have been addressed by several authors. Vasilopoulos and Stamboulis (1978) modified the control limits. Montgomery and Mastrangelo (1991) and Mastrangelo and Montgomery (1995) used the EWMA with a moving center line (MCEWMA). The behavior of the mean was modelled by Vander Wiel (1996) with an IMA(1,1) time series.

Although tailored to meet different process monitoring needs, the Shewhart chart and its successors all involve a direct plot of the actual data. The objective is to expose deviations from statistical stability of a totally *unexpected* kind in the case of the Shewhart chart or a sustained step shift in the case of the CUSUM chart. However, because of these purposes, they will not be as sensitive to certain other *specific* deviations. Box and Ramírez (1992) proposed that a Cuscore chart be used to identify suspected deviations known to be characteristic of (or peculiar to) the monitored system. They suggested a model expressed in the form

$$a_i = a_i(y_i, x_i, \gamma) \qquad i = 1, 2, \ldots, t, \tag{1}$$

where the y_i are observations, the x_i are known quantities, and γ is some unknown parameter. With standard normal theory models, one assumes that when γ is the true value of the unknown parameter, then the a_i's are independent normal random variables with zero mean and variance σ^2. If σ is known and does not depend on γ, then the log likelihood function is given by

$$l = -\frac{1}{2\sigma^2}\sum_{i=1}^{t} a_i^2 + c, \tag{2}$$

where c is a constant that does not depend on γ.

Following Fisher (1925), the efficient score statistic is obtained from Equation (2) by differentiating with respect to γ at $\gamma = \gamma_0$. Thus,

$$\left.\frac{\partial l}{\partial \gamma}\right|_{\gamma=\gamma_0} = \frac{1}{\sigma^2}\sum_{i=1}^{t} a_{i0} r_i \quad \text{with} \quad r_i = -\left.\frac{\partial a_i}{\partial \gamma}\right|_{\gamma=\gamma_0},$$

where the null values, a_{i0}, are obtained by setting $\gamma = \gamma_0$ in Equation (1). We say that

$$Q = \sum_{i=1}^{t} a_{i0} r_i$$

is the *Cuscore* associated with the parameter value $\gamma = \gamma_0$, and r_i is the *detector*. Box and Luceño (1997) showed that the Cuscore statistic detects a specific signal f which is present when $\gamma \neq \gamma_0$.

Box and Ramírez (1992) applied the Cuscore statistic to detecting a sine wave hidden in noise, to detecting a change in gradient, and to constructing approximate significance tests. They also detected changes in the parameter of a time series model and detected non-stationarity in time series models. They observed that the procedure is quite general and could be applied to other situations as an adjunct to the Shewhart chart. For example, Box, Jenkins, and Reinsel (1994) and Box and Luceño (1997) suggested monitoring the parameters of time series models for disturbances using Cuscores.

Joint Monitoring and Adjustment

In many industrial systems, using only SPC to monitor a process will not be sufficient to achieve acceptable output. Real processes tend to drift away from target, use input material from different suppliers, and are run by operators who may use different techniques. For these and many other reasons, a system of active adjustment using engineering process control (EPC) is often necessary. Many authors have addressed the integration of SPC and EPC to jointly monitor and adjust industrial processes. Box and Jenkins (1962) pioneered these attempts by demonstrating the inter-relationships of adaptive optimization, adaptive quality control, and prediction.

Following Box and Ramírez (1992), Box and Kramer (1992) discussed the complementary roles of SPC and EPC. Since then, many other authors have addressed the joint monitoring and adjustment of industrial processes, including Shao (1998), Nembhard (1998), Nembhard and Mastrangelo (1998), Tsung, Shi, and Wu (1999), Tsung and Shi (1999), Ruhhal, Runger, and Dumitrescu (2000), Nembhard (2001), and Nembhard, Mastrangelo, and Kao (2001).

In all of these investigations on joint monitoring and adjustment, the working model that has been employed is of a *dynamic* system that has input X_t and output y_t. In a dynamic system, there will be a period of delay between the time X is changed and the time that change is realized in the output y. (By contrast, in a *responsive* (or steady-state) system, a change in X is realized immediately in y.) Figure 2 shows a block diagram that illustrates this working model. In addition, it shows that both a controller, represented by the control equation, and a monitor, represented by the Cuscore chart, use the information available at the output to test and adjust the input. An input variable X_t has certain impact on another variable, y_t, as expressed by

$$y_t = \frac{L_2(B)B^{d+1}}{L_1(B)}X_t,$$

where B is the backshift operator such that $BX_t = X_{t-1}$, L_1 and L_2 are polynomials in B with roots outside the unit circle to assure stability and invertibility of the process, and d is an integer indicating the delay of a change in X to induce a change in y (Box, Jenkins, and Reinsel (1994)). (In a responsive system, $d = 0$; in a dynamic system, $d \geq 1$.) *After* and independently of this, we find noise *plus* an anticipated *signal* that could appear at some time, namely

$$z_t = \frac{\theta(B)}{\varphi(B)}a_t + \gamma f(t),$$

or equivalently

$$a_t = \frac{\varphi(B)[z_t - \gamma f(t)]}{\theta(B)},$$

where $\varphi(B)$ and $\theta(B)$ are polynomials in the backshift operator with roots outside the unit circle. At

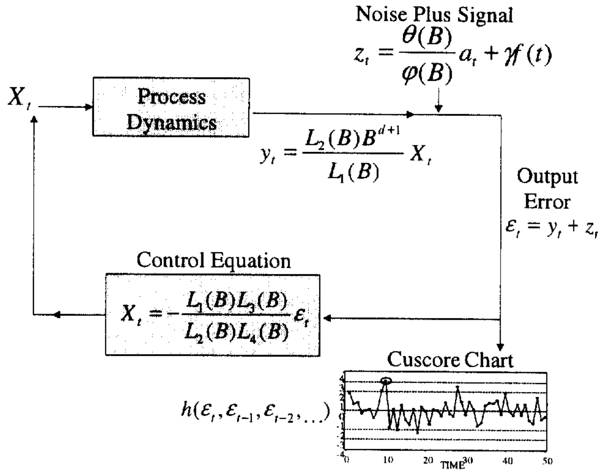


FIGURE 2. A Block Diagram Showing the Input, Output, and Noise Components and the Relationship Between Feedback Control and Cuscore Monitoring of an Anticipated Signal.

the output, we observe the error that we use in two ways: (1) to specify the next level of X via

$$X_t = -\frac{L_1(B)L_3(B)}{L_2(B)L_4(B)}\varepsilon_t,$$

where L_3 and L_4 are polynomials in B with roots outside the unit circle; and (2) to construct the Cuscore chart, $h(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots)$, to monitor the process output.

The Cuscore statistic for detecting the signal f, hidden in the noisy dynamic process, is given by

$$Q_t = \sum_A \frac{1}{L_4(B)}\varepsilon_t \frac{\varphi(B)}{\theta(B)} f(t), \qquad (3)$$

where $\varepsilon_t = y_t + z_t$ is the output error at time t, A is a set containing the time values when the signal appears, $a_t = 1/[L_4(B)]\varepsilon_t$ is the white noise expressed in terms of the output error, and $\varphi(B)$ and $\theta(B)$ are polynomials in B with roots outside the unit circle. Appendix A contains a derivation of Equation (3). Box and Luceño (1997) gave an intuitive explanation of how the Cuscore generally works. In our context, we suppose that we are looking for a spike of size s hidden in an IMA(1,1) disturbance with parameter λ. Then, during the normal operation of a responsive process, the Cuscore chart displays observations normally distributed with a mean 0 and a standard deviation $s\sigma_a$, where σ_a is the standard deviation of the white noise generating the IMA(1,1). At the moment the spike appears, the corresponding observation has a normal distribution with mean s^2 and standard deviation $s\sigma_a$. This mean, s^2, allows us to observe the spike in the chart.

In short, this model illustrates the essence of how we conceive of the joint monitoring and controlling of a given dynamic process. In a dynamic system, a change may take several time periods to be fully realized, whereas a change to a responsive system is realized in the next time period. In other words, dynamic behavior occurs when the output does not respond instantaneously to a change in the input. The modeling problem becomes more difficult in the presence of a noise process that affects the output. Noise may occur anywhere in the system. The complete effect of the noise is measured at the output and hence represented as affecting the output. The noise process, z_t, may be a combination of random shocks plus a distinct signal or load.

Applying the DOE-Cuscore Framework in Industry

Problem Definition

Our efforts at HomeWindows focused on the defect known as "out-of-registration" that can occur in cellular blinds. The pleating and gluing manufacturing process is diagrammed in Figure 3 (the numbers in the following discussion refer to this figure). It starts with the fabric roll or "web" (which is about 2,400 ft. long by 10 ft. wide) on the Unwind Stand (1). It then passes through the first Accumulator Roll (2) which helps stabilize the web feed-out. Before going through the second Accumulator Roll (3), it passes through the Pleater Web Tensioner Roll (3), a weighted roller with a support air cylinder that controls the amount of weight applied to the web, thereby varying web tension. The Pleater Web Tensioner basically pulls on the web to achieve proper alignment. The Guiding Table (4) is used to maintain proper feed-through alignment. The Skew Roll (5) is adjusted to correct the non-perpendicularity of the striping to the web direction (known as bias). The Damper Bar Assembly (6) is a horizontal bar that acts as a damper to the indexed (noncontinuous) feed of fabric into the pleater blades. This bar can be adjusted for its static weight, parallellism, and the angle of the counterweights. This assembly also has the following (not pictured) parts: Damper Weight (7a); Damper Angle (7b); and Damper Counterbalance (7c). The Bow Bar (8) is a bar located just before the pleater blades and has two functions. First, it keeps the web spread outwardly to reduce the effect of any web center bagginess. Second, it brings the stripe "into line" when raised to pull the center of the fabric. The Pleater Blades (9) are overlapping flat blades that pull and fold the fabric into a zigzag pattern. The Shuttle Blades (10) and Gluer Web Tensioner (11) provide for the correct pleat-by-pleat feed into the gluing process. The Gluer Nozzles (12) deposit a bead of glue across the web face, parallel to the pleat crests. Finally, the Gluer Backpressure Bar (13) is a weighted swing-bar that applies backpressure to the gluer blades to prevent pullouts.

Just above the Bow Bar, a camera (not pictured) makes an image, like the one amplified in Figure 4, that goes to the computer screen. The computer compares (in pixels) the edge of the colored band with the target position given by the operator. If the two lines match, then the blind is said to be "in-registration." The air cylinder pressure is automatically adjusted by the computer in an attempt to maintain stripe-edge to pleat-crest alignment at stage 9.

Design of Experiment and Factor to Control

To apply DOE according to the framework required several steps. We met with the operators,

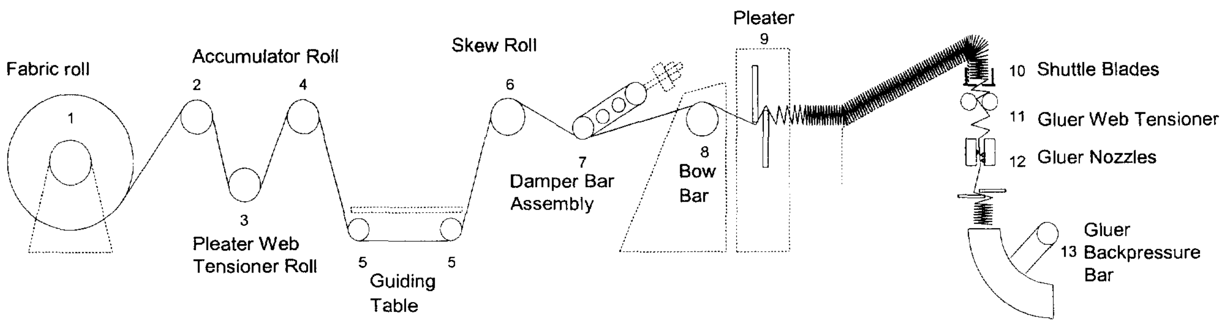


FIGURE 3. Schematic Diagram of the Pleating and Gluing Manufacturing Process.

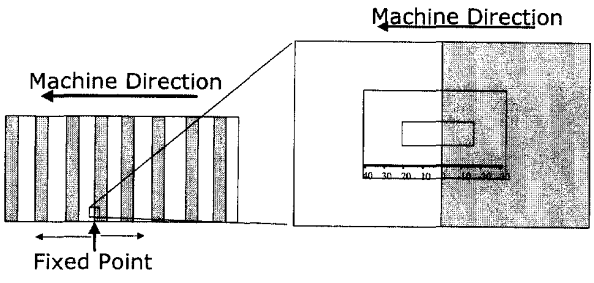


FIGURE 4. Representation of the Measurement of the Displacement of the Leading Edge of the Fabric with Respect to the Fixed Point Given by the Operator.

troubleshooters, crew leaders, machine builder representatives, and managers to decide which factors and levels to consider for the experiment. We had to define scales and operating ranges for some of the factors that were changed only by experience. Based on the advice and experience of the engineers and operators who worked with the process, we conducted a design with the Pleater Web Tensioner (3), Damper Weight (7a), and Damper Angle (7b) as factors. In order to measure the registration as a response variable, we placed a scale under the camera that transmitted the image to a computer screen (as in Figure 4). We let the fabric, from a single roll, run as shown in Figure 3 and measured the displacement of the leading edge of the stripe from a fixed point after 20 pleats (Figure 4). The experimental unit was a piece of fabric long enough to process 20 pleats. Table 1 shows the experimental design employed. The design was a completely randomized full $4 \times 2 \times 2$ factorial that was replicated three times (we reset all the levels of the factors, even when the next combination shared the same level). Because of the way we read the displacement, we knew the cross-web variation was not important, and, with respect to the down-web variation, we relied on the evidence given by the supplier assuring us that this was not a problem. Table 2 shows the results of this experiment.

The analyses of the results from this experiment are summarized in Figure 5 and Figure 6. As Figure 5 shows, we found a statistically significant interaction between the Damper Weight and the Pleater Web Tensioner. Figure 6 shows that in order for the operator to better set the conditions for the controller, he or she should know that a change in the Damper Weight must accompany a change in the Pleater Web Tensioner to maintain the same speed rate. The darker line (a linear interpolation) shows the combinations of Damper Weight and Pleater Web Tensioner where the feed rate is about zero. We can also see that an increment of about one unit in the Damper Weight should be accompanied by a decrement of about three units in the Pleater Web Tensioner to maintain the same feed rate.

According to the DOE results we should *control* the Pleater Web Tensioner and the Damper Weight to drive the position of the leading edge of the printed fabric to maintain registration.

Controller and Output Error

Designing the controller requires modeling the process data. In the case where there is no controller the modeling can be done using open loop data. In the case where there is a controller in place we can use closed loop data, which is based on the output error from the existing controller. In our case there was already a controller using the Pleater Web Tensioner in place. Unfortunately, the current controller performed very poorly. Figure 7 shows that the output deviations, even after this controller had made its adjustments, were unacceptable. Appendix B gives the actual data for the 580 observations. Since propietary issues did not allow us to modify the existing controller, we modeled the output error of the existing controller using the statistical methods for closed loop data (i.e., controlled process) described by Box and MacGregor (1974).

Using a dataset of 150 observations from one of the production lines, we modeled the output error as a function of the net effect of the accumulated adjustments, y_t, the air pressure level, X_t, and the

TABLE 1. Factors and Levels Used in the Experimental Design

Factors	Levels			
Pleater Web Tensioner (3)	26lb	29lb	35lb	38lb
Damper Weight (7a)	25lb	30lb		
Damper Angle (7b)	155°	183°		

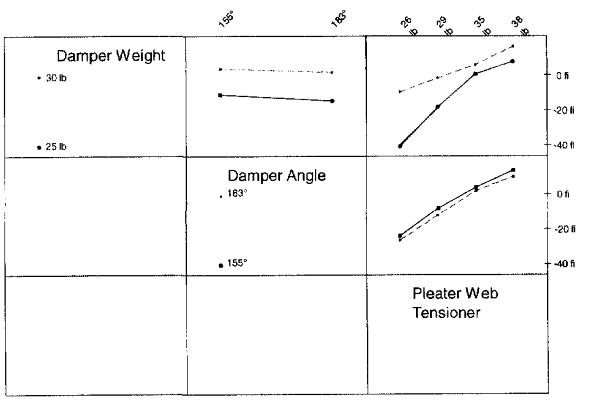
TABLE 2. Observations Obtained from the DOE

Pleater Web Tensioner (in pounds)	Damper Weight (in pounds)	Damper Angle 155°	Damper Angle 183°
26	25	−37, −41, −41	−40, −42, −42
	30	−8, 0, −16	−13, −1, −19
29	25	−17, −11, −19	−21, −14, −27
	30	2, 0, −4	−2, −1, −6
35	25	−2, 8, 1	−3, −5, 3
	30	11, 7, 1	11, 6, 0
38	25	12, 13, 4	4, 11, 1
	30	17, 23, 12	14, 20, 11

disturbance, z_t, as

$$\varepsilon_t = y_t + z_t = \frac{-(0.77 + 0.82B + 0.56B^2)B^2}{1 + 1.51B + 0.97B^2} X_t + \frac{1}{1 - 0.84B + 0.14B^2} a_t. \quad (4)$$

We note that the displacement, y_t, is given by the first term in Equation (4) as a transfer function of the air pressure level, X_t, representing the dynamics of the process. Also notice that, in this case, $L_1(B) = 1 + 1.51B + 0.97B^2$, and $L_2(B) = -0.77 - 0.82B - 0.56B^2$. The second term is the disturbance, the behavior of the output if no control were in place, given by an AR(2) time series model. This gives the correlation structure of the process, with $\theta(B) = 1$ and $\varphi(B) = 1 - 0.84B - 0.14B^2$. (The interested reader may refer to Box, Jenkins, and Reinsel (1994) for details on how to fit this type of transfer function equation to a set of data.)



(The damper angle is measured in degrees (°), the damper weight and the pleater web tensioner are measured in pounds (lb), and the displacement is measured in units of 1/32 of an inch (fi) (where displacement in the machine direction is considered positive).

FIGURE 5. Interaction Plot Showing How the Registration for a Given Damper Weight Depends on the Pleater Web Tensioner.

As shown in Appendix C, the adjustment equation to correct for this output error is

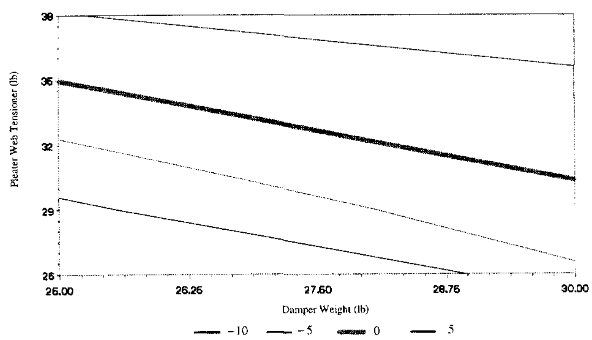


FIGURE 6. Contour Plot of the Displacement as a Function of Damper Weight and Pleater Web Tensioner.

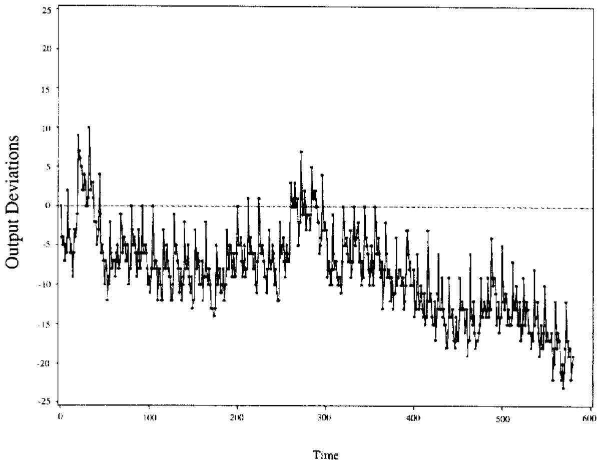


FIGURE 7. Data from Line 15 Showing the Output from the Current Controller.

$$x_t = X_t - X_{t-1} \quad (5)$$
$$= \frac{-(1+1.51B+0.97B^2)\frac{0.8456+0.1176B}{1-0.84B-0.14B^2}(1-B)}{(-0.77-0.82-0.56B^2)(1+0.84B)}\varepsilon_t.$$

Equivalently, after eliminating the backshift operator, the adjustment equation can be written as

$$x_t = -1.06x_{t-1} + 0.12x_{t-2} + 1.02x_{t-3} + 0.74x_{t-4} + 0.09x_{t-5} + 1.10\varepsilon_t + 0.71\varepsilon_{t-1} - 0.52\varepsilon_{t-2} - 1.15\varepsilon_{t-3} - 0.15\varepsilon_{t-4}, \quad (6)$$

where x_t is the adjustment to the pressure in the Pleater Web Tensioner at pleat number t, and ε_t is the output error in pleat number t after applying this adjustment.

Cuscore Chart

As discussed in Section 2, the Cuscore chart is a very specialized tool for detecting an anticipated signal. Since we are looking for a registration problem, we anticipated a signal in the form of a spike. That is, when the fabric is out of registration we expect a single anomalous observation at that time. A single spike signal is represented by

$$f(t) = \begin{cases} 1 & \text{if } t \in A \\ 0 & \text{if } t \notin A \end{cases} \quad \text{where} \quad A = \{t_0\},$$

and appears as shown in Figure 8.

The Cuscore statistic in the case of a spike is found from Equation (3) as follows:

$$Q_t = \sum_A \frac{1}{L_4(B)}\varepsilon_t \frac{\varphi(B)}{\theta(B)} f(t)$$
$$= \sum_A \frac{1}{L_4(B)}\varepsilon_t \frac{1}{1+\psi_1 B + \psi_2 B^2 + \cdots} f(t)$$
$$= \sum_A \frac{1}{L_4(B)}\varepsilon_t [f(t) + \pi_1 f(t-1) + \pi_2 f(t-2) + ..]$$
$$= \frac{1}{L_4(B)}\varepsilon_{t_0} f(t_0).$$

Since A contains only one value for the spike which can appear at any time, we may write

$$Q_t = \frac{1}{L_4(B)}\varepsilon_t.$$

We observe that for the special case when $d = 0$ (a responsive system), the Cuscore is simply the output error, ε_t, which is equivalent to using a Shewhart chart. However, in this pleating and gluing process $d = 1$, and

$$Q_t = \frac{1}{1+0.84B}\varepsilon_t. \quad (7)$$

We applied our new adjustment Equation (6) to the output error data (Figure 7) of the controller for the pleating and gluing process line. Then we used the output error from this controller to construct the Cuscore chart in Figure 9 using Equation (7). Note that the Cuscore chart identifies spike signals at pleat numbers 8, 20, 32, etc. Appendix D shows detailed calculations for the first 8 time periods of Figure 9.

In tracking down this problem, it appeared that the printing cylinder used by the supplier to print the fabric was the cause. In that process, the printing consists of passing the fabric over a screen roll with 12 channels. However, one of the twelve stripes had a different width, probably because the printing cylinder was not joined properly at the seam.

A New Framework for Quality Improvement

In the introduction, we discussed the development of our framework for aligning DOE, feedback control, and the Cuscore chart (Figure 1). Initially, we identified these elements as important components to ad-

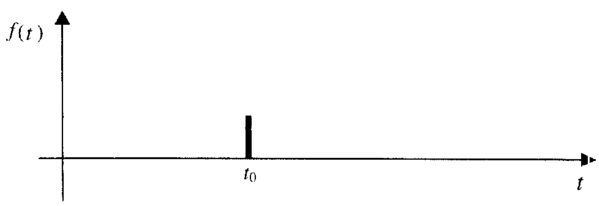


FIGURE 8. Graph of a Spike Occuring at Time t_0.

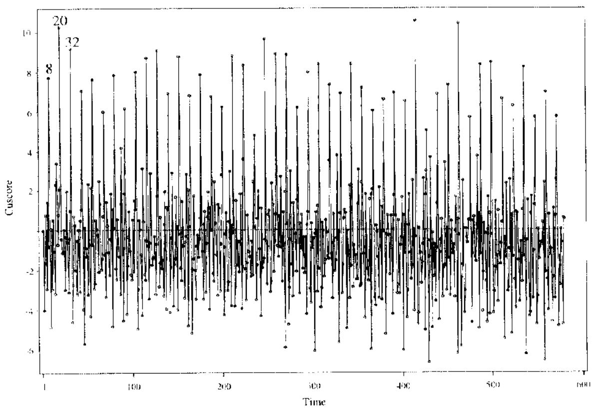


FIGURE 9. Cuscore Chart Detecting Spike Signals at Every Twelfth Pleat.

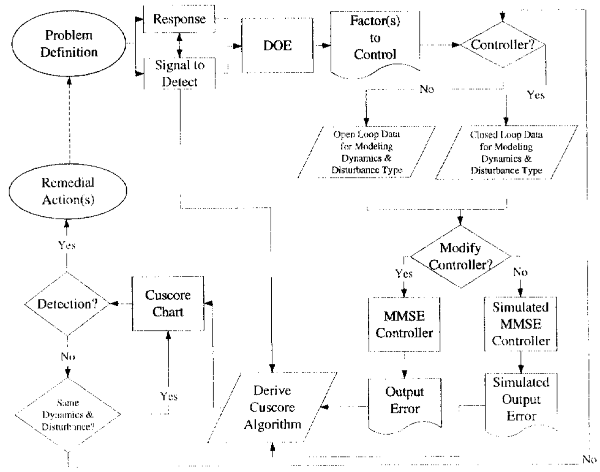


FIGURE 10. Revised Framework for Using DOE with Monitoring and Control.

dress the industry problem at hand with HomeWindows. However, as our project developed over the next 20 months, our framework developed as well. In this section, we address several important aspects that enable the framework to work and in essence expand the framework. In reflecting on this overall experience, we believe that we can impart an effective methodology that would be helpful in other industrial applications. Figure 10 shows this new framework for quality improvement.

Response and Signal

These intermediate nodes between the Problem Definition and DOE steps reflect our experience with the HomeWindows project. Early in the project's development, we realized that in order for the Cuscore to be applicable we should be able to describe how the signal might modify the response and, therefore, the output error. In our case study, the displacement was affected by the stripe width which manifested as a spike signal. Consequently, the output error was larger than expected. In general, we would expect that some consideration would need to be given to the system in order to establish a clear understanding of the response, the expected signal to try to detect, and the relationship between the two.

Modeling Dynamics and Disturbance

A second aspect we wish to highlight is the modeling of the transfer function, or dynamics of the process, and the disturbance. In the modeling stage we need to collect data from the process working in an open loop or in a closed loop mode. In our case study we used data produced in a closed loop mode, since HomeWindows already had a controller on the system. As Equation (4) demonstrates, the models can be far from intuitive or simple. Furthermore, even though we now have statistical software that can handle those medels easily, we still need to be careful in judging the goodness of the fit, as well as in using all the subject matter knowledge of the engineers working with the process.

Simulated MMSE Controller and Output Error

Ordinarily, we might like to be able to directly modify (or create) a process controller. However, in our case this was not possible due to restrictions on the use of the system controller. As we have informally discussed this issue with other engineers, this situation seems to be somewhat common. Given this, we were motivated to establish an alternative procedure that would allow us to continue with the development of the Cuscore chart. We treated the closed loop data (in the form of output error) as the raw output from the process, and used it for simulation modeling of the dynamics and disturbance and for deriving the Cuscore statistic.

Derive Cuscore Statistic

Equation (3) shows that the Cuscore statistic depends on the output error and the signal to be detected. However, *finding* the statistic to apply for a given process requires some derivation that depends on the actual process under consideration, as shown in the equations leading up to Equation (7). In other words, for each application this derivation may be different. With the addition of this block, we emphasize this point as well as the connection to the earlier issue of thoroughly understanding the response and signal.

Detection and Remedial Action

In our case study, after discovering the spikes in the new data we contacted the supplier and asked them to check their printing cylinder for the uniformity of the width of the stripes. The faulty cylinder was discarded once they verified that, in effect, one of the widths was different. This experience underscored the general need to establish a procedure for what to do after the application of the Cuscore chart has revealed a signal.

We note also that during the monitoring phase it is possible that there is no detection while using the Cuscore chart. This could be due to an absence of a signal in the process, in which case we continue the monitoring. Or it could be due to too much change in the underlying dynamics or disturbance with respect to the initial model, in which case we should gather data to remodel the dynamics and disturbance.

Summary

We have developed a methodology for combining DOE and Cuscore charts to control and monitor an industrial process. We started with an initial framework for this methodology, and refined it based on our industry case work with a dynamic manufacturing system that involved processing a fabric web to make window blinds. DOE is critical in this framework because it can be used to screen out the most important process variable from a larger set of variables, which helps to focus the process control effort. The Cuscore chart is also critical because it incorporates information and expectations about the system dynamics disturbance and signal into the monitoring effort. However, there are several other important elements that we added to this framework to make it feasible and practical in industrial applications. These include understanding the response and signal, modeling the open or closed loop process data, designing the appropriate controller (to work with the Cuscore), deriving the Cuscore statistic, and considering the possible outcomes of detection. During these phases, we use several statistical and mathematical tools, including SAS (PROC ARIMA), Minitab, and transfer functions, in order to establish a working model. These tools require an intermediate level of ability. Initially, our industry partners were hesitant to use them, but over the course of the project they came to appreciate their benefits.

In our industry case, the DOE phase led us to identify a process factor called the Pleater Web Tensioner as having the most significant impact on the output registration quality. So, we used that variable as a compensating factor in the design of an MMSE feedback controller. We determined that a problem with output quality would appear in the process as a spike signal. To address this issue, we showed the derivation of a Cuscore statistic that will identify spike signals in a dynamic system that is subject to feedback control, and we applied it to the output error from the control scheme. The resulting Cuscore chart performed quite well, identifying a problem that would affect output quality (that was eventually traced back to a supplier).

The estimated cost of conducting the DOE for this project was \$15,000. This cost included the scrap, machine downtime, and labor. The payoff for this investment is a significant improvement in quality and production. Company reports showed that at the beginning of year 2000, one of the problematic areas had a scrap rate of 41%. At the end of that year this scrap rate was about 21%. Also, the finished goods yield increased in the same period from 43% to 68%. Although there are many activities that have been initiated to reach these levels, company managers conservatively estimate that \$100,000 in savings can be directly related to our efforts on this project.

In the future, we plan to develop Cuscore statistics for other likely industry system characterizations and examine their performance in detecting signals. We expect this work to deliver improved quality performance through better process understanding, monitoring, and control.

Appendix A: Derivation of the Cuscore Statistic

In this Appendix, we provide the derivation of the Cuscore statistic in Equation (3). As indicated in Section 3, the process dynamics dictate how the output will behave in response to a change in the input. The relationship between the input, X_t, and the output, y_t, can be represented by the dynamic transfer function model

$$y_t = \frac{L_2(B)B^{d+1}}{L_1(B)} X_t, \quad \text{(A1)}$$

where B is the backshift operator, such that $BX_t = X_{t-1}$, L_1 and L_2 are polynomials in B with roots outside the unit circle, and d is an integer indicating the delay of a change in X to induce a change in y (Box, Jenkins, and Reinsel (1994)).

Disturbance and Output Error

We consider a disturbance, or deviation from target, that may affect a system (before or after adjustment). We can model the disturbance as the ARIMA process

$$\begin{aligned} z_{t+l} &= \sum_{j=0}^{\infty} \psi_j a_{t+l-j} \\ &= a_{t+l} + \psi_1 a_{t+l-1} + \cdots + \psi_{l-1} a_{t+1} \\ &\quad + \psi_l a_t + \psi_{l+1} a_{t-1} + \cdots, \end{aligned}$$

where $\psi_0 = 1$ and the a_t's are independent random variables with mean zero and standard deviation σ_a (this representation is used in the next section).

We assume that the disturbance can be represented by the following ARIMA model

$$z_t = \frac{\theta(B)}{\varphi(B)} a_t = \left(1 + \sum_{j=1}^{\infty} \psi_j B^j\right) a_t.$$

Then, the output error $\varepsilon_{t+d+1} = Y_{t+d+1} - T$, at time $t+d+1$, can be written as

$$\varepsilon_{t+d+1} = y_{t+d+1} + z_{t+d+1} = \frac{L_2(B)}{L_1(B)} X_t + z_{t+d+1}. \quad \text{(A2)}$$

Minimum Mean Square Error Forecast and Feedback Control

Now, we make a forecast, at time t, of the observation l steps ahead using the current and previous shocks $a_t, a_{t-1}, a_{t-2}, \ldots$, i.e.,

$$\hat{z}_t(l) = \psi_l^* a_t + \psi_{l+1}^* a_{t-1} + \psi_{l+2}^* a_{t-2} + \cdots,$$

where the coefficients $\psi_l^*, \psi_{l+1}^*, \psi_{l+2}^*, \ldots$ are to be determined. Then,

$$\begin{aligned} E[z_{t+l} - \hat{z}_t(l)]^2 &= (1 + \psi_1^2 + \cdots + \psi_{l-1}^2)\sigma_a^2 \\ &\quad + \sigma_a^2 \sum_{j=0}^{\infty} (\psi_{l+j} - \psi_{l+j}^*)^2 \end{aligned}$$

is minimized by taking $\psi_{l+j}^* = \psi_{l+j}$, $j \geq 0$.

Hence, the minimum mean square error (MMSE) forecast, at origin t for an observation l steps ahead, is given by

$$\hat{z}_t(l) = \psi_l a_t + \psi_{l-1} a_{t-1} + \psi_{l+2} a_{t-2} + \cdots,$$

and the forecast error is given by

$$e_t(l) = z_{t+1} - \hat{z}(l) = a_{t+1} + \psi_1 a_{t+l-1} + \cdots + \psi_{l-1} a_{t+1}.$$

For just one step ahead ($l = 1$), the MMSE forecast is

$$\hat{z}_t 1) = \psi_1 a_t + \psi_2 a_{t-1} + \psi_3 a_{t-2} + \cdots,$$

and the forecast error is

$$e_t(1) = z_{t+1} - \hat{z}_t(1) = a_{t+1}.$$

Expressing the disturbance as a forecast plus the forecasting error $z_{t+d+1} = \hat{z}_t(d+1) + e_t(d+1)$, we obtain, using Equation (A2),

$$\varepsilon_{t+d-1} = \frac{L_2(B)}{L_1(B)} X_t + \hat{z}_t(d+1) + e_t(d+1).$$

So by making $X_t = -\{[L_1(B)]/[L_2(B)]\}\hat{z}_t(d+1)$, we cancel out the forecast of the noise, and the output error at time $t+d+1$ is equivalent to the $d+1$-step ahead forecast error, i.e., $\varepsilon_{t+d+1} = e_t(d+1)$. In order to express the forecast $\hat{z}_t(d+1)$ in terms of the output error ε_t, we write

$$\begin{aligned} \varepsilon_t &= e_{t-d-1}(d+1) \\ &= a_t + \psi_1 a_{t-1} + \cdots + \psi_d a_{t-d} \\ &= (1 + \psi_1 B + \cdots + \psi_d B^d) a_t \\ &= L_4(B) a_t, \end{aligned}$$

or $a_t = \varepsilon_t / L_4(B)$.

Then, we have

$$\begin{aligned} \hat{z}_t(d+1) &= \psi_{d+1} a_t + \psi_{d+2} a_{t-1} + \cdots \\ &= (\psi_{d+1} + \psi_{d+2} B + \cdots) a_t \\ &= L_3(B) a_t \\ &= \frac{L_3(B)}{L_4(B)} \varepsilon_t. \end{aligned}$$

Therefore, the MMSE control equation and the adjustment equation are given, respectively, by

$$X_t = -\frac{L_1(B)L_3(B)}{L_2(B)L_4(B)} \varepsilon_t$$

and

$$x_t = -\frac{L_1(B)L_3(B)(1-B)}{L_2(B)L_4(B)} \varepsilon_t.$$

Suppose now that, as given in Figure 2,

$$\begin{aligned}\varepsilon_t &= y_t + \dot{z}_t \\ &= L_1^{-1}(B)L_2(B)B^{d+1}X_t + \dot{z}_t \\ &= \frac{L_2(B)}{L_1(B)}B^{d+1}X_t + \frac{\theta(B)}{\varphi(B)}a_t + \gamma f(t),\end{aligned}$$

where

$$f(t) = \begin{cases} k_t & \text{if } t \in A \\ 0 & \text{if } t \notin A \end{cases}$$

is the signal we want to detect, and A is a set of discrete time values for which the signal appears.

In the MMSE case we have

$$\begin{aligned}\varepsilon_t &= \frac{L_2(B)}{L_1(B)}B^{d+1}\left(-\frac{L_1(B)L_3(B)}{L_2(B)L_4(B)}\varepsilon_t\right) \\ &\quad + \frac{\theta(B)}{\varphi(B)}a_t + \gamma f(t)I_A(t) \\ &= -\frac{L_3(B)}{L_4(B)}\varepsilon_{t-d-1} + \frac{\theta(B)}{\varphi(B)}a_t + \gamma f(t).\end{aligned}$$

That is, we have

$$a_t = \frac{\varphi(B)}{\theta(B)}\left\{\varepsilon_t + \frac{L_3(B)}{L_4(B)}\varepsilon_{t-d-1} - \gamma f(t)\right\},$$

which can be expressed as

$$\begin{aligned}a_t &= \frac{\varphi(B)}{\theta(B)}\left\{\varepsilon_t + \frac{L_3(B)}{L_4(B)}\varepsilon_{t-d-1} - \gamma f(t)\right\} \\ &= \frac{\varphi(B)}{\theta(B)}\left\{1 + \frac{L_3(B)}{L_4(B)}B^{d+1}\right\}\varepsilon_t - \gamma\frac{\varphi(B)}{\theta(B)}f(t) \\ &= \frac{\varphi(B)}{\theta(B)}\frac{L_4(B) + L_3(B)B^{d+1}}{L_4(B)}\varepsilon_t - \gamma\frac{\varphi(B)}{\theta(B)}f(t) \\ &= \left(\frac{1}{1 + \psi_1 B + \psi_2 B^2 + \cdots}\right)\left(\frac{1 + \psi_1 B + \cdots + \psi_d B^d}{L_4(B)}\right. \\ &\quad \left. + \frac{(\psi_{d+1} + \psi_{d+2}B + \cdots)B^{d+1}}{L_4(B)}\right)\varepsilon_t \\ &\quad - \gamma\frac{\varphi(B)}{\theta(B)}f(t) \\ &= \frac{1}{L_4(B)}\varepsilon_t - \gamma\frac{\varphi(B)}{\theta(B)}f(t).\end{aligned} \tag{A3}$$

Thus, the null model, when $\gamma = 0$ in Equation (A3), is given by

$$a_{t0} = \frac{1}{L_4(B)}\varepsilon_t.$$

Taking the derivative of Equation (A3) with respect to γ (at $\gamma = 0$), we obtain

$$\left.\frac{da_t}{d\gamma}\right|_{\gamma=0} = -\frac{\varphi(B)}{\theta(B)}f(t).$$

The detector becomes

$$r_t = \frac{\varphi(B)}{\theta(B)}f(t).$$

Finally, the Cuscore statistic for detecting a signal $f(t)$ hidden in an ARIMA(p,d,q) disturbance $z_t = \theta(B)a_t/\varphi(B)$ is given by

$$Q_f = \sum_A \frac{1}{L_4(B)}\varepsilon_t\frac{\varphi(B)}{\theta(B)}f(t),$$

as written in Equation (3).

Appendix B: Data for Figure 7

t	z_t	t	z_t	t	z_t	t	z_t	t	z_t	t	z_t	t	z_t
1	0	51	−7	101	−11	151	−12	201	−5	251	−5	301	−3
2	−4	52	−8	102	−9	152	−3	202	−5	252	−4	302	−9
3	−5	53	−12	103	−8	153	−6	203	−5	253	−7	303	−8
4	−4	54	−9	104	0	154	−8	204	−4	254	−9	304	−7
5	−7	55	−10	105	−6	155	−6	205	−7	255	−6	305	−10
6	−5	56	−2	106	−8	156	−7	206	−9	256	−5	306	−8
7	−6	57	−6	107	−7	157	−10	207	−8	257	−7	307	−10
8	2	58	−8	108	−7	158	−10	208	−5	258	−7	308	−1
9	−4	59	−7	109	−8	159	−10	209	−9	259	−6	309	−6
10	−3	60	−7	110	−12	160	−7	210	−8	260	3	310	−8
11	−6	61	−7	111	−8	161	−12	211	−8	261	1	311	−7
12	−5	62	−9	112	−8	162	−9	212	1	262	0	312	−7
13	−6	63	−6	113	−10	163	−9	213	−4	263	1	313	−9
14	−9	64	−5	114	−12	164	−2	214	−6	264	0	314	−10
15	−6	65	−7	115	−12	165	−8	215	−4	265	3	315	−9
16	−3	66	−8	116	−3	166	−9	216	−6	266	0	316	−9
17	−4	67	−7	117	−5	167	−10	217	−6	267	0	317	−10
18	−3	68	−1	118	−8	168	−8	218	−8	268	1	318	−11
19	−1	69	−3	119	−8	169	−10	219	−8	269	−5	319	−7
20	9	70	−6	120	−5	170	−13	220	−6	270	−2	320	0
21	7	71	−4	121	−7	171	−13	221	−10	271	−2	321	−2
22	6	72	−4	122	−8	172	−13	222	−11	272	7	322	−5
23	5	73	−5	123	−9	173	−14	223	−7	273	1	323	−5
24	5	74	−7	124	−9	174	−14	224	1	274	−1	324	−4
25	2	75	−7	125	−12	175	−13	225	−2	275	−1	325	−6
26	2	76	−5	126	−12	176	−5	226	−5	276	2	326	−7
27	4	77	−10	127	−10	177	−7	227	−6	277	0	327	−7
28	3	78	−8	128	−1	178	−10	228	−5	278	−3	328	−3
29	0	79	−8	129	−5	179	−9	229	−6	279	−1	329	−9
30	0	80	0	130	−6	180	−8	230	−7	280	−1	330	−8
31	1	81	−3	131	−5	181	−11	231	−9	281	−1	331	−7
32	10	82	−5	132	−7	182	−11	232	−9	282	−3	332	0
33	4	83	−4	133	−8	183	−10	233	−11	283	−1	333	−4
34	2	84	−5	134	−11	184	−8	234	−8	284	5	334	−5
35	3	85	−6	135	−10	185	−12	235	−8	285	1	335	−4
36	3	86	−9	136	−8	186	−10	236	−3	286	2	336	−3
37	1	87	−7	137	−12	187	−10	237	−7	287	1	337	−2
38	−2	88	−3	138	−10	188	−3	238	−8	288	2	338	−7
39	−2	89	−8	139	−9	189	−6	239	−7	289	0	339	−6
40	−2	90	−8	140	−2	190	−8	240	−5	290	0	340	−7
41	−5	91	−6	141	−7	191	−5	241	−6	291	−2	341	−10
42	−4	92	0	142	−8	192	−5	242	−10	292	−2	342	−7
43	−3	93	−5	143	−8	193	−6	243	−8	293	−6	343	−9
44	4	94	−6	144	−5	194	−9	244	−10	294	−5	344	0
45	−1	95	−6	145	−9	195	−7	245	−11	295	−4	345	−4
46	−6	96	−5	146	−10	196	−6	246	−12	296	4	346	−4
47	−5	97	−6	147	−11	197	−9	247	−12	297	0	347	−5
48	−6	98	−8	148	−9	198	−9	248	−2	298	−3	348	−6
49	−7	99	−10	149	−13	199	−6	249	−6	299	−2	349	−8
50	−10	100	−9	150	−12	200	0	250	−6	300	−3	350	−10

TABLE B1. Continued

t	z_t	t	z_t	t	z_t	t	z_t	t	z_t
351	−8	401	−14	451	−17	501	−10	551	−16
352	−5	402	−12	452	−9	502	−12	552	−17
353	−8	403	−13	453	−13	503	−13	553	−17
354	−10	404	−6	454	−13	504	−11	554	−17
355	−7	405	−11	455	−12	505	−12	555	−17
356	0	406	−13	456	−13	506	−15	556	−16
357	−2	407	−12	457	−13	507	−13	557	−22
358	−6	408	−10	458	−16	508	−14	558	−18
359	−6	409	−11	459	−15	509	−15	559	−20
360	−4	410	−13	460	−13	510	−15	560	−12
361	−6	411	−14	461	−19	511	−14	561	−15
362	−8	412	−11	462	−17	512	−7	562	−18
363	−9	413	−15	463	−17	513	−13	563	−16
364	−7	414	−14	464	−6	514	−12	564	−17
365	−13	415	−14	465	−13	515	−13	565	−17
366	−10	416	−3	466	−14	516	−10	566	−21
367	−8	417	−9	467	−15	517	−13	567	−22
368	−2	418	−12	468	−12	518	−15	568	−20
369	−6	419	−11	469	−16	519	−15	569	−23
370	−9	420	−12	470	−18	520	−12	570	−21
371	−8	421	−14	471	−17	521	−17	571	−18
372	−8	422	−15	472	−17	522	−15	572	−12
373	−9	423	−15	473	−17	523	−16	573	−17
374	−12	424	−12	474	−16	524	−9	574	−17
375	−9	425	−17	475	−15	525	−12	575	−19
376	−10	426	−14	476	−9	526	−15	576	−18
377	−13	427	−11	477	−14	527	−13	577	−18
378	−11	428	−6	478	−14	528	−12	578	−22
379	−11	429	−13	479	−13	529	−13	579	−20
380	−4	430	−13	480	−12	530	−16	580	−19
381	−10	431	−13	481	−13	531	−16		
382	−9	432	−9	482	−14	532	−16		
383	−9	433	−14	483	−13	533	−18		
384	−8	434	−13	484	−9	534	−16		
385	−8	435	−15	485	−14	535	−17		
386	−10	436	−15	486	−12	536	−8		
387	−10	437	−18	487	−13	537	−15		
388	−9	438	−18	488	−4	538	−14		
389	−13	439	−16	489	−9	539	−14		
390	−12	440	−9	490	−10	540	−12		
391	−10	441	−14	491	−9	541	−16		
392	−3	442	−14	492	−9	542	−19		
393	−3	443	−14	493	−11	543	−17		
394	−6	444	−13	494	−15	544	−15		
395	−8	445	−16	495	−14	545	−18		
396	−7	446	−17	496	−12	546	−18		
397	−10	447	−18	497	−16	547	−16		
398	−10	448	−14	498	−14	548	−10		
399	−10	449	−17	499	−14	549	−15		
400	−8	450	−16	500	−5	550	−17		

Appendix C: Derivation of the MMSE Adjustment Equation

In this section, we provide the development of the MMSE adjustment in Equation (5).

From Equation (4), we obtain two of the elements to construct the adjustment equation, $L_1(B)$ and $L_2(B)$, as identified in the Figure 2 block diagram. Specifically, $L_1(B) = 1 + 1.51B + 0.97B^2$ and $L_2(B) = -0.77 - 0.82B - 0.56B^2$.

In order to forecast the behavior of the displacement, we write the model for the noise as

$$z_t = \frac{\theta(B)}{\varphi(B)}a_t = \frac{1}{1 - 0.84B - 0.14B^2}a_t,$$

and so

$$z_t - 0.84z_{t-1} - 0.14z_{t-2} = a_t. \qquad \text{(C1)}$$

The one-step ahead forecast for Equation (C1) is

$$\hat{z}_t(1) = 0.84z_t + 0.14z_{t-1}.$$

In order to complete the control equation as represented in Figure 2, we need to find $L_3(B)$ and $L_4(B)$. From the one-step-ahead forecast we can find the two-step-ahead forecast, which gives us

$$L_3(B) = \frac{0.8456 + 0.1176B}{1 - 0.84B - 0.14B^2}.$$

Using the two-step-ahead forecast error and its relationship with the output error, we find

$$L_4(B) = \frac{1 - 0.8456B^2 - 0.1176B^3}{1 - 0.84B - 0.14B^2} = 1 + 0.84B.$$

Combining $L_1(B)$, $L_2(B)$, $L_3(B)$, and $L_4(B)$, we can now write the MMSE control equation for the process as

$$X_t = \frac{-(1 + 1.51B + 0.97B^2)\frac{0.8456+0.1176B}{1-0.84B-0.14B^2}}{(-0.77 - 0.82B - 0.56B^2)(1 + 0.84B)}\varepsilon_t.$$

We take the difference of this expression at t and $t - 1$ to determine the adjustment equation given in Equation (5).

Appendix D: Calculations for the Cuscore Chart in Figure 9

In this case, the z_t's are given by the data in Appendix B, the net effect of the accumulated adjustments is given, according to Equation (4), by

$$y_t = -1.51y_{t-1} - 0.97y_{t-2} - 0.77X_{t-2} - 0.82X_{t-3} - 0.56X_{t-4},$$

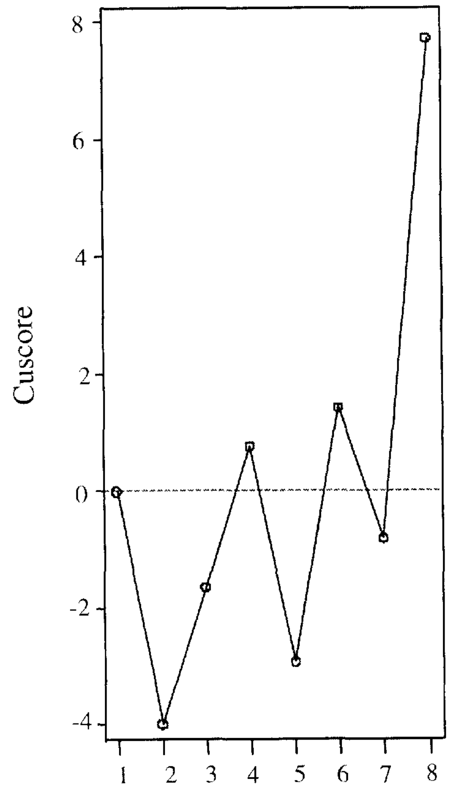


FIGURE D1. First Eight Points in the Cuscore Plot of Figure 9.

the adjustment equation is given by

$$x_t = -1.06x_{t-1} + 0.12x_{t-2} + 1.02x_{t-3} + 0.74x_{t-4} + 0.09x_{t-5} + 1.10\varepsilon_t + 0.71\varepsilon_{t-1} - 0.52\varepsilon_{t-2} - 1.15\varepsilon_{t-3} - 0.15\varepsilon_{t-4},$$

and the Cuscore is given by $Q_t = \varepsilon_t - 0.84Q_{t-1}$ (according to Equation (7)). We show the detailed calculations for the first eight time periods plotted in Figure 9. These are redrawn in more detail in Figure D1.

For $t = 1$,

$$y_1 = 0, \quad z_1 = 0, \quad \varepsilon_1 = y_1 + z_1 = 0 + 0 = 0,$$
$$x_1 = 1.10\varepsilon_1 = 1.10(0) = 0, \quad X_1 = x_1 = 0, \quad \text{and}$$
$$Q_1 = \varepsilon_1 = 0.$$

For $t = 2$,

$$y_2 = -1.51y_1 = -1.51(0) = 0, \quad z_2 = -4,$$
$$\varepsilon_2 = y_2 + z_2 = 0 - 4 = -4,$$
$$x_2 = -1.06x_1 + 1.10\varepsilon_2 + 0.71\varepsilon_1$$

$$= -1.06(0) + 1.10(-4) + 0.71(0) = -4.4,$$
$$X_2 = X_1 + x_2 = 0 - 4.4 = -4.4, \quad \text{and}$$
$$Q_2 = \varepsilon_2 - 0.84Q_1 = -4 - 0.84(0) = -4.$$

For $t = 3$,

$$\begin{aligned}
y_3 &= -1.51y_2 - 0.97y_1 - 0.77X_1 \\
&= -1.51(0) - 0.97(0) - 0.77(0) = 0, \\
z_3 &= -5, \quad \varepsilon_3 = y_3 + z_3 = 0 - 5 = -5, \\
x_3 &= -1.06x_2 + 0.12x_1 + 1.10\varepsilon_3 + 0.71\varepsilon_2 - 0.52\varepsilon_1 \\
&= -1.06(-4.4) + 0.12(0) + 1.10(-5) \\
&\quad + 0.71(-4) - 0.52(0) \\
&= -3.676 \\
X_3 &= X_2 + x_3 = -4.4 - 3.676 = -8.076, \quad \text{and} \\
Q_3 &= \varepsilon_3 - 0.84Q_2 = -5 - 0.84(-4) = -1.64.
\end{aligned}$$

For $t = 4$,

$$\begin{aligned}
y_4 &= -1.51y_3 - 0.97y_2 - 0.77X_2 - 0.82X_1 \\
&= -1.51(0) - 0.97(0) - 0.77(-4.4) - 0.82(0) \\
&= 3.388, \\
z_4 &= -4, \quad \varepsilon_4 = y_4 + z_4 = 3.388 - 4 = -0.612, \\
x_4 &= -1.06x_3 + 0.12x_2 + 0.02x_1 + 1.10\varepsilon_4 + 0.71\varepsilon_3 \\
&\quad - 0.52\varepsilon_2 - 1.15\varepsilon_1 \\
&= -1.06(-3.676) + 0.12(-4.4) + 0.02(0) \\
&\quad + 1.10(-0.612) + 0.71(-5) - 0.52(-4) \\
&\quad - 1.15(0) \\
&= 1.22536 \\
X_4 &= X_3 + x_4 = -8.076 + 1.22536 = -6.8506, \quad \text{and} \\
Q_4 &= \varepsilon_4 - 0.84Q_3 = -0.612 - 0.84(-1.64) = 0.7656.
\end{aligned}$$

For $t = 5$,

$$\begin{aligned}
y_5 &= -1.51y_4 - 0.97y_3 - 0.77X_3 - 0.82X_2 - 0.56X_1 \\
&= -1.51(3.388) - 0.97(0) - 0.77(-8.076) \\
&\quad - 0.82(-4.4) - 0.56(0) \\
&= 4.7106 \\
z_5 &= -7, \quad \varepsilon_5 = y_5 + z_5 = 4.7106 - 7 = -2.2894, \\
x_5 &= -1.06x_4 + 0.12x_3 + 1.02x_2 + 0.74x_1 + 1.10\varepsilon_5 \\
&\quad + 0.71\varepsilon_4 - 0.52\varepsilon_3 - 1.15\varepsilon_2 - 0.15\varepsilon_1 \\
&= -1.06(1.22536) + 0.12(-3.676) + 1.02(-4.4) \\
&\quad + 0.74(0) + 1.10(-2.2894) + 0.71(-0.612) \\
&\quad - 0.52(-5) - 1.15(-4) - 0.15(0) \\
&= -1.9808 \\
X_5 &= X_4 + x_5 = -6.8506 - 1.9808 = -8.8315, \quad \text{and} \\
Q_5 &= \varepsilon_5 - 0.84Q_4 \\
&= -2.2894 - 0.84(0.7656) = -2.9325.
\end{aligned}$$

For $t = 6$,

$$\begin{aligned}
y_6 &= -1.51y_5 - 0.97y_4 - 0.77X_4 - 0.82X_3 - 0.56X_2 \\
&= -1.51(4.7106) - 0.97(3.388) - 0.77(-6.8506) \\
&\quad - 0.82(-8.076) - 0.56(-4.4) \\
&= 3.9619 \\
z_6 &= -5, \quad \varepsilon_6 = y_6 + z_6 = 3.9619 - 5 = -1.0381, \\
x_6 &= -1.06x_5 + 0.12x_4 + 1.02x_3 + 0.74x_2 + 0.09x_1 \\
&\quad + 1.10\varepsilon_6 + 0.71\varepsilon_5 - 0.52\varepsilon_4 - 1.15\varepsilon_3 - 0.15\varepsilon_2 \\
&= -1.06(-1.9808) + 0.12(1.2254) + 1.02(-3.676) \\
&\quad + 0.74(-4.4) + 0.09(0) + 1.10(-1.0381) \\
&\quad + 0.71(-2.2894) - 0.52(-0.612) - 1.15(-5) \\
&\quad - 0.15(-4)) \\
&= -0.8579 \\
X_6 &= X_5 + x_6 = -8.8315 - 0.8579 = -9.6894, \quad \text{and} \\
Q_6 &= \varepsilon_6 - 0.84Q_5 \\
&= -1.0381 - 0.84(-2.9325) = 1.4252.
\end{aligned}$$

For $t = 7$,

$$\begin{aligned}
y_7 &= -1.51y_6 - 0.97y_5 - 0.77X_5 - 0.82X_4 - 0.56X_3 \\
&= -1.51(3.9619) - 0.97(4.7106) - 0.77(-8.8315) \\
&\quad - 0.82(-6.8506) - 0.56(-8.076) \\
&= 6.3885 \\
z_7 &= -6, \quad \varepsilon_7 = y_7 + z_7 = 6.3885 - 6 = 0.3885, \\
x_7 &= -1.06x_6 + 0.12x_5 + 1.02x_4 + 0.74x_3 + 0.09x_2 \\
&\quad + 1.10\varepsilon_7 + 0.71\varepsilon_6 - 0.52\varepsilon_5 - 1.15\varepsilon_4 - 0.15\varepsilon_3 \\
&= -1.06(0.8579) + 0.12(-1.9808) + 1.02(1.2254) \\
&\quad + 0.74(-3.676) + 0.09(-4.4) + 1.10(0.3885) \\
&\quad + 0.71(-1.0381) - 0.52(-2.2894) - 1.15(-0.612) \\
&\quad - 0.15(-5) \\
&= 1.1399 \\
X_7 &= X_6 + x_7 = -9.6894 + 1.1399 = -8.5495, \quad \text{and} \\
Q_7 &= \varepsilon_7 - 0.84Q_6 \\
&= 0.3885 - 0.84(1.4252) = -0.8086.
\end{aligned}$$

For $t = 8$,

$$\begin{aligned}
y_8 &= -1.51y_7 - 0.97y_6 - 0.77X_6 - 0.82X_5 - 0.56X_4 \\
&= -1.51(6.3885) - 0.97(3.9619) - 0.77(-9.6894) \\
&\quad - 0.82(-8.8315) - 0.56(-6.8506) \\
&= 5.0493 \\
z_8 &= 2, \quad \varepsilon_8 = y_8 + z_8 = 5.0493 + 2 = 7.0493, \\
x_8 &= -1.06x_7 + 0.12x_6 + 1.02x_5 + 0.74x_4 + 0.09x_3
\end{aligned}$$

$$
\begin{aligned}
&+ 1.10\varepsilon_8 + 0.71\varepsilon_7 - 0.52\varepsilon_6 \\
&- 1.15\varepsilon_5 - 0.15\varepsilon_4 \\
&= -1.51(1.1399) + 0.12(-0.8579) + 1.02(-1.9808) \\
&+ 0.74(1.2254) + 0.09(-3.676) + 1.10(7.0493) \\
&+ 0.71(0.3885) - 0.52(-1.0381) \\
&- 1.15(-2.2894) - 0.15(-0.612) \\
&= 8.5386
\end{aligned}
$$

$X_8 = X_7 + x_8 = -8.5495 + 8.5386 = -0.0108,$ and

$$
\begin{aligned}
Q_8 &= \varepsilon_8 - 0.84Q_7 \\
&= 7.0493 - 0.84(-0.8086) = 7.7285.
\end{aligned}
$$

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