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Statistics as a Catalyst to Learning by Scientific Method Part I

AUTHOR:GEORGE E. P. BOX; PATRICK Y. T. LIU

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ABSTRACT

Innovation in the design and manufacture of processes and products usually comes about as a result of careful investigation--a directed process of sequential learning. Many practitioners, although familiar with "one-shot" statistical procedures, have little knowledge of the power of statistical techniques designed to catalyze investigation itself. A simple means of demonstrating and experiencing this learning process is illustrated using response surface methods to find an improved design for a paper helicopter.

Key Words: Design of Experiments, Investigation, Response Surface Methodology, Screening Experiments, Steepest Ascent.

INTRODUCTION

IT has long been emphasized that the statistician should be involved not merely in the analysis of data, but also in the design of the experiments which generate the data. As a result, it is now common for students taking a course in experimental design to be required to plan and perform a real experiment. The concept of the statistician as one who analyzes someone else's data is flawed, but equally inappropriate is the idea of the statistician engaged only in the design and analysis of an individual experiment.

An industrial innovation of major importance, such as the development of a new drug or the design of a new engineering system, comes about as the result of an investigation requiring a sequence of experiments. Such research and development is a process of learning: dynamic, not stationary; adaptive, not one-shot. The route by which the objective can be reached is discovered only as the investigation progresses, each subset of experimental runs supplying a basis for deciding the next. Also, the objective itself can change as new knowledge is brought to light. To catalyze such innovation, the statistician must be part of the investigational team. Unfortunately, many statistics students (and their professors) have little or no training or experience to qualify them for this important role.

Consider a statistician who is analyzing some data coming from, say, a factorial arrangement that has been designed in collaboration with an experimenter. The statistician knows that, even though the data are subject to observational error, certain probability statements can be made and conclusions drawn. However, it may not be realized how such conclusions are conditional on the experimental environment. For example, if the statistician had been working on the same problem with a different experimenter, then

a) the design would almost certainly have contained somewhat different factors,

b) different ranges for the factors would have been chosen,

c) different transformations for the factors might be used (such as the length, l, and width, w, of an aircraft wing or its area, A = wl, and length to width ratio, l/w), and/or

d) different tentative models might have been considered.

Such differences would affect the conclusions drawn from this single experiment far more than would observational error.

However, in an investigational sequence of experiments, although different experimenters will take different routes and begin from different starting points, they nevertheless can arrive at similar solutions or equally satisfactory solutions. Like mathematical iteration, scientific iteration tends to be self-correcting. The concept of iterative investigation requires a mind set unfamiliar to many students of statistics. It is difficult to teach and to illustrate, and it needs to be experienced to be appreciated. The purpose of this paper is to illustrate, using a paper helicopter, how response surface methodology (RSM) may be used to practice the process of investigation itself. For detailed descriptions of RSM, see, for example, Box and Draper (1987), Khuri and Cornell (1996), Myers and Montgomery (1995), and the many references contained therein.

Exercises of this kind can be conducted for any system of iterative experimentation; they can be carried out with paper helicopters employed in the investigation described below or with any other convenient experimental device. However, the device must be one that can be subjected to design changes that are not predetermined. A device for which only a fixed number of built-in factors can be changed is of no use for this purpose. The important thing is that the student, using factors whose nature is only limited by the extent of his or her imagination, should start from some prototype design that seems reasonable and then experience the process of experimental learning needed to improve it. It is particularly valuable for several groups of students to conduct independent investigations simultaneously. Each group should then discuss their adventures and conclusions with the other groups. Remember that, in the context of continuous never-ending improvement, there is no such thing as optimality. The best helicopters described in this paper are not optimal. Better designs will be found when factors not previously considered are tested. Already a correspondent, Donald Olsson, has reported designs with longer flight times. These were constructed from a special drawing paper called "rough newsprint."

Students should be warned that there are at least two important respects in which this helicopter experiment does not provide a true picture of a real investigation. In practice, not one but a number of responses will be measured, recorded, and jointly considered. Furthermore, in this example progress is made almost entirely empirically. If we were really in the business of making helicopters, there would be aerodynamicists and other engineers on the team; their help in interpreting the results from the designed experiments would undoubtedly have produced better helicopters quicker.

DESIGN I: AN INITIAL SCREENING EXPERIMENT

The prototype design for a paper helicopter, shown in Figure 1, was kindly made available to us some years ago by Kipp Rogers of Digital Equipment Corporation. The objective of this investigation was to find an improved helicopter design giving longer flight times. We limited our designs to those that could be constructed from readily available office supplies and, in particular, from standard paper 11 × 8 1/2 inches in size. Our test flights were carried out in a room with a ceiling 102 inches (8' 6") from the floor. The wings of each tested helicopter were initially held against the ceiling, and the flight time was measured with a digital stop watch.

We began by running a screening experiment to get some idea of what factors might be important. After considerable discussion it was decided to begin by testing the eight factors (input variables) listed in Table 1. Each factor was tested at two levels with the plus and minus limits shown there. The response (output variable) was the flight time. The initial experimental plan defined sixteen helicopter types set out in Table 2. The experimental design is a 2[sup8-4[sub[subIV] fractional factorial (see, e.g., Box, Hunter, and Hunter (1978)). Each of the sixteen types of helicopters was dropped four times, and the flight times recorded in centiseconds (units of one hundredth of a second). The mean flight times, y, and the standard deviations, s, are also shown in Table 2, together with the quantity, 100 log(s), which we will call the dispersion. Remember it is the design that is being tested, not the individual helicopter "manufactured" according to that design. In earlier work, Sandra Martin showed that indeed a small variance component associated with manufacturing could be detected. The dispersions given here and calculated from repeat runs of the same helicopter are therefore slight underestimates. The analyses of mean flight times with normal plots are, however, not affected since we use the averages as data. It is well known (Bartlett and Kendall (1946)) that for the analysis of variation there are considerable advantages in using the logarithm of the sample standard deviation rather than s itself. To avoid decimals, we have used 100 log(s) in our analysis. The effects calculated from the mean flight time y will be called location effects. Effects calculated using the dispersion will be called dispersion effects. Visual observation suggested that larger variations of flight times were usually associated with instability of the helicopter design.

The effects are shown in Table 3 as regression coefficients; thus, the constant term is the overall average, and each of the remaining coefficients is one half of the usual factor effect. Normal plots for these effects are shown in Figures 2(a) and 2(b). Figure 2(a) for location effects suggests that factors describing three of the dimensions of the helicopter--wing length l, body length L, and body width W--all have distinguishable effects on mean flight time, but, of the five remaining "qualitative" variables, only factor C (corresponding to the application of a paper clip to the body of the helicopter) is applicable and is not negative.

The plot for dispersion effects in Figure 2(b) suggests effects for l, L, W, and C and for the string of two-factor interactions, PL + lC + WT + FM. The signs of the coefficients are such that the changes in the dimensional variables l, L, and W, which gave increases in the mean flight time, are also associated with reductions in dispersion. However, the addition of a paper clip, while reducing the dispersion, also decreased the flight time. We made a judgment that, for the moment, we would concentrate on increasing flight times and not use the paper clip. We could reconsider this later if instability became a problem. Also, we decided that we would not attempt to interpret, or to separate out by additional runs, the interaction string at this time.

On this basis, a linear model for estimating mean flight times in the immediate neighborhood of the experimental design was

^y = 223 + 28x[sub2] - 13x[sub3] - 8x[sub4], (1)

where the coefficients are those, suitably rounded, in Table 3 and x[sub2], x[sub3], and x[sub4] are the coded levels of l, L, and W (coded as in Table 2).

Equation (1) is usually called a fitted linear regression model and the coefficients are those obtained by the method of least squares. The contour diagram of Figure 3 is a convenient way of conveying visually what is implied by Equation (1); for example, those combinations of x[sub2], x[sub3], and x[sub4] corresponding to points on the 240 contour plane should all produce alternative helicopter designs with flight times of about 240 centiseconds.

STEEPEST ASCENT USING THE RESULTS FROM DESIGN I

Now, since increasing the wing length and reducing the body length and body width all had positive effects on mean flight time, it might be expected that helicopter designs with greater wing lengths and with reduced body lengths and body widths might give even longer flights. We can best determine such helicopter designs by exploring the direction at right angles to the contour planes indicated by the arrow in Figure 3. In the units of x[sub2], x[sub3], and x[sub4], this is the direction of greatest increase at a given distance from the design center and is called the direction of steepest ascent.

To calculate a series of points along the direction of steepest ascent, you don't need a contour plot. You can do this by starting at the center of the design and changing the factors in proportion to the coefficients of the fitted equation. Thus, the relative changes in x[sub2], x[sub3], and x[sub4] are such that for every increase of 28 units in x[sub2], x[sub3] is reduced by 13 units and x[sub4] by 8 units. The units are the scale factors, s[subl] = 0.875, s[subL] = 0.875, and s[subW] = 0.375, which looking at Table 1 are the changes in l, L, and W corresponding to a change of one unit in x[sub2], x[sub3], and x[sub4], respectively.

In our investigation we chose the first point, P[sub1], to give a helicopter with a 4 inch wing length, and we then increased l by 3/4 inch increments, adjusting the other dimensions accordingly. This produced the designs corresponding to P[sub2], P[sub3], P[sub4], and P[sub5] shown in Figure 4. In our investigation we ran experiments in sequence at all the five points making ten repeat drops at each point. Alternatively, experiments along such a path could have been run sequentially with the choice of the points along the path a matter of judgment guided by results as they occurred. For example, we might have decided to take a large jump initially and to try the design P[sub5] right away. This would have given a disappointingly low result causing us to back track and to test designs in the neighborhood of P[sub2] or P[sub3]. In any case, we would have ended up with more or less the same conclusion. As you see from Figure 4, P[sub3] gave the longest average flight time of 347 centiseconds--while designs P[sub4] and P[sub5] appeared to give not only lesser mean flight times, but also higher standard deviations.

Since none of the qualitative variables we tried in this and previous experimentation (including heavy paper, fold at the wing tip, fold at the base, etc.) seemed to produce any positive effects, we decided that, at least at this stage, we would fix these features and explore more thoroughly the effects of the dimensional variables--wing length, wing width, body length, and body width using a full factorial experiment.

DESIGN II: A FACTORIAL EXPERIMENT IN WINGS AND BODY DIMENSIONS

At about this time, discussion with an engineer led to the suggestion that a better way to characterize the dimensions of the wing might be in terms of wing area, $A = lw$, and length to width ratio, $Q = l/w$. Therefore, in subsequent experimentation this reparameterization was adopted.

A 2^4 factorial in the four dimensional variables A, Q, W, and L, centered close to the previous best conditions is set out in Table 4 with the data given in Table 5. The normal plot for mean flight times in Figure 5(a) showed large location effects for wing area and body length, but the normal plot for dispersion did not show any evidence of real effects. It was decided, therefore, to try to gain further improvement of flight times by using steepest ascent based on the two large effects, see Table 6, using the model

$$\hat{y} = 326 + 8x_1 - 17x_4,$$

where x_1 and x_4 are recoded variables for A and L, respectively.

The path was explored by making ten drops at each of the five different conditions set out in Figure 6. Interpolation suggests that the best design along this path required A to be about 12.4 and L to be about 2.0, at which the average flight time was 370 centiseconds--a further valuable improvement. These helicopters were extremely stable, and the dispersions for the five tested helicopters on this path were smaller than those obtained before. To allow comparison with previous results, wing lengths and wing widths are also shown in Figure 6.

After this investigation had been completed, a review of the results showed that the path of ascent had been slightly miscalculated. The relative changes in x_1 and x_4 should have been 8:17 but were mistakenly taken to be 8:11. This deviation is unlikely to have made much difference. (The error we made arose from accidentally switching certain experimental runs. It underlines the importance of checking and rechecking experimental procedures. It also illustrates that in an iterative scheme of this kind, errors tend to be self-correcting.)

DESIGN III: A SEQUENTIALLY ASSEMBLED COMPOSITE DESIGN

It seemed likely at this stage of the investigation that further advance from this improved experimental region might not be possible with first order steepest ascent and that a full second degree equation might be needed to represent the flight times. This was not certain, however. Therefore, a new 2^4 factorial experiment in A, Q, W, and L was run with two added center points using the (-1, 0, 1) levels shown in Table 7 and centered close to the best point so far reached. We will call this Design IIIa. It consists of the first block shown in Table 8 with the calculated effect coefficients in Table 9. A normal plot is shown in Figure 7. The plot for dispersion effects failed to show anything of interest and is not given. We see from Figure 7 that some two-factor interactions are quite large and approaching the size of certain main effects, which suggests that we should add further runs to provide for estimation of the remaining second order (quadratic) terms. A further block, referred to as Design IIIb, was therefore added consisting of eight axial points set at conditions corresponding with the levels -2 and +2 in Table 7, with four additional center points. The results from this second block are shown in Table 8.

An analysis of variance of average flight times for the completed design is given in Table 10. There is some evidence of lack of fit. Nevertheless, for this analysis we have used the overall residual mean square of 9.9 as the error variance. The overal F ratio for the fitted second degree equation is 21.0. This exceeds its five percent significance level of $F_{0.05,14,14} = 2.48$ by a factor of 8.5, thus complying with the argument of Box and Wetz (1973) that a factor of at least four is needed to ensure that the fitted equation is worthy of further interpretation. See also Box and Draper (1987) and Draper and Smith (1998).

Proceeding further with the analysis we find that the fitted second-degree equation is

$$\hat{y} = 372.06 - 0.08x_1 + 5.08x_2 + 0.25x_3 - 6.08x_4 - 2.04x_1^2 - 1.66x_2^2 - 2.54x_3^2 - 0.16x_4^2 - 2.88x_1x_2 - 3.75x_1x_3 + 4.38x_1x_4 + 4.63x_2x_3 - 1.50x_2x_4 - 2.13x_3x_4. \quad (2)$$

We have shown the constant term and the four linear terms on the first line, the four quadratic terms on the second line, and the six interaction terms on the third and fourth lines. The standard errors for these linear, quadratic, and interaction effects are shown in Table 11. This second degree equation in four variables (x_1, x_2, x_3, x_4) contains 15 coefficients, and in its "raw" form is not easily understood. We briefly review methods of analysis which can make its meaning clear and allow further progress. A fuller account of such analysis is given in the texts referred to in the introduction. We first illustrate the analysis for constructed examples in just two variables, x_1 and x_2.

Look at Figure 8. Suppose that, in the circle indicated in Figure 8(c), a suitable design has been run centered on the point O ($x_{10} = 0$, $x_{20} = 0$), yielding the second degree equation shown in Figure 8(a). Figure 8(b) shows a computer plot of the corresponding response surface which contains a maximum. A plot of the $\hat{y}$ contours of the surface is shown in Figure 8(c) with dashed lines indicating their unreliability outside the immediate region of experimentation. Contour plots of this kind are very helpful in understanding the meaning of a second degree equation when there are only two or three input variables (x), but for more variables such methods are not available. Canonical analysis, which we now explain, makes it easy to understand the meaning of any fitted second degree equation for any number of such variables. The mathematics is sketched in Figure 8(d) and illustrated geometrically in Figure 8(c). There are two steps:

i) the origin of measurement is shifted from O to S, where S is the center of the contour system (in this case the maximum);

ii) the axes are rotated about S so that they lie along the axes of the elliptical contours which are denoted by X_1 and X_2.

In this way the quadratic equation of Figure 8(a) is expressed in terms of a new system of coordinates X_1 and X_2 in the simpler form,

$\hat{y} = 87.7 - 9.0X_1^2 - 2.1X_2^2$. (3)

By inspection of this canonical form you can understand the meaning of the quadratic equation without a contour plot. In this case, since the coefficients, -9.0 and -2.1, which measure the quadratic curvatures along the X_1 and X_2 axes are both negative, the point S (at which $\hat{y}_s = 87.7$) must be a maximum. Also, if you move away from S in either direction along the X_1 axis, $\hat{y}$ falls off much more rapidly than if you move similarly along the X_2 axis. This indicates that the contours are drawn out (attenuated) along the X_2 axis, which has the smaller coefficient.

Now look at Figure 9. Figure 9(a) produces the response surface shown in Figure 9(b), which represents a "saddle" or minimax whose contours are shown in Figure 9(c). Again it is easy to understand the nature of the surface without any graphical aid using the canonical form of the equation. This turns out to be $\hat{y} = 87.7 - 9.0X_1^2 + 2.1X_2^2$. Because the coefficient of X_1^2 is negative and that of X_2^2 is positive, the center of the system S is a maximum along the X_1 axis but is a minimum along the X_2 axis. Thus, we know at once that the surface is a minimax. In particular, this implies that movement away from S along the X_2 axis in either direction gives larger values of $\hat{y}$, suggesting the existence of more than one maximum. In response surface studies such saddles are rather rare, but, as we shall see, they can occur.

ANALYSIS FOR THE HELICOPTER DATA

If we apply the canonical analysis outlined above to Equation (2) obtained for the helicopter data, then we get

Position of S: $x_{1s} = 0.86$ $x_{2s} = -0.33$

$x_{3s} = -0.84$ $x_{4s} = -0.12$ $\hat{y}_s = 371.4$;

Shift of Origin: $\tilde{x}_1 = x_1 - 0.86$ $\tilde{x}_2 = x_2 + 0.33$

$\tilde{x}_3 = x_3 + 0.84$ $\tilde{x}_4 = x_4 + 0.12$;

Rotation of Axes: $X_1 = 0.39\tilde{x}_1 - 0.45\tilde{x}_2 + 0.80\tilde{x}_3 - 0.07\tilde{x}_4$

$X_2 = -0.76\tilde{x}_1 - 0.50\tilde{x}_2 + 0.12\tilde{x}_3 + 0.39\tilde{x}_4$

$X_3 = 0.52\tilde{x}_1 - 0.45\tilde{x}_2 - 0.45\tilde{x}_3 \_ 0.57\tilde{x}_4$

$X_4 = -0.04\tilde{x}_1 + 0.58\tilde{x}_2 - 0.37\tilde{x}_3 - 0.72\tilde{x}_4$;

Canonical Form: $\hat{y} = 371.4 - 4.66X_1^2 - 3.81X_2^2 + 3.27X_3^2 - 1.20X_4^2$. (4)

Now we had thought it likely that we would find a maximum at S, in which case all four squared terms in Equation (4) would have had negative coefficients. However, the coefficient (+3.27) of X_3^2 is positive, and its standard error is roughly the same as that of a quadratic coefficient in Equation (2), that is, about 0.61. This implies that the response surface almost certainly has a minimum in the direction represented by X_3. If this is so, we will be able to move from the point S in either direction along the X_3 axis and get increased flight times.

In terms of the centered $\tilde{x}$'s,

$X_3 = 0.52\tilde{x}_1 - 0.45\tilde{x}_2 - 0.45\tilde{x}_3 + 0.57\tilde{x}_4$.

Thus, beginning at S, one direction of ascent along the X_3 axis is such that for each increase in $\tilde{x}_1$ of 0.52 units, $\tilde{x}_2$ must be reduced by 0.45 units, $\tilde{x}_3$ reduced by 0.45 units, and $\tilde{x}_4$ increased by 0.57 units. The units are those of the design given in Table 7. To follow the opposite direction of ascent you must make precisely the opposite changes. Before we explore these possibilities further, we consider a somewhat different form of analysis.

RIDGE ANALYSIS

In the original paper by Box and Wilson (1951), the application of the method of steepest ascent to response surfaces was discussed in general (not just for linear models) and in particular for second degree equations. For two variables the general concept can be understood by considering again the two dimensional contour representation of the minimax surface in Figure 9(c). As shown in Figure 10, suppose a series of concentric circles are drawn centered at the point O with increasing radius, r. It can be shown that, as r is increased, the circles will touch the contours of any response surface at a series of points at which the rate of increase or decrease in response with respect to r will be greatest. In units of x, the path formed by such points is thus one of maximum gradient and hence, of steepest ascent or descent. For a first degree equation, such as Equation (1), this is a straight line path at right angles to the planar contour surfaces, as in Figure (3). More generally, the path is curved. In particular, it was shown that, for a second degree equation, points along the paths of maximum gradient can be found for different values of r by solving a series of linear equations. A. E. Hoerl (1959) developed an extended technique of this kind under the general heading of "Ridge Analysis" and illustrated its use with many applications (see also R. W. Hoerl (1985). For more on the underlying theory, see Draper (1963).

Figure 10 shows, for the minimax surface of Figure 9, the paths of maximum gradient (two of steepest ascent and two of steepest descent originating from S). In this example, where O is close to S, these paths converge very rapidly onto the axes of the canonical variables X[sub1] and X[sub2]. Indeed, if we start at S instead of O, these axes are themselves the paths of steepest gradient. For the helicopter example, the paths of ascent can be followed either by ridge analysis from the origin O or by following the X[sub3] axis from the origin S. For this example, we obtain almost identical results by either method.

Mean flight times and dispersion for a series of helicopter designs along the X[sub3] ridge are summarized in Table 12. To better understand these results, we also show the dimensions of the tested helicopters in terms of the original variables of wing length, wing width, body length, and body width.

These tests fully confirm what was implied by the earlier canonical and ridge analyses--that we can indeed get longer flight times by proceeding in either of two directions. We can increase w and L as we reduce W and l or precisely the reverse. For sixteen helicopter designs along this path, Figure 11 shows graphically the mean flight times and standard deviations of flight times together with the dimensions of the associated helicopters. It will be seen that, in either direction, mean flight times of over 400 centiseconds can be obtained. These are almost twice the flight time of the original helicopter design. The plot shows that in both directions mean flight times go through a maximum and that the standard deviations remain reasonably constant except at the extremes where instability causes rapid increases.

Obviously, the process we have described could have been continued. However, we decided to quit at this point. In particular, we resisted the temptation to investigate further an analysis of individual degrees of freedom of the sum of squares for lack of fit. In Table 10, this lack of fit sum of squares was combined with that for "pure error" to provide that for "residual error." A more detailed analysis showed a large AQL interaction ($t = 4.3$) and a large component due to differences between curvature checks ($t = 3.8$). See, for example, Box and Draper (1987, p. 459). Both t values have 4 degrees of freedom. The reader may want to look further into these phenomena.

Of much greater importance is our hope that this example will encourage others to run their own iterative investigations. If paper helicopters are used, they can test their own ideas employing different starting points, varying different factors, and so forth. Also, devices other than the paper helicopter may be tried, and perhaps other methods of iterative investigation developed. Most of all we plead that the above may not be treated as providing just one more "data set" for students to further analyze and reanalyze. The art of investigation cannot be acquired by playing with someone else's data. You need to know what it feels like to make discoveries using your own.

CONCLUSIONS

The purpose of this paper is to provide an example of the use of statistics to catalyze investigation and discovery. The iterative use of appropriate statistical methods resulted in stable helicopters with flight times almost twice those of the original prototype. Two very different helicopter designs of this kind with almost equal performance were discovered. (In practice, considerations of cost, convenience of manufacture, and so forth, might decide the better choice at this point.) We believe that actual conduct of exercises of this kind can help students experience and understand scientific method and its catalysis using statistics.

ADDED MATERIAL

GEORGE E. P. BOX University of Wisconsin--Madison, Madison, WI 53705

PATRICK Y. T. LIU Kohler Company, Kohler, WI 53044

Dr. Box is the Director of Research at the Center for Quality and Productivity Improvement. He is also Professor Emeritus in the Department of Statistics and the Department of Industrial Engineering.

Mr. Liu is a Quality Engineer at the Kohler Company. He is a Member of ASQ.

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TABLE 1. Factor Levels Used in Design I: An Initial 2[sup8-4[sub[subIV] Screening Experiment

	Factor	Symbol	-1	+1
1.	Paper Type	P	regular	bond
2.	Wing Length	l	3.00 in.	4.75 in.
3.	Body Length	L	3.00 in.	4.75 in.
4.	Body Width	W	1.25 in.	2.00 in.
5.	Fold	F	no	yes
6.	Taped Body	T	no	yes
7.	Paper Clip	C	no	yes
8.	Taped Wing	M	no	yes

Here in. stands for Inches

TABLE 2. Design I: Layout and Data for 2[sup-8-4[sub[subIV] Screening Design

Run	P	l	L	W	F	T	C	M	y	s	100log(s)
1	-1	-1	-1	-1	-1	-1	-1	-1	236	2.1	31
2	1	-1	-1	-1	-1	1	1	1	185	4.7	67
3	-1	1	-1	-1	1	-1	1	1	259	2.7	42
4	1	1	-1	-1	1	1	-1	-1	318	5.3	72
5	-1	-1	1	-1	1	1	1	-1	180	7.7	89
6	1	-1	1	-1	1	-1	-1	1	195	7.7	89
7	-1	1	1	-1	-1	1	-1	1	246	9.0	96
8	1	1	1	-1	-1	-1	1	-1	229	3.2	450
9	-1	-1	-1	1	1	1	-1	1	196	11.5	106
10	1	-1	-1	1	1	-1	1	-1	203	10.0	100
11	-1	1	-1	1	-1	1	1	-1	230	2.9	46
12	1	1	-1	1	-1	-1	-1	1	261	15.3	118
13	-1	-1	1	1	-1	-1	1	1	168	11.3	105
14	1	-1	1	1	-1	1	-1	-1	197	11.7	107
15	-1	1	1	1	1	-1	-1	-1	220	16.0	120
16	1	1	1	1	1	1	1	1	241	6.8	83

TABLE 3. Design I: Estimates for a 2[sup8-4[sub[subIV] Screening Design

	Location	Dispersion
Constant	222.8	82.7
P	5.8	3.2
l	27.7	-4.1
L	-13.2	9.7
W	-8.3	15.6
F	3.7	5.1
T	1.4	0.6
C	-10.9	-9.8
M	-4.0	5.7
Pl + LC + WM + FT	6.0	-0.7
PL + lC + WT + FM	0.2	-13.4
PW + lM + LT + FC	5.0	0.6

PF + lT + LM + WC	7.0	-4.9
PT + lF + LW + CM	5.2	-4.0
PC + lL + WF + TM	-3.3	-0.9
PM + lW + LF + TC	-4.2	-2.1

TABLE 4. Design II: Factor Levels Used in 2[sup4] Experiment

	Factor	Symbol	-1	+1
1.	Wing Area (lw)	A	9.00 inch[sup2]	12.96 inch[sup2]
2.	Wing Length/Width Ratio (l/w)	Q	2.25	2.78
3.	Body Width	W	1.25 inches	2.00 inches
4.	Body Length	L	2.00 inches	3.00 inches

TABLE 5. Design II: Layout and Data for a 2[sup4] Design

Run	A	Q	W	L	y	s	100 log(s)
1	-1	-1	-1	-1	331	9.0	95
2	1	-1	-1	-1	339	22.6	136
3	-1	1	-1	-1	335	14.3	116
4	1	1	-1	-1	348	17.3	124
5	-1	-1	1	-1	330	9.1	96
6	1	-1	1	-1	354	11.9	108
7	-1	1	1	-1	355	14.9	118
8	1	1	1	-1	346	15.1	118
9	-1	-1	-1	1	301	11.9	108
10	1	-1	-1	1	326	14.9	117
11	-1	1	-1	1	313	37.7	158
12	1	1	-1	1	327	25.5	141
13	-1	-1	1	1	299	30.3	148
14	1	-1	1	1	319	3.0	48
15	-1	1	1	1	277	23.9	138
16	1	1	1	1	310	10.5	102

TABLE 6. Design II: Estimates for a 2[sup4] Design

	Location	Dispersion
Constant	325.6	116.8
A	8.1	-5.2
Q	0.7	9.8
W	-1.8	-7.3
L	-16.7	3.2
AQ	-1.6	-0.3
AW	0.6	-10.3
AL	3.6	-12.7
QW	-2.4	-0.4
QL	-3.1	4.8
WL	-5.8	-3.5
AQW	-0.9	7.0
AQL	1.8	5.0
AWL	1.3	-5.7
QWL	-3.1	-3.3
AQWL	3.9	4.3

TABLE 7. Factor Levels. The Levels (-1, 0, 1) Were Used in a 2[sup4] Factorial, Design IIa, With Center Points. Design IIIa is a Second Block Adding Later Axial and Center Points With Levels (-2, 0, 2) Producing a Central Composite Design

Factor	Symbol	-2	-1	0	+1	+2
Wing Area (lw [inch[sup2]])	A	11.20	11.80	12.40	13.00	13.60
Wing Length/Width Ratio (l/w)	Q	1.98	2.25	2.52	2.78	3.04
Body Width [inch]	W	0.75	1.00	1.25	1.50	1.75
Body Length [inch]	L	1.00	1.50	2.00	2.50	3.00

TABLE 8. Central Composite Design and Data; Block 1: Design IIIa, Block 2: Design IIIb

Run	Block	A	Q	W	L	y	100 log(s)
1	1	-1	-1	-1	-1	367	72
2	1	1	-1	-1	-1	369	72
3	1	-1	1	-1	-1	374	74
4	1	1	1	-1	-1	370	79
5	1	-1	-1	1	-1	372	72
6	1	1	-1	1	-1	355	81
7	1	-1	1	1	-1	397	72
8	1	1	1	1	-1	377	99
9	1	-1	-1	-1	1	350	90
10	1	1	-1	-1	1	373	86
11	1	-1	1	-1	1	358	92
12	1	1	1	-1	1	363	112
13	1	-1	-1	1	1	344	76
14	1	1	-1	1	1	355	69
15	1	-1	1	1	1	370	91
16	1	1	1	1	1	362	71
17	1	0	0	0	0	377	51
18	1	0	0	0	0	375	74
19	2	-2	0	0	0	361	111
20	2	2	0	0	0	364	93
21	2	0	-2	0	0	355	100
22	2	0	2	0	0	373	80
23	2	0	0	-2	0	361	71
24	2	0	0	2	0	360	98
25	2	0	0	0	-2	380	69
26	2	0	0	0	2	360	74
27	2	0	0	0	0	370	86
28	2	0	0	0	0	368	74
29	2	0	0	0	0	369	89
30	2	0	0	0	0	366	76

TABLE 9. Design IIIa: Estimated Coefficients for Mean Flight Times

	Coefficients
Constant	367.2
A	-0.4
Q	5.4
W	0.5
L	-6.6
AQ	-3.0
AW	-3.7
AL	4.3
QW	4.7
QL	-1.5
WL	-2.0
AQW	0.0

AQL	-1.8
AWL	0.7
QWL	-0.3
AQWL	-0.2

TABLE 10. Design III: Analysis of Variance for Completed Composite Design

Source	DF	SS	MS	F	P
Blocks	1	66.7	66.7	6.71	0.021
Regression	14	2907.3	207.6	20.88	<0.001
Linear Terms	4	1515.2	378.7	38.09	<0.001
Interaction Terms	6	1104.7	184.1	18.52	<0.001
Square Terms	4	287.4	71.8	7.23	0.002
Residual Error	14	139.2	9.9		
Lack-of-Fit	10	126.6	12.6	4.03	0.096
Pure Error	4	12.5	3.1		
Total	29	3113.2			

TABLE 11. Central Composite Design: Estimated Coefficients for Mean Flight Times

	Coefficients	Std. Error
Constant	372.06	1.29
A	-0.08	0.64
Q	5.08	0.64
W	0.25	0.64
L	-6.08	0.64
A[sup2]	-2.04	0.60
Q[sup2]	-1.66	0.60
W[sup2]	-2.54	0.60
L[sup2]	-0.16	0.60
AQ	-2.88	0.78
AW	-3.75	0.78
AL	4.38	0.78
QW	4.63	0.78
QL	-1.50	0.78
WL	-2.13	0.78

TABLE 12. Experimental Data on Second Order Steepest Ascent Path

Factor	A	Q	W	L	w	l	W	L	y	s
Coded Factor	x[sub1]	x[sub2]	x[sub3]	x[sub4]	in.	in.	in.	in.	cent-sec.	cent-sec.
Coefficient	0.52	-0.46	-0.45	0.57						
X[sub3] = 5.50	3.73	-2.92	-3.34	2.95	2.91	5.03	0.42	3.48	332	12.7
X[sub3] = 4.80	3.37	-2.60	-3.02	2.55	2.82	5.12	0.50	3.28	373	5.8
X[sub3] = 4.20	3.05	-2.32	-2.75	2.20	2.74	5.19	0.56	3.10	395	5.9
X[sub3] = 3.30	2.59	-1.91	-2.35	1.69	2.64	5.29	0.66	2.85	402	7.5
X[sub3] = 2.67	2.25	-1.62	-2.06	1.33	2.57	5.35	0.74	2.67	395	6.6
X[sub3] = 1.86	1.83	-1.25	-1.70	0.87	2.49	5.43	0.83	2.44	385	9.0
X[sub3] = 0.70	1.23	-0.71	-1.18	0.21	2.38	5.53	0.96	2.11	374	10.2
X[sub3] = 0.30	1.03	-0.53	-1.00	-0.02	2.34	5.56	1.00	1.99	372	7.6
X[sub3] = 0.00	0.87	-0.39	-0.86	-0.19	2.31	5.59	1.04	1.91	370	6.9
X[sub3] =-0.70	0.51	-0.07	-0.55	-0.59	2.25	5.64	1.11	1.71	376	6.3
X[sub3] =-1.05	0.32	0.09	-0.39	-0.79	2.22	5.66	1.15	1.61	379	8.4
X[sub3] =-1.82	-0.17	0.53	0.04	-1.33	2.15	5.72	1.26	1.34	387	9.0
X[sub3] =-2.51	-0.43	0.76	0.27	-1.62	2.11	5.75	1.32	1.19	406	5.4
X[sub3] =-3.47	-0.93	1.21	0.70	-2.17	2.04	5.81	1.43	0.92	416	6.2

X[sub3] =-3.70	-1.05	1.31	0.81	-2.30	2.02	5.82	1.45	0.85	399	8.8
X[sub3] =-4.22	-1.32	1.55	1.04	-2.60	1.99	5.84	1.51	0.70	350	33.2

The numbers in bold represent "Best Helicopters."

FIGURE 1. The Initial Helicopter Design.

FIGURE 2. Design I--Normal Plots for: (a) Location Effects From y and (b) Dispersion Effects from 100 log(s).

FIGURE 3. Design I: Contours of Mean Flight Times.

FIGURE 4. Data for 5 Helicopters on the Path of Steepest Ascent Calculated From Design I.

FIGURE 5. Design II--Normal Plots for: (a) Location Effects and (b) Dispersion Effects.

FIGURE 6. Data for 5 Helicopters on the Path of Steepest Ascent Calculated From Design II.

FIGURE 7. Design IIIa: Normal Plot for Location Effects.

FIGURE 8. Canonical Analysis of Second Degree Equation Representing a Maximum.

FIGURE 9. Canonical Analysis of Second Degree Equation Representing a Saddle.

FIGURE 10. Second Order Steepest Ascent and Ridge Analysis for the Example of Figure 9.

FIGURE 11. Characteristics of Helicopters Along the X[sub3] Axis.

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