Chapter 6 Exercise Solutions

Notes:
1. New exercises are denoted with an "☺".
2. For these solutions, we follow the MINITAB convention for determining whether a point is out of control. If a plot point is *within* the control limits, it is considered to be in control. If a plot point is *on* or *beyond* the control limits, it is considered to be out of control.
3. MINITAB defines some sensitizing rules for control charts differently than the standard rules. In particular, a run of *n* consecutive points on one side of the center line is defined as 9 points, not 8. This can be changed under Tools > Options > Control Charts and Quality Tools > Define Tests. Also fewer special cause tests are available for attributes control charts.

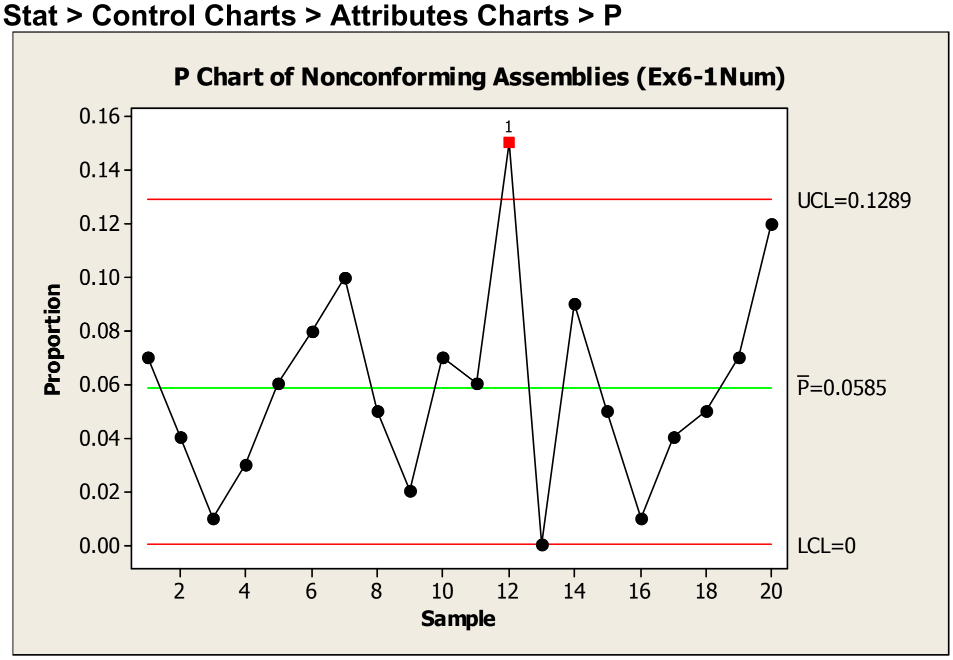
6-1.

$$n = 100;\quad m = 20;\quad \sum_{i=1}^{m} D_i = 117;\quad \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{117}{20(100)} = 0.0585$$

$$\text{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 + 3\sqrt{\frac{0.0585(1-0.0585)}{100}} = 0.1289$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0585 - 3\sqrt{\frac{0.0585(1-0.0585)}{100}} = 0.0585 - 0.0704 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex6-1Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

6-1 continued

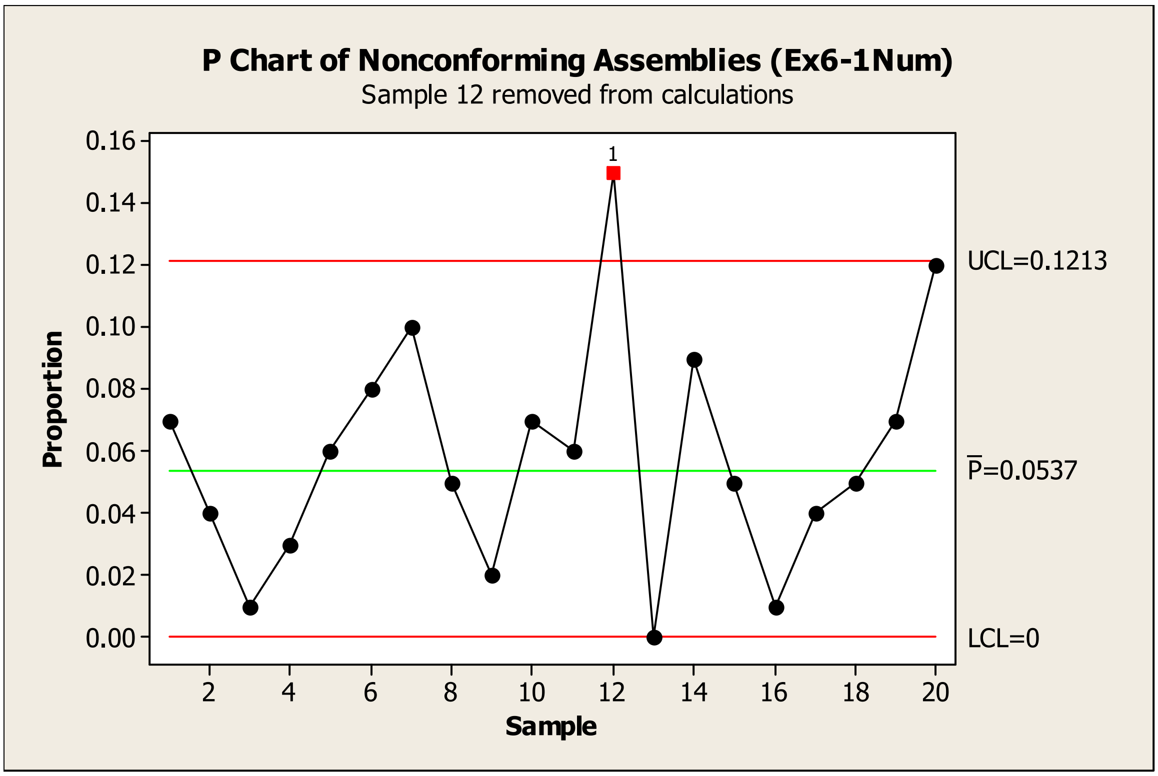
Sample 12 is out-of-control, so remove from control limit calculation:

$$n = 100; \quad m = 19; \quad \sum_{i=1}^{m} D_i = 102; \quad \overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{102}{19(100)} = 0.0537$$

$$\text{UCL}_p = 0.0537 + 3\sqrt{\frac{0.0537(1-0.0537)}{100}} = 0.1213$$

$$\text{LCL}_p = 0.0537 - 3\sqrt{\frac{0.0537(1-0.0537)}{100}} = 0.0537 - 0.0676 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P




```
Test Results for P Chart of Ex6-1Num
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  12
```

Chapter 6 Exercise Solutions

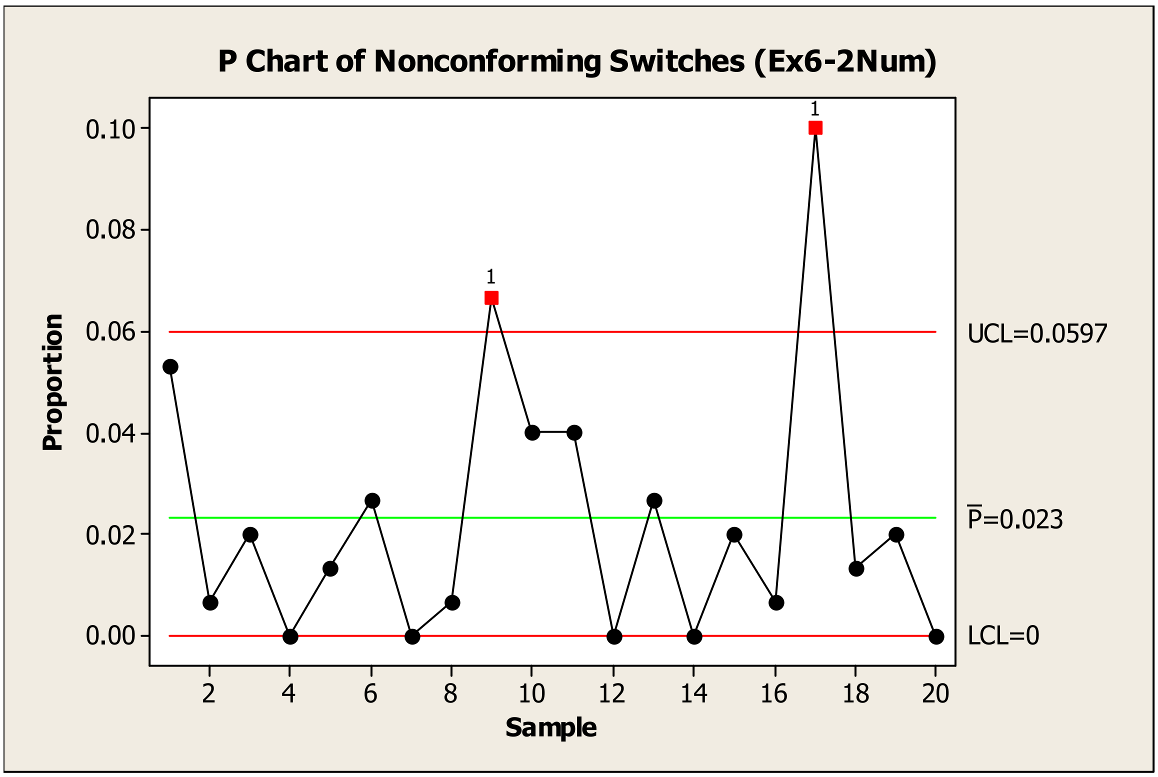
6-2.

$$n = 150; \quad m = 20; \quad \sum_{i=1}^{m} D_i = 69; \quad \bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{69}{20(150)} = 0.0230$$

$$\text{UCL}_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0230 + 3\sqrt{\frac{0.0230(1-0.0230)}{150}} = 0.0597$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0230 - 3\sqrt{\frac{0.0230(1-0.0230)}{150}} = 0.0230 - 0.0367 \Rightarrow 0$$

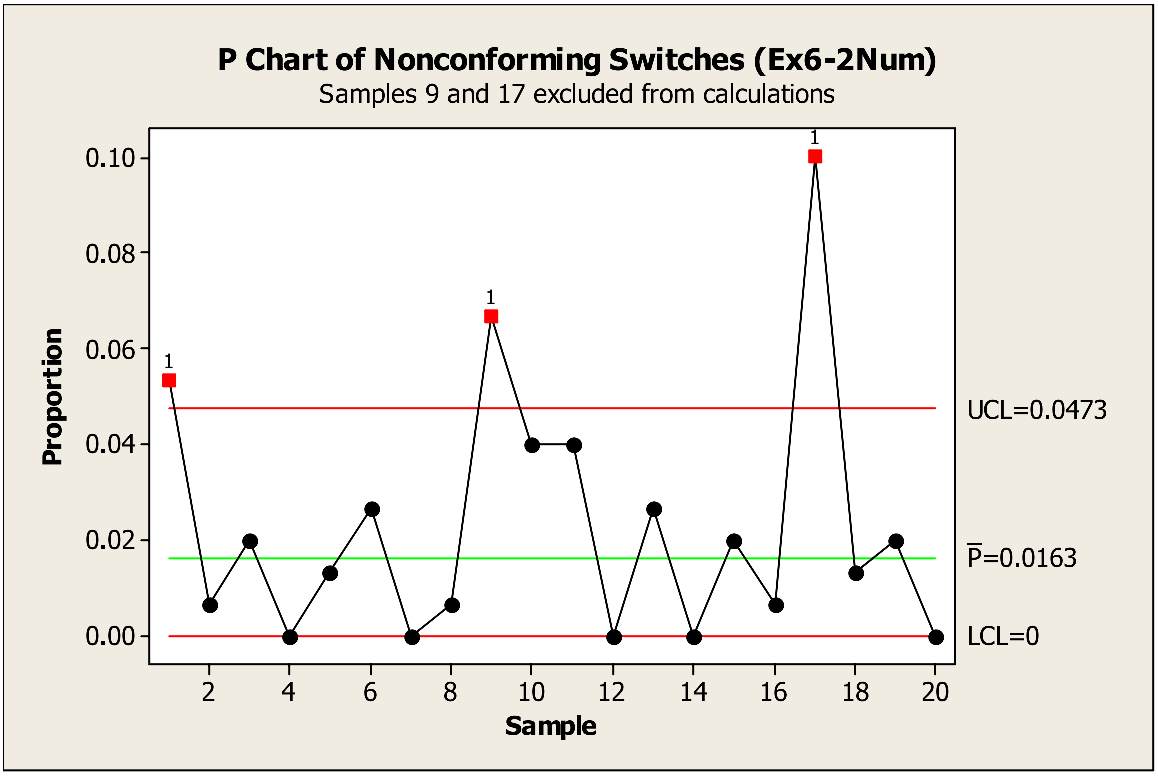
MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex6-2Num
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points: 9, 17

6-2 continued
Re-calculate control limits without samples 9 and 17:

MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex6-2Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 9, 17
```

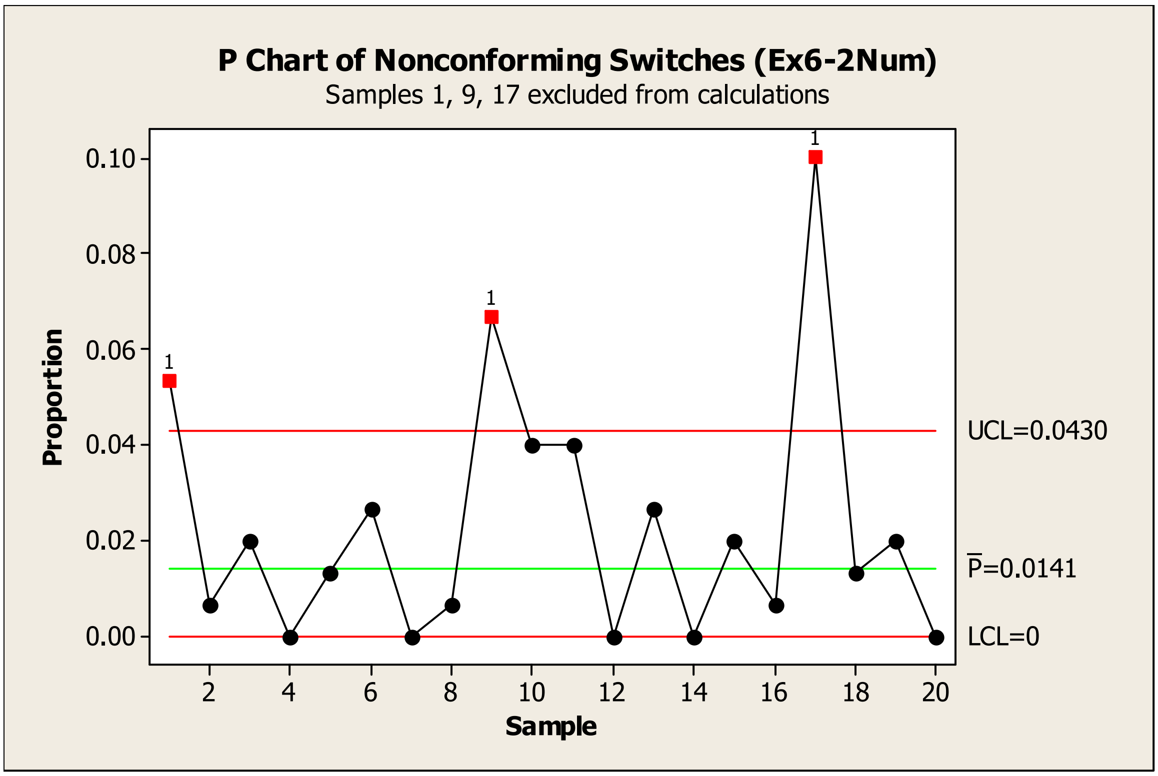
6-2 continued
Also remove sample 1 from control limits calculation:

$$n = 150;\quad m = 17;\quad \sum_{i=1}^{m} D_i = 36;\quad \overline{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{36}{17(150)} = 0.0141$$

$$\text{UCL}_p = 0.0141 + 3\sqrt{\frac{0.0141(1-0.0141)}{150}} = 0.0430$$

$$\text{LCL}_p = 0.0141 - 3\sqrt{\frac{0.0141(1-0.0141)}{150}} = 0.0141 - 0.0289 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex6-2Num
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 9, 17
```

6-3.
NOTE: There is an error in the table in the textbook. The Fraction Nonconforming for Day 5 should be 0.046.

$$m = 10;\ \sum_{i=1}^{m} n_i = 1000;\ \sum_{i=1}^{m} D_i = 60;\ \bar{p} = \sum_{i=1}^{m} D_i \Big/ \sum_{i=1}^{m} n_i = 60/1000 = 0.06$$

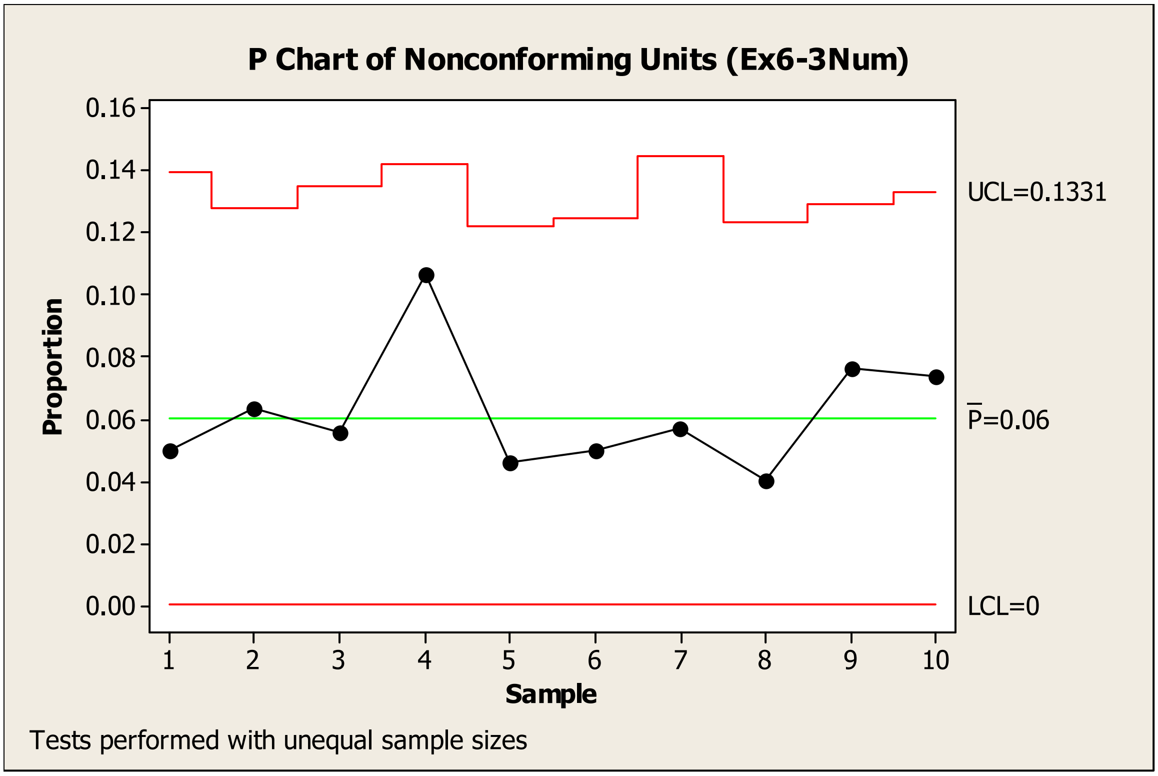
$$\text{UCL}_i = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n_i} \text{ and } \text{LCL}_i = \max\{0, \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n_i}\}$$

As an example, for $n = 80$:

$$\text{UCL}_1 = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n_1} = 0.06 + 3\sqrt{0.06(1-0.06)/80} = 0.1397$$

$$\text{LCL}_1 = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n_1} = 0.06 - 3\sqrt{0.06(1-0.06)/80} = 0.06 - 0.0797 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P



The process appears to be in statistical control.

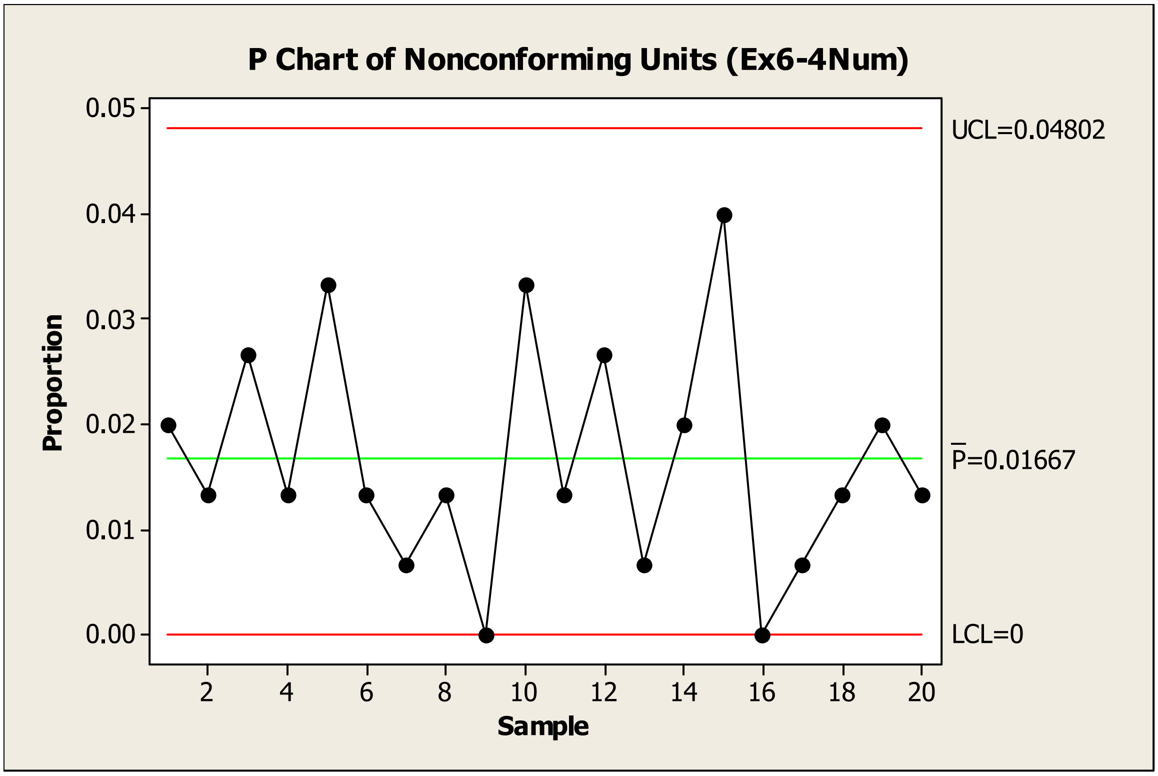
6-4.
(a)

$$n = 150;\quad m = 20;\quad \sum_{i=1}^{m} D_i = 50;\quad \overline{p} = \sum_{i=1}^{m} D_i \Big/ mn = 50/20(150) = 0.0167$$

$$\text{UCL} = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.0167 + 3\sqrt{0.0167(1-0.0167)/150} = 0.0480$$

$$\text{LCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.0167 - 3\sqrt{0.0167(1-0.0167)/150} = 0.0167 - 0.0314 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > P



The process appears to be in statistical control.

(b)
Using Equation 6-12,

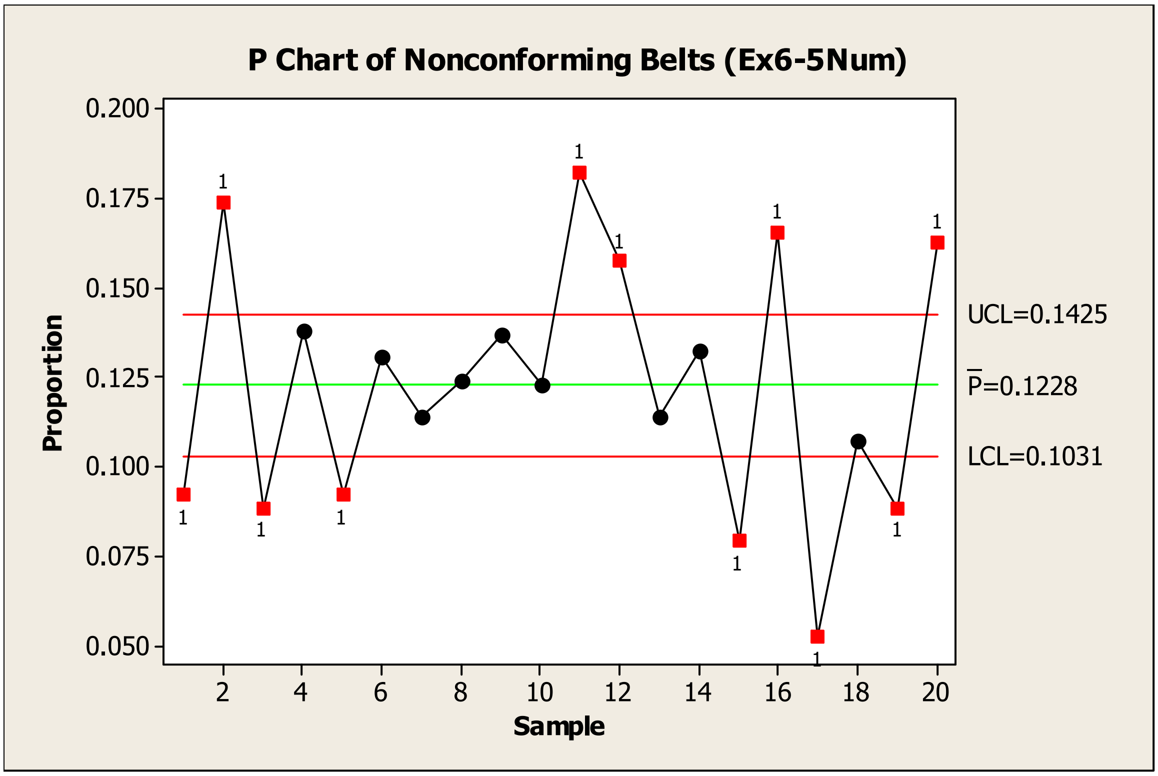
$$n > \frac{(1-p)}{p} L^2$$
$$> \frac{(1-0.0167)}{0.0167} (3)^2$$
$$> 529.9 \qquad \text{Select } n = 530.$$

6-5.
(a)

$$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.1228 + 3\sqrt{0.1228(1-0.1228)/2500} = 0.1425$$

$$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.1228 - 3\sqrt{0.1228(1-0.1228)/2500} = 0.1031$$

MTB > Stat > Control Charts > Attributes Charts > P



Test Results for P Chart of Ex6-5Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 3, 5, 11, 12, 15, 16, 17, 19, 20
```

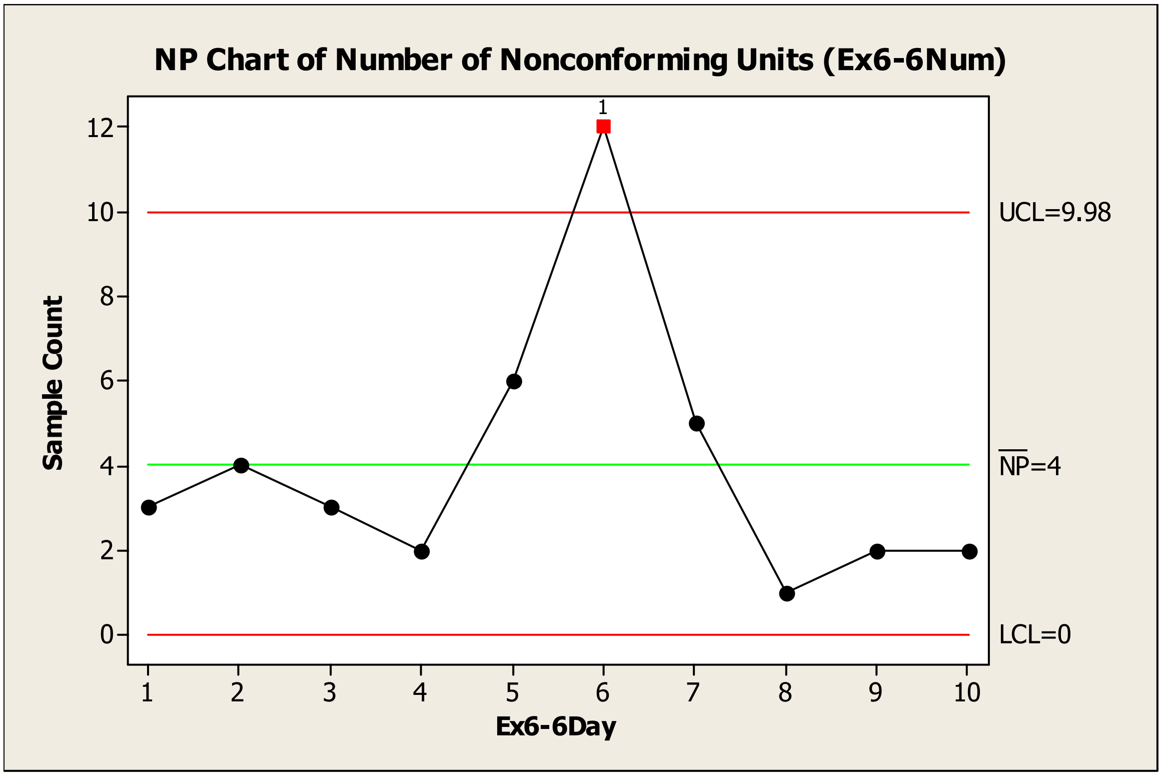
(b)
So many subgroups are out of control (11 of 20) that the data should not be used to establish control limits for future production. Instead, the process should be investigated for causes of the wild swings in *p*.

6-6.

$$\text{UCL} = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})} = 4 + 3\sqrt{4(1-0.008)} = 9.976$$

$$\text{LCL} = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})} = 4 - 3\sqrt{4(1-0.008)} = 4 - 5.976 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > NP

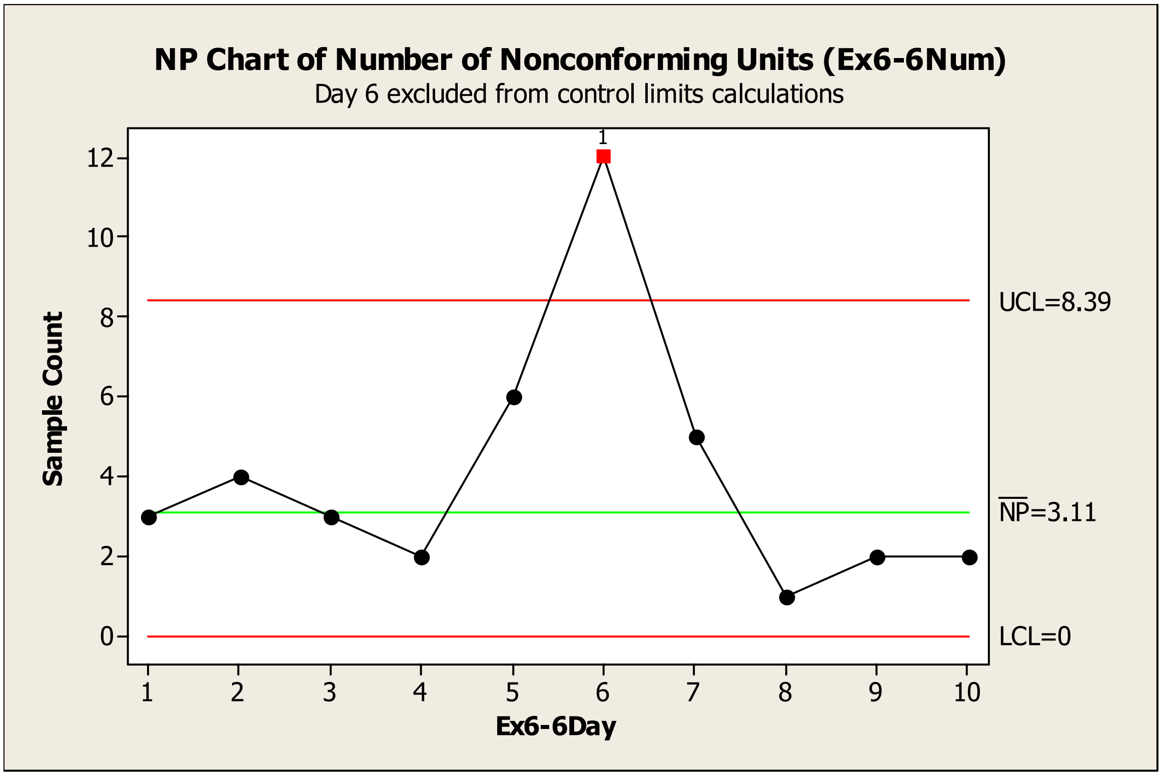


Test Results for NP Chart of Ex6-6Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  6
```

6.6 continued

Recalculate control limits without sample 6:



Test Results for NP Chart of Ex6-6Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  6
```

Recommend using control limits from second chart (calculated less sample 6).

6-7.

$\overline{p} = 0.02; n = 50$

$$\text{UCL} = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.02 + 3\sqrt{0.02(1-0.02)/50} = 0.0794$$

$$\text{LCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.02 - 3\sqrt{0.02(1-0.02)/50} = 0.02 - 0.0594 \Rightarrow 0$$

Since $p_{\text{new}} = 0.04 < 0.1$ and $n = 50$ is "large", use the Poisson approximation to the binomial with $\lambda = np_{\text{new}} = 50(0.04) = 2.00$.

Pr{detect|shift}

= 1 – Pr{not detect|shift}

= $1 - \beta$

= $1 - [\Pr\{D < n\text{UCL} \mid \lambda\} - \Pr\{D \le n\text{LCL} \mid \lambda\}]$

= $1 - \Pr\{D < 50(0.0794) \mid 2\} + \Pr\{D \le 50(0) \mid 2\}$

= 1 – POI(3,2) + POI(0,2) = 1 – 0.857 + 0.135 = 0.278

where POI(·) is the cumulative Poisson distribution.

Pr{detected by 3rd sample} = 1 – Pr{detected after 3rd} = $1 - (1 - 0.278)^3 = 0.624$

6-8.

$$m = 10;\ \ n = 250;\ \ \sum_{i=1}^{10} \hat{p}_i = 0.0440;\ \ \overline{p} = \frac{0.0440}{10} = 0.0044$$

$$\text{UCL} = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.0044 + 3\sqrt{0.0044(1-0.0044)/250} = 0.0170$$

$$\text{UCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.0044 - 3\sqrt{0.0044(1-0.0044)/250} = 0.0044 - 0.0126 \Rightarrow 0$$

No. The data from the shipment do not indicate statistical control. From the 6th sample, $(\hat{p}_6 = 0.020) > 0.0170$, the UCL.

6-9.

$\overline{p} = 0.10; n = 64$

$\text{UCL} = \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.10 + 3\sqrt{0.10(1-0.10)/64} = 0.2125$

$\text{LCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n} = 0.10 - 3\sqrt{0.10(1-0.10)/64} = 0.10 - 0.1125 \Rightarrow 0$

$$\begin{aligned}\beta &= \Pr\{D < n\text{UCL} \mid p\} - \Pr\{D \le n\text{LCL} \mid p\} \\ &= \Pr\{D < 64(0.2125) \mid p\} - \Pr\{D \le 64(0) \mid p\} \\ &= \Pr\{D < 13.6) \mid p\} - \Pr\{D \le 0 \mid p\}\end{aligned}$$

p	$\Pr\{D \le 13 \mid p\}$	$\Pr\{D \le 0 \mid p\}$	β
0.05	0.999999	0.037524	0.962475
0.10	0.996172	0.001179	0.994993
0.20	0.598077	0.000000	0.598077
0.21	**0.519279**	**0.000000**	**0.519279**
0.22	0.44154	0.000000	0.44154
0.215	0.480098	0.000000	0.480098
0.212	0.503553	0.000000	0.503553

Assuming $L = 3$ sigma control limits,

$$\begin{aligned}n &> \frac{(1-p)}{p}L^2 \\ &> \frac{(1-0.10)}{0.10}(3)^2 \\ &> 81\end{aligned}$$

6-10.

$np = 16.0;\ \ n = 100;\ \ \bar{p} = 16/100 = 0.16$

$\text{UCL} = np + 3\sqrt{np(1-\bar{p})} = 16 + 3\sqrt{16(1-0.16)} = 27.00$

$\text{LCL} = np - 3\sqrt{np(1-\bar{p})} = 16 - 3\sqrt{16(1-0.16)} = 5.00$

(a)

np_{new} = 20.0 > 15, so use normal approximation to binomial distribution.

$$
\begin{aligned}
\Pr\{\text{detect shift on 1st sample}\} &= 1-\beta \\
&= 1-[\Pr\{D<\text{UCL} \mid p\} - \Pr\{D \le \text{LCL} \mid p\}] \\
&= 1-\Phi\left(\frac{\text{UCL}+1/2-np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL}-1/2-np}{\sqrt{np(1-p)}}\right) \\
&= 1-\Phi\left(\frac{27+0.5-20}{\sqrt{20(1-0.2)}}\right) + \Phi\left(\frac{5-0.5-20}{\sqrt{20(1-0.2)}}\right) \\
&= 1-\Phi(1.875)+\Phi(-3.875) \\
&= 1-0.970+0.000 \\
&= 0.030
\end{aligned}
$$

Pr{detect by at least 3rd}

= 1 – Pr{detected after 3rd}

= 1 – $(1 - 0.030)^3$

= 0.0873

(b)

Assuming L = 3 sigma control limits,

$$
\begin{aligned}
n &> \frac{(1-p)}{p}L^2 \\
&> \frac{(1-0.16)}{0.16}(3)^2 \\
&> 47.25
\end{aligned}
$$

So, n = 48 is the minimum sample size for a positive LCL.

6-11.

$p = 0.10;\ \ p_{\text{new}} = 0.20;$ desire $\Pr\{\text{detect}\} = 0.50;$ assume $k = 3$ sigma control limits

$\delta = p_{\text{new}} - p = 0.20 - 0.10 = 0.10$

$$n = \left(\frac{k}{\delta}\right)^2 p(1-p) = \left(\frac{3}{0.10}\right)^2 (0.10)(1-0.10) = 81$$

6-12.
n = 100, p = 0.08, UCL = 0.161, LCL = 0

(a)

$$np = 100(0.080) = 8$$
$$\text{UCL} = np + 3\sqrt{np(1-p)} = 8 + 3\sqrt{8(1-0.080)} = 16.14$$
$$\text{LCL} = np - 3\sqrt{np(1-p)} = 8 - 3\sqrt{8(1-0.080)} = 8 - 8.1388 \Rightarrow 0$$

(b)
p = 0.080 < 0.1 and n =100 is large, so use Poisson approximation to the binomial.

Pr{type I error} = α
= Pr{D < LCL | p} + Pr{D > UCL | p}
= Pr{D < LCL | p} + [1 – Pr{$D \leq$ UCL | p}]
= Pr{D < 0 | 8} + [1 – Pr{$D \leq$ 16 | 8}]
= 0 + [1 – POI(16,8)]
= 0 + [1 – 0.996]
= 0.004
where POI(·) is the cumulative Poisson distribution.

(c)
np_{new} = 100(0.20) = 20 > 15, so use the normal approximation to the binomial.

$$\begin{aligned}\Pr\{\text{type II error}\} = \beta \\ &= \Pr\{\hat{p} < \text{UCL} \mid p_{\text{new}}\} - \Pr\{\hat{p} \leq \text{LCL} \mid p_{\text{new}}\} \\ &= \Phi\left(\frac{\text{UCL} - p_{\text{new}}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{\text{LCL} - p_{\text{new}}}{\sqrt{p(1-p)/n}}\right) \\ &= \Phi\left(\frac{0.161 - 0.20}{\sqrt{0.08(1-0.08)/100}}\right) - \Phi\left(\frac{0 - 0.20}{\sqrt{0.08(1-0.08)/100}}\right) \\ &= \Phi(-1.44) - \Phi(-7.37) \\ &= 0.07494 - 0 \\ &= 0.07494\end{aligned}$$

(d)
Pr{detect shift by at most 4th sample}
= 1 – Pr{not detect by 4th}
= 1 – $(0.07494)^4$
= 0.99997

6-13.

(a)

$\overline{p} = 0.07$; $k = 3$ sigma control limits; $n = 400$

$$\text{UCL} = \overline{p} + 3\sqrt{p(1-p)/n} = 0.07 + 3\sqrt{0.07(1-0.07)/400} = 0.108$$

$$\text{LCL} = \overline{p} - 3\sqrt{p(1-p)/n} = 0.07 - 3\sqrt{0.07(1-0.07)/400} = 0.032$$

(b)

$np_{\text{new}} = 400(0.10) => 40$, so use the normal approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{detect on 1st sample}\} &= 1 - \Pr\{\text{not detect on 1st sample}\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} - p}{\sqrt{p(1-p)/n}}\right) + \Phi\left(\frac{\text{LCL} - p}{\sqrt{p(1-p)/n}}\right) \\
&= 1 - \Phi\left(\frac{0.108 - 0.1}{\sqrt{0.1(1-0.1)/400}}\right) + \Phi\left(\frac{0.032 - 0.1}{\sqrt{0.1(1-0.1)/400}}\right) \\
&= 1 - \Phi(0.533) + \Phi(-4.533) \\
&= 1 - 0.703 + 0.000 \\
&= 0.297
\end{aligned}$$

(c)

Pr{detect on 1st or 2nd sample}

= Pr{detect on 1st} + Pr{not on 1st}×Pr{detect on 2nd}

= 0.297 + (1 – 0.297)(0.297)

= 0.506

6-14.

$p = 0.20$ and $L = 3$ sigma control limits

$$\begin{aligned}
n &> \frac{(1-p)}{p} L^2 \\
&> \frac{(1-0.20)}{0.20}(3)^2 \\
&> 36
\end{aligned}$$

For Pr{detect} = 0.50 after a shift to $p_{\text{new}} = 0.26$,

$$\delta = p_{\text{new}} - p = 0.26 - 0.20 = 0.06$$

$$n = \left(\frac{k}{\delta}\right)^2 p(1-p) = \left(\frac{3}{0.06}\right)^2 (0.20)(1-0.20) = 400$$

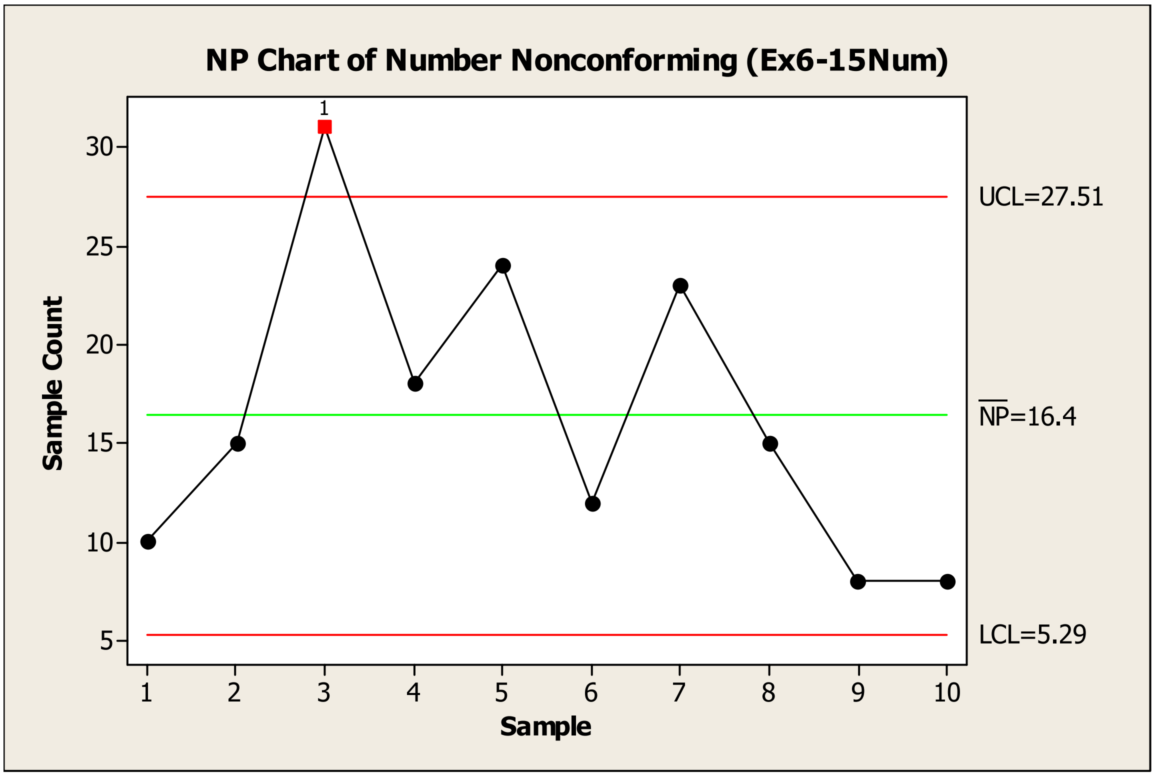
6-15.
(a)

$$m = 10; \quad n = 100; \quad \sum_{i=1}^{10} D_i = 164; \quad \overline{p} = \sum_{i=1}^{10} D_i \Big/ (mn) = 164/[10(100)] = 0.164; \quad n\overline{p} = 16.4$$

$$\text{UCL} = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})} = 16.4 + 3\sqrt{16.4(1-0.164)} = 27.51$$

$$\text{LCL} = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})} = 16.4 - 3\sqrt{16.4(1-0.164)} = 5.292$$

MTB > Stat > Control Charts > Attributes Charts > NP

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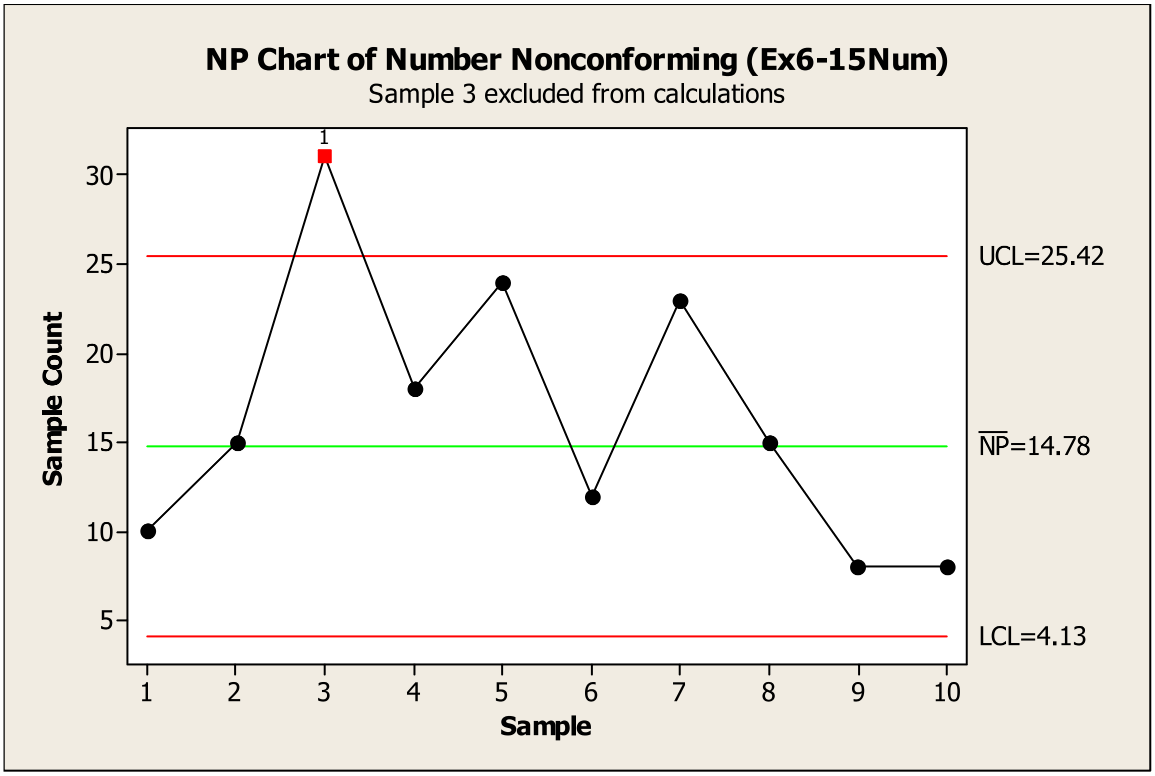
Test Results for NP Chart of Ex6-15Num
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  3
```

6-15 continued

Recalculate control limits less sample 3:



Test Results for NP Chart of Ex6-15Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  3
```

6-15 continued
(b)
$p_{\text{new}} = 0.30$. Since $p = 0.30$ is not too far from 0.50, and $n = 100 > 10$, the normal approximation to the binomial can be used.

$$\begin{aligned}
\Pr\{\text{detect on 1st}\} &= 1 - \Pr\{\text{not detect on 1st}\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{D < \text{UCL} \mid p\} - \Pr\{D \le \text{LCL} \mid p\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} + 1/2 - np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL} - 1/2 - np}{\sqrt{np(1-p)}}\right) \\
&= 1 - \Phi\left(\frac{25.42 + 0.5 - 30}{\sqrt{30(1-0.3)}}\right) + \Phi\left(\frac{4.13 - 0.5 - 30}{\sqrt{30(1-0.3)}}\right) \\
&= 1 - \Phi(-0.8903) + \Phi(-5.7544) \\
&= 1 - (0.187) + (0.000) \\
&= 0.813
\end{aligned}$$

6-16.
(a)

$$\text{UCL}_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.03 + 3\sqrt{0.03(1-0.03)/200} = 0.0662$$

$$\text{LCL}_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.03 - 3\sqrt{0.03(1-0.03)/200} = 0.03 - 0.0362 \Rightarrow 0$$

(b)
$p_{\text{new}} = 0.08$. Since $(p_{\text{new}} = 0.08) < 0.10$ and n is large, use the Poisson approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{detect on 1st sample} \mid p\} &= 1 - \Pr\{\text{not detect} \mid p\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}] \\
&= 1 - \Pr\{D < n\text{UCL} \mid np\} + \Pr\{D \le n\text{LCL} \mid np\} \\
&= 1 - \Pr\{D < 200(0.0662) \mid 200(0.08)\} + \Pr\{D \le 200(0) \mid 200(0.08)\} \\
&= 1 - \text{POI}(13,16) + \text{POI}(0,16) \\
&= 1 - 0.2745 + 0.000 \\
&= 0.7255
\end{aligned}$$

where POI($\cdot$) is the cumulative Poisson distribution.
Pr{detect by at least 4th} = 1 – Pr{detect after 4th} = 1 – $(1 – 0.7255)^4$ = 0.9943

6-17.

(a)

$$\bar{p} = \sum_{i=1}^{m} D_i \Big/ (mn) = 1200 \Big/ [30(400)] = 0.10; \quad n\bar{p} = 400(0.10) = 40$$

$$\text{UCL}_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 40 + 3\sqrt{40(1-0.10)} = 58$$

$$\text{LCL}_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 40 - 3\sqrt{40(1-0.10)} = 22$$

(b)

np_{new} = 400 (0.15) = 60 > 15, so use the normal approximation to the binomial.

$$\begin{aligned}
\Pr\{\text{detect on 1st sample} \mid p\} &= 1 - \Pr\{\text{not detect on 1st sample} \mid p\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{D < \text{UCL} \mid np\} - \Pr\{D \le \text{LCL} \mid np\}] \\
&= 1 - \Phi\left(\frac{\text{UCL} + 1/2 - np}{\sqrt{np(1-p)}}\right) + \Phi\left(\frac{\text{LCL} - 1/2 - np}{\sqrt{np(1-p)}}\right) \\
&= 1 - \Phi\left(\frac{58 + 0.5 - 60}{\sqrt{60(1-0.15)}}\right) + \Phi\left(\frac{22 - 0.5 - 60}{\sqrt{60(1-0.15)}}\right) \\
&= 1 - \Phi(-0.210) + \Phi(-5.39) \\
&= 1 - 0.417 + 0.000 \\
&= 0.583
\end{aligned}$$

6-18.
(a)

$$\text{UCL} = p + 3\sqrt{p(1-p)/n}$$

$$n = p(1-p)\left(\frac{3}{\text{UCL}-p}\right)^2 = 0.1(1-0.1)\left(\frac{3}{0.19-0.1}\right)^2 = 100$$

(b)
Using the Poisson approximation to the binomial, $\lambda = np = 100(0.10) = 10$.

$$\begin{aligned}\Pr\{\text{type I error}\} &= \Pr\{\hat{p} < \text{LCL} \mid p\} + \Pr\{\hat{p} > \text{UCL} \mid p\} \\ &= \Pr\{D < n\text{LCL} \mid \lambda\} + 1 - \Pr\{D \le n\text{UCL} \mid \lambda\} \\ &= \Pr\{D < 100(0.01) \mid 10\} + 1 - \Pr\{D \le 100(0.19) \mid 10\} \\ &= \text{POI}(0,10) + 1 - \text{POI}(19,10) \\ &= 0.000 + 1 - 0.996 \\ &= 0.004\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(c)
$p_{\text{new}} = 0.20$.
Using the Poisson approximation to the binomial, $\lambda = np_{\text{new}} = 100(0.20) = 20$.

$$\begin{aligned}\Pr\{\text{type II error}\} &= \beta \\ &= \Pr\{D < n\text{UCL} \mid \lambda\} - \Pr\{D \le n\text{LCL} \mid \lambda\} \\ &= \Pr\{D < 100(0.19) \mid 20\} - \Pr\{D \le 100(0.01) \mid 20\} \\ &= \text{POI}(18,20) - \text{POI}(1,20) \\ &= 0.381 - 0.000 \\ &= 0.381\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

6-19.
NOTE: There is an error in the textbook. This is a continuation of Exercise 6-17, not 6-18.

from 6-17(b), $1 - \beta = 0.583$
$\text{ARL}_1 = 1/(1-\beta) = 1/(0.583) = 1.715 \cong 2$

6-20.
from 6-18(c), $\beta = 0.381$
$\text{ARL}_1 = 1/(1-\beta) = 1/(1-0.381) = 1.616 \cong 2$

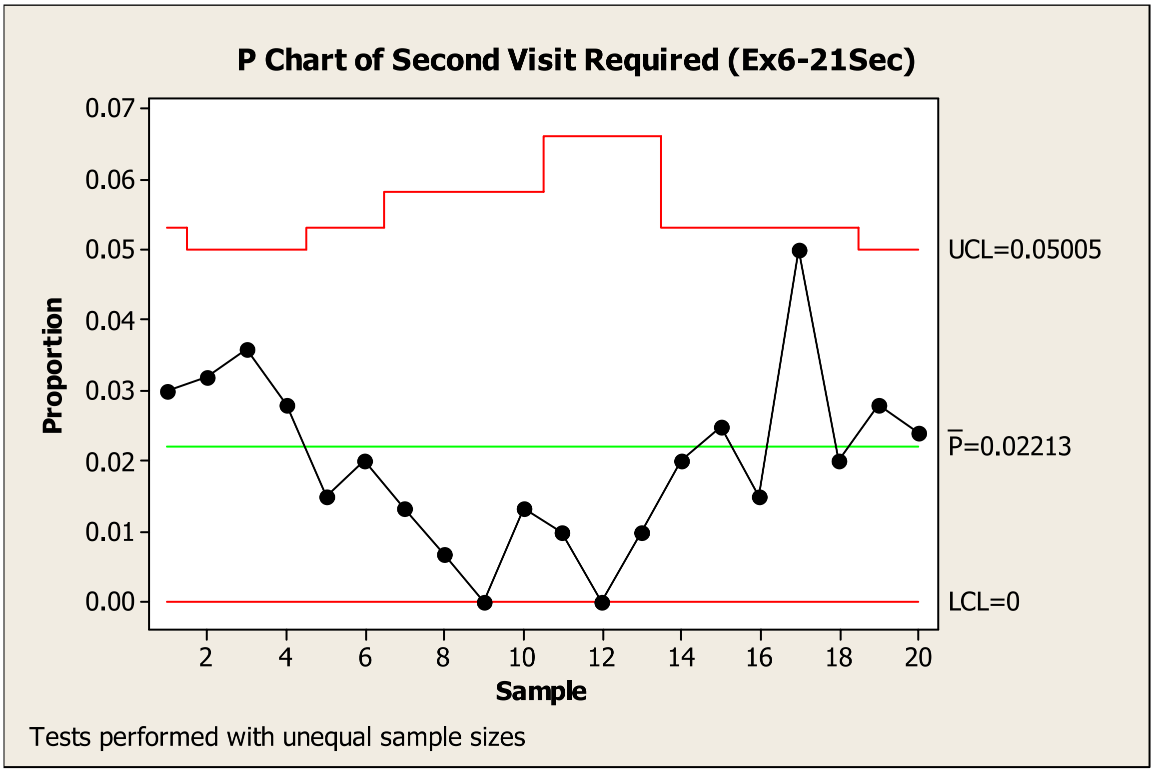
6-21.

(a)

For a p chart with variable sample size: $\bar{p} = \sum_i D_i / \sum_i n_i = 83/3750 = 0.0221$ and control limits are at $\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i}$

n_i	[LCL_i, UCL_i]
100	[0, 0.0662]
150	[0, 0.0581]
200	[0, 0.0533]
250	[0, 0.0500]

MTB > Stat > Control Charts > Attributes Charts > P



Process is in statistical control.

(b)

There are two approaches for controlling future production. The first approach would be to plot $\hat{p}_i$ and use constant limits unless there is a different size sample or a plot point near a control limit. In those cases, calculate the exact control limits by $\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i} = 0.0221 \pm 3\sqrt{0.0216/n_i}$. The second approach, preferred in many cases, would be to construct standardized control limits with control limits at ± 3, and to plot $Z_i = (\hat{p}_i - 0.0221)\Big/\sqrt{0.0221(1-0.0221)/n_i}$.

6-22.

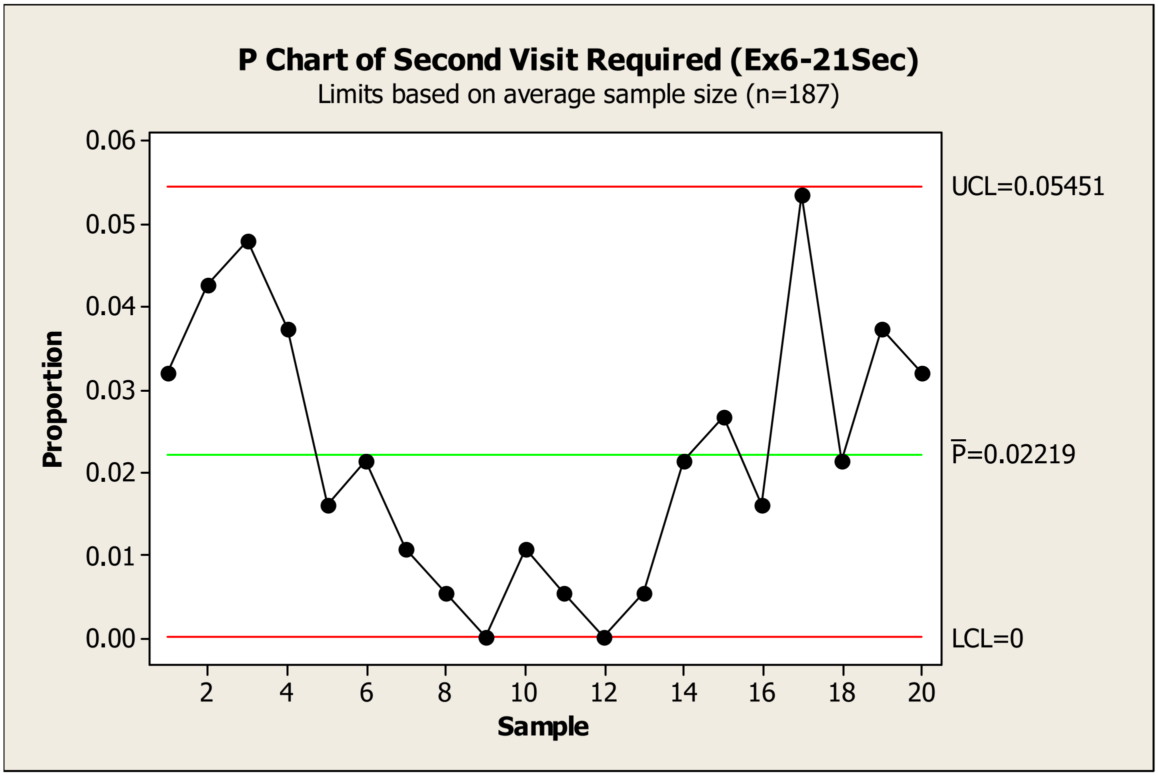
MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex6-21Req

```
Variable    N   Mean
Ex6-21Req  20  187.5
```

Average sample size is 187.5, however MINITAB accepts only integer values for n. Use a sample size of $n = 187$, and carefully examine points near the control limits.

MTB > Stat > Control Charts > Attributes Charts > P

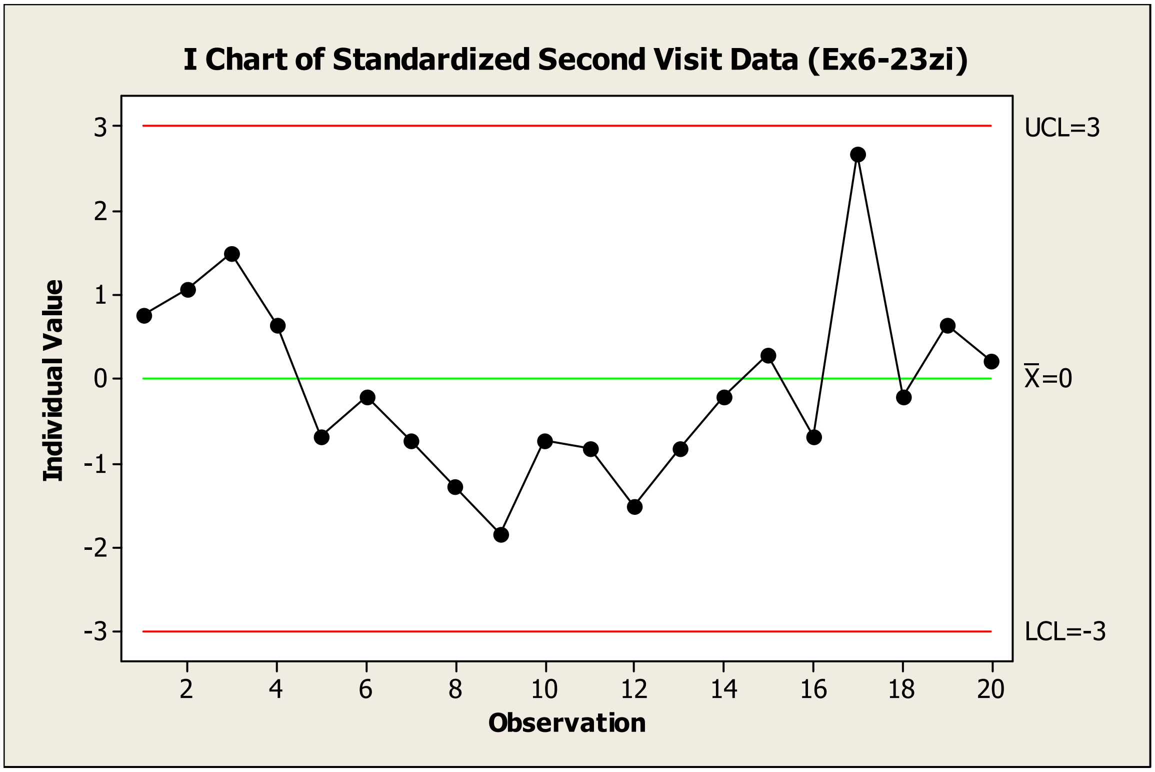


Process is in statistical control.

6-23.

$$z_i = (\hat{p}_i - \overline{p}) \Big/ \sqrt{\overline{p}(1-\overline{p})/n_i} = (\hat{p}_i - 0.0221) \Big/ \sqrt{0.0216/n_i}$$

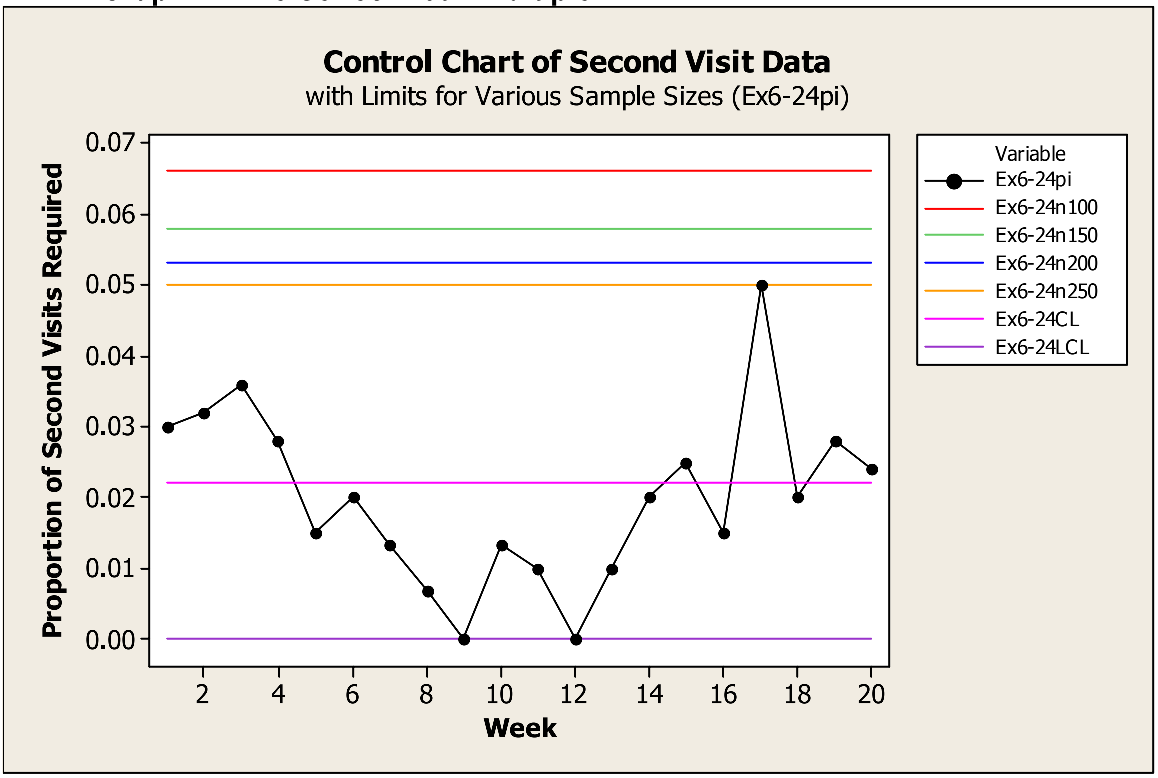
MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



Process is in statistical control.

6-24.
CL = 0.0221, LCL = 0
$UCL_{100} = 0.0662$, $UCL_{150} = 0.0581$, $UCL_{200} = 0.0533$, $UCL_{250} = 0.0500$

MTB > Graph > Time Series Plot > Multiple



6-25.
UCL = 0.0399; $\bar{p}$ = CL = 0.01; LCL = 0; n = 100

$$
\begin{aligned}
n &> \left(\frac{1-p}{p}\right)L^2 \\
&> \left(\frac{1-0.01}{0.01}\right)3^2 \\
&> 891 \\
&\geq 892
\end{aligned}
$$

6-26.
The *np* chart is inappropriate for varying sample sizes because the centerline (process center) would change with each n_i.

6-27.
n = 400; UCL = 0.0809; p = CL = 0.0500; LCL = 0.0191

(a)

$$
\begin{aligned}
0.0809 &= 0.05 + L\sqrt{0.05(1-0.05)/400} = 0.05 + L(0.0109) \\
L &= 2.8349
\end{aligned}
$$

(b)

$$
\begin{aligned}
\text{CL} &= np = 400(0.05) = 20 \\
\text{UCL} &= np + 2.8349\sqrt{np(1-p)} = 20 + 2.8349\sqrt{20(1-0.05)} = 32.36 \\
\text{LCL} &= np - 2.8349\sqrt{np(1-p)} = 20 - 2.8349\sqrt{20(1-0.05)} = 7.64
\end{aligned}
$$

(c)
n = 400 is large and p = 0.05 < 0.1, use Poisson approximation to binomial.

$$
\begin{aligned}
&\Pr\{\text{detect shift to 0.03 on 1st sample}\} \\
&= 1 - \Pr\{\text{not detect}\} \\
&= 1 - \beta \\
&= 1 - [\Pr\{D < \text{UCL} \mid \lambda\} - \Pr\{D \leq \text{LCL} \mid \lambda\}] \\
&= 1 - \Pr\{D < 32.36 \mid 12\} + \Pr\{D \leq 7.64 \mid 12\} \\
&= 1 - \text{POI}(32,12) + \text{POI}(7,12) \\
&= 1 - 1.0000 + 0.0895 \\
&= 0.0895
\end{aligned}
$$

where POI(·) is the cumulative Poisson distribution.

6-28.
(a)

$$\text{UCL} = p + L\sqrt{p(1-p)/n}$$
$$0.0962 = 0.0500 + L\sqrt{0.05(1-0.05)/400}$$
$$L = 4.24$$

(b)
p = 15, λ = np = 400(0.15) = 60 > 15, use normal approximation to binomial.

$$\begin{aligned}
&\Pr\{\text{detect on 1st sample after shift}\}\\
&= 1 - \Pr\{\text{not detect}\}\\
&= 1 - \beta\\
&= 1 - [\Pr\{\hat{p} < \text{UCL} \mid p\} - \Pr\{\hat{p} \le \text{LCL} \mid p\}]\\
&= 1 - \Phi\left(\frac{\text{UCL} - p}{\sqrt{p(1-p)/n}}\right) + \Phi\left(\frac{\text{LCL} - p}{\sqrt{p(1-p)/n}}\right)\\
&= 1 - \Phi\left(\frac{0.0962 - 0.15}{\sqrt{0.15(1-0.15)/400}}\right) + \Phi\left(\frac{0.0038 - 0.15}{\sqrt{0.15(1-0.15)/400}}\right)\\
&= 1 - \Phi(-3.00) + \Phi(-8.19)\\
&= 1 - 0.00135 + 0.000\\
&= 0.99865
\end{aligned}$$

6-29.
$p = 0.01;\ L = 2$

(a)

$$\begin{aligned}
n &> \left(\frac{1-p}{p}\right)L^2\\
&> \left(\frac{1-0.01}{0.01}\right)2^2\\
&> 396\\
&\ge 397
\end{aligned}$$

(b)
δ = 0.04 – 0.01 = 0.03

$$n = \left(\frac{L}{\delta}\right)^2 p(1-p) = \left(\frac{2}{0.03}\right)^2 (0.01)(1-0.01) = 44$$

6-30.

(a)

$$
\begin{aligned}
&\Pr\{\text{type I error}\}\\
&= \Pr\{\hat{p} < \text{LCL} \mid p\} + \Pr\{\hat{p} > \text{UCL} \mid p\}\\
&= \Pr\{D < n\text{LCL} \mid np\} + 1 - \Pr\{D \leq n\text{UCL} \mid np\}\\
&= \Pr\{D < 100(0.0050) \mid 100(0.04)\} + 1 - \Pr\{D \leq 100(0.075) \mid 100(0.04)\}\\
&= \text{POI}(0,4) + 1 - \text{POI}(7,4)\\
&= 0.018 + 1 - 0.948\\
&= 0.070
\end{aligned}
$$

where POI(·) is the cumulative Poisson distribution.

(b)

$$
\begin{aligned}
&\Pr\{\text{type II error}\}\\
&= \beta\\
&= \Pr\{D < n\text{UCL} \mid np\} - \Pr\{D \leq n\text{LCL} \mid np\}\\
&= \Pr\{D < 100(0.075) \mid 100(0.06)\} - \Pr\{D \leq 100(0.005) \mid 100(0.06)\\
&= \text{POI}(7,6) - \text{POI}(0,6)\\
&= 0.744 - 0.002\\
&= 0.742
\end{aligned}
$$

where POI(·) is the cumulative Poisson distribution.

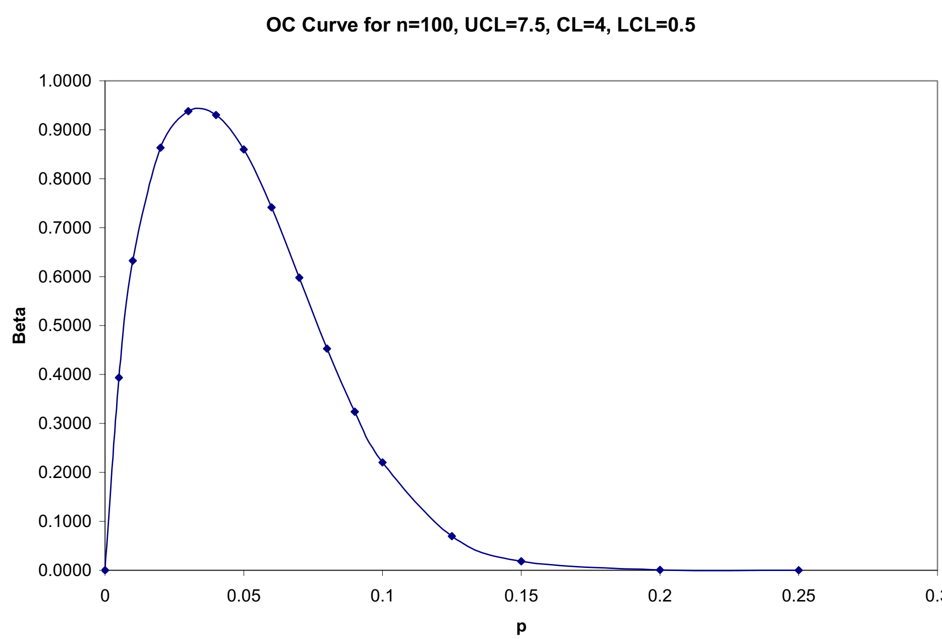
6-30 continued

(c)

$$\beta = \Pr\{D < n\text{UCL} \mid np\} - \Pr\{D \le n\text{LCL} \mid np\}$$
$$= \Pr\{D < 100(0.0750) \mid 100p\} - \Pr\{D \le 100(0.0050) \mid 100p\}$$
$$= \Pr\{D < 7.5 \mid 100p\} - \Pr\{D \le 0.5 \mid 100p\}$$

Excel : workbook Chap06.xls : worksheet Ex6-30

p	np	Pr{D<7.5|np}	Pr{D<=0.5|np}	beta
0	0	1.0000	1.0000	0.0000
0.005	0.5	1.0000	0.6065	0.3935
0.01	1	1.0000	0.3679	0.6321
0.02	2	0.9989	0.1353	0.8636
0.03	3	0.9881	0.0498	0.9383
0.04	4	0.9489	0.0183	0.9306
0.05	5	0.8666	0.0067	0.8599
0.06	6	0.7440	0.0025	0.7415
0.07	7	0.5987	0.0009	0.5978
0.08	8	0.4530	0.0003	0.4526
0.09	9	0.3239	0.0001	0.3238
0.1	10	0.2202	0.0000	0.2202
0.125	12.5	0.0698	0.0000	0.0698
0.15	15	0.0180	0.0000	0.0180
0.2	20	0.0008	0.0000	0.0008
0.25	25	0.0000	0.0000	0.0000



(d)

from part (a), $\alpha = 0.070$: $ARL_0 = 1/\alpha = 1/0.070 = 14.29 \cong 15$

from part (b), $\beta = 0.0742$: $ARL_1 = 1/(1-\beta) = 1/(1-0.742) = 3.861 \cong 4$

6-31.

$n = 100$; $\bar{p} = 0.02$

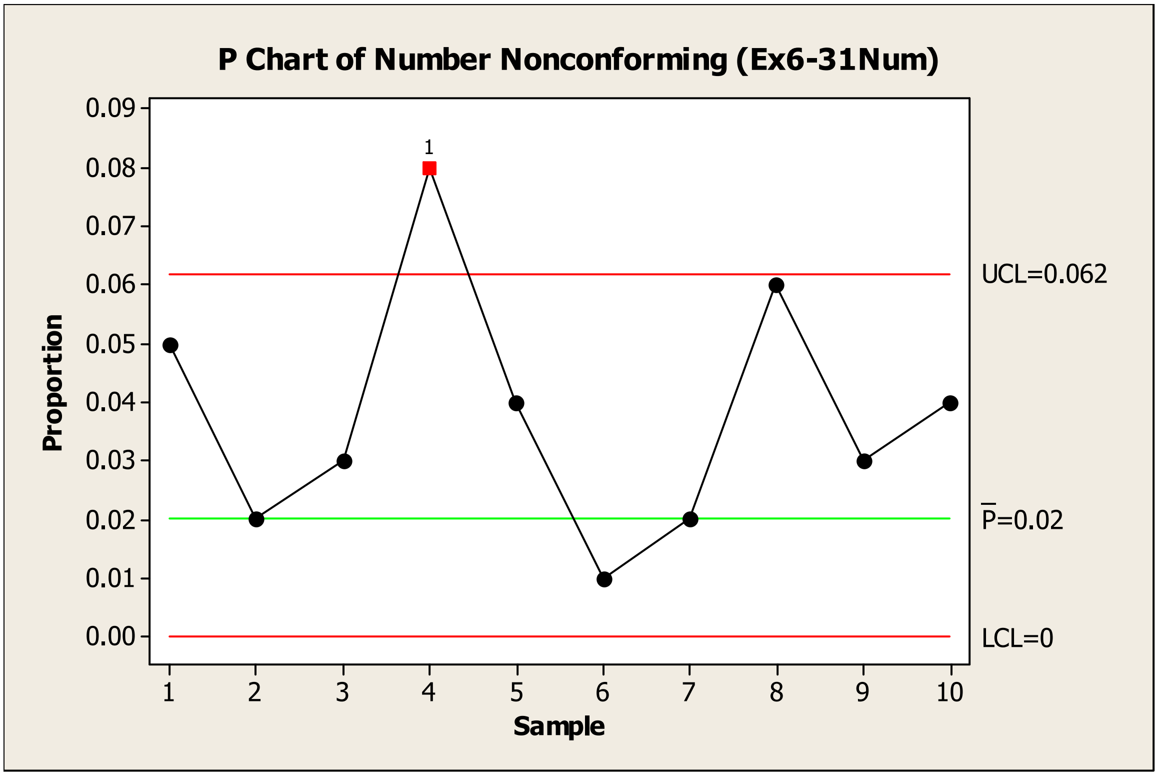
(a)

$$\text{UCL} = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.02 + 3\sqrt{0.02(1-0.02)/100} = 0.062$$

$$\text{LCL} = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n} = 0.02 - 3\sqrt{0.02(1-0.02)/100} \Rightarrow 0$$

(b)

MTB > Stat > Control Charts > Attributes Charts > P




```
Test Results for P Chart of Ex6-31Num
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  4
```

Sample 4 exceeds the upper control limit.

$\bar{p} = 0.038$ and $\hat{\sigma}_p = 0.0191$

6-32.

$$\text{LCL} = n\bar{p} - k\sqrt{n\bar{p}(1-\bar{p})} > 0$$

$$n\bar{p} > k\sqrt{n\bar{p}(1-\bar{p})}$$

$$n > k^2\left(\frac{1-\bar{p}}{\bar{p}}\right)$$

6-33.

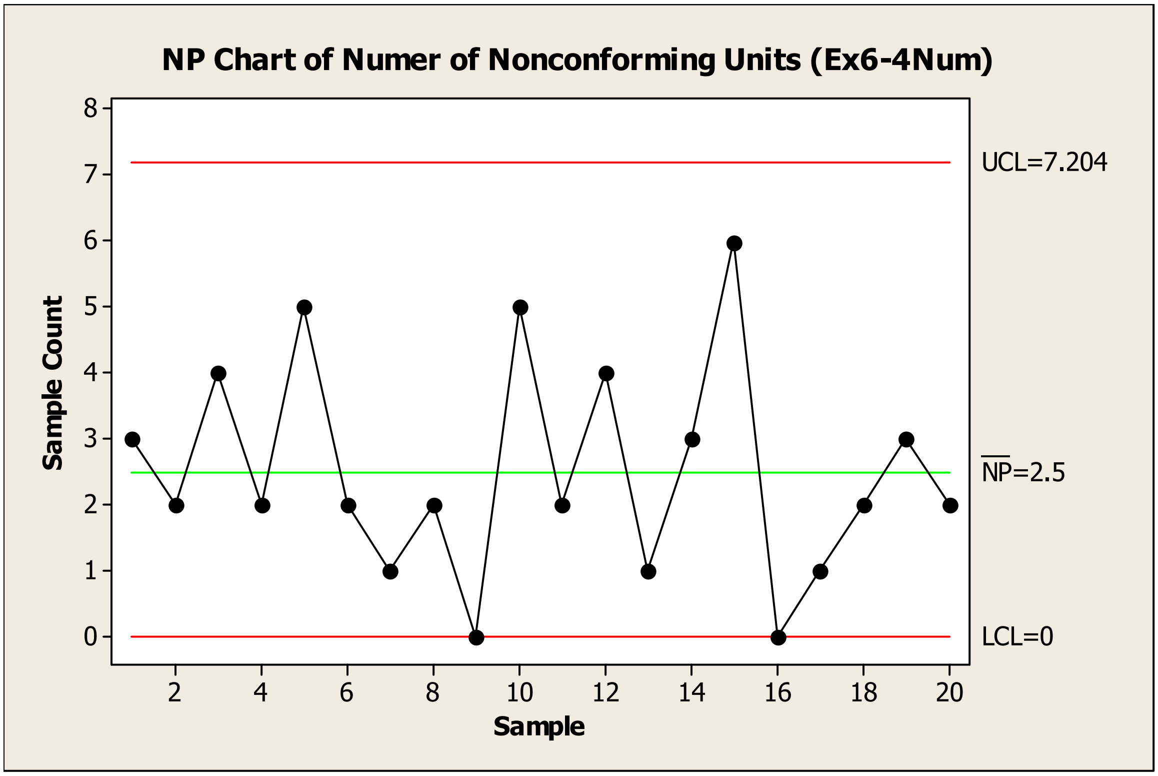
$$n = 150; \quad m = 20; \quad \sum D = 50; \quad \overline{p} = 0.0167$$

$$\text{CL} = n\overline{p} = 150(0.0167) = 2.505$$

$$\text{UCL} = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})} = 2.505 + 3\sqrt{2.505(1-0.0167)} = 7.213$$

$$\text{LCL} = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})} = 2.505 - 4.708 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > NP



The process is in control; results are the same as for the *p* chart.

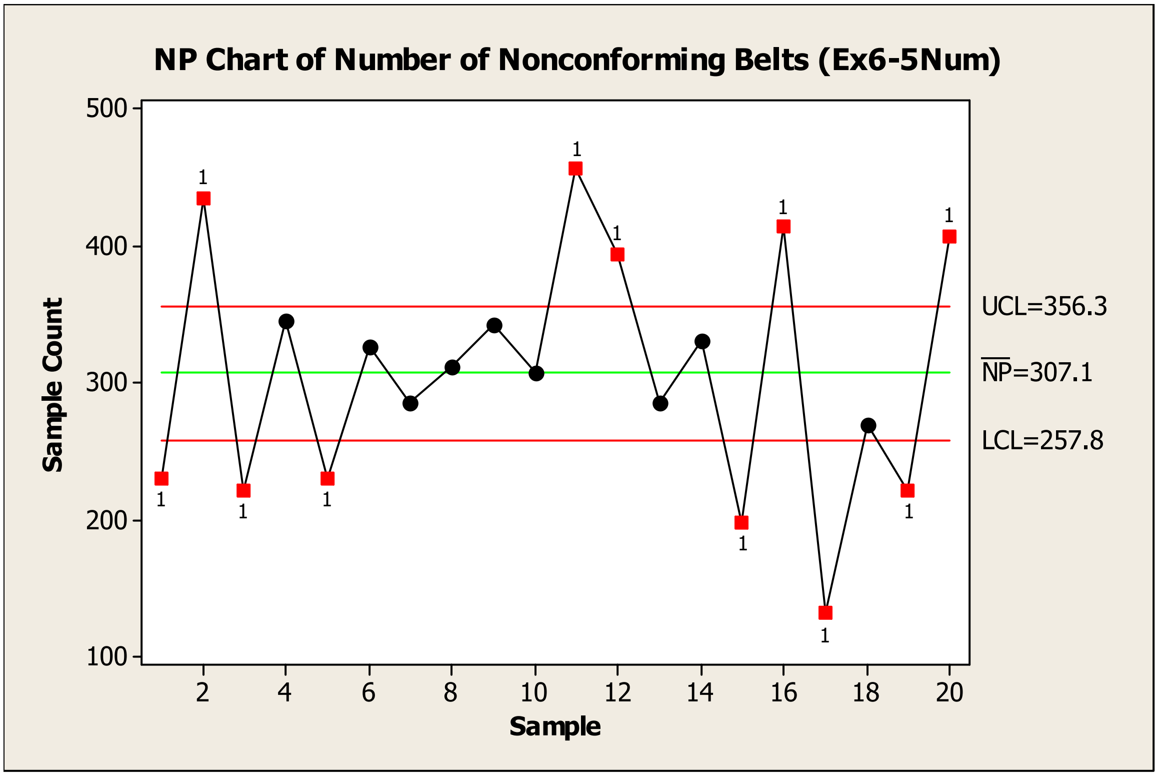
6-34.

$$\text{CL} = n\bar{p} = 2500(0.1228) = 307$$

$$\text{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 307 + 3\sqrt{307(1-0.1228)} = 356.23$$

$$\text{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 307 - 3\sqrt{307(1-0.1228)} = 257.77$$

MTB > Stat > Control Charts > Attributes Charts > NP



Test Results for NP Chart of Ex6-5Num

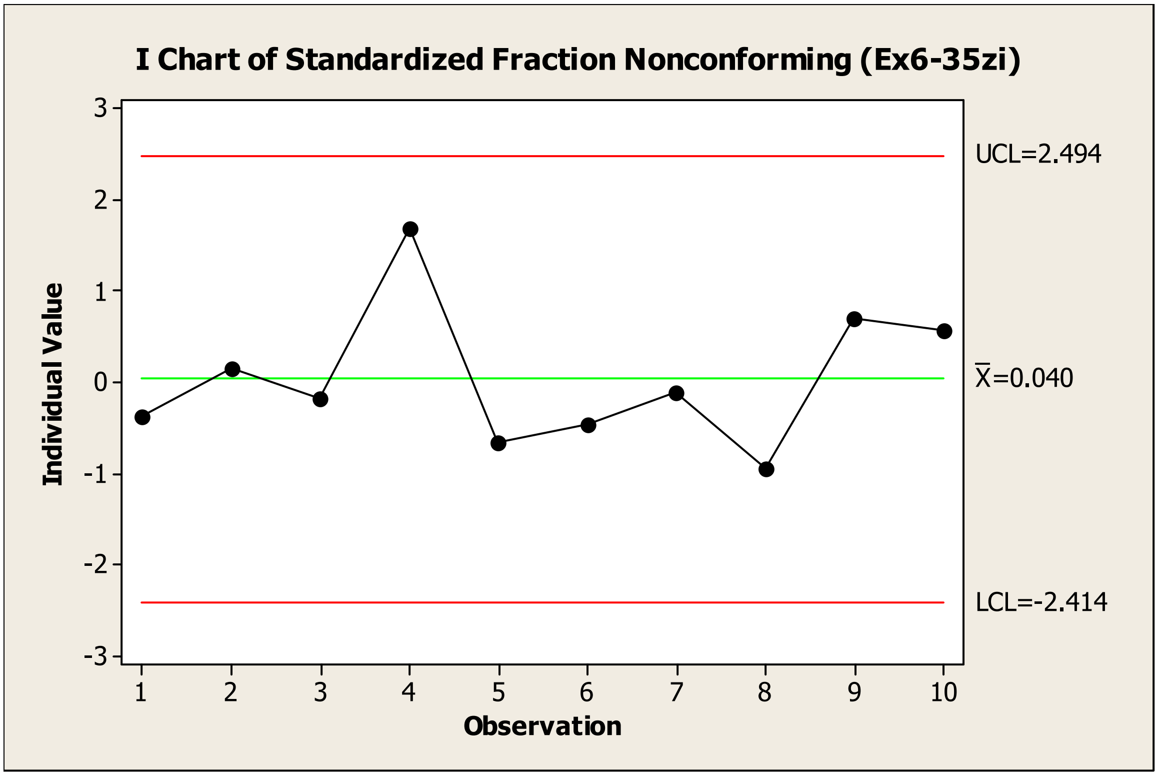
```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 3, 5, 11, 12, 15, 16, 17, 19, 20
```

Like the p control chart, many subgroups are out of control (11 of 20), indicating that this data should not be used to establish control limits for future production.

6-35.
$\overline{p} = 0.06$

$z_i = (\hat{p}_i - 0.06)\Big/\sqrt{0.06(1-0.06)/n_i} = (\hat{p}_i - 0.06)\Big/\sqrt{0.0564/n_i}$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



The process is in control; results are the same as for the *p* chart.

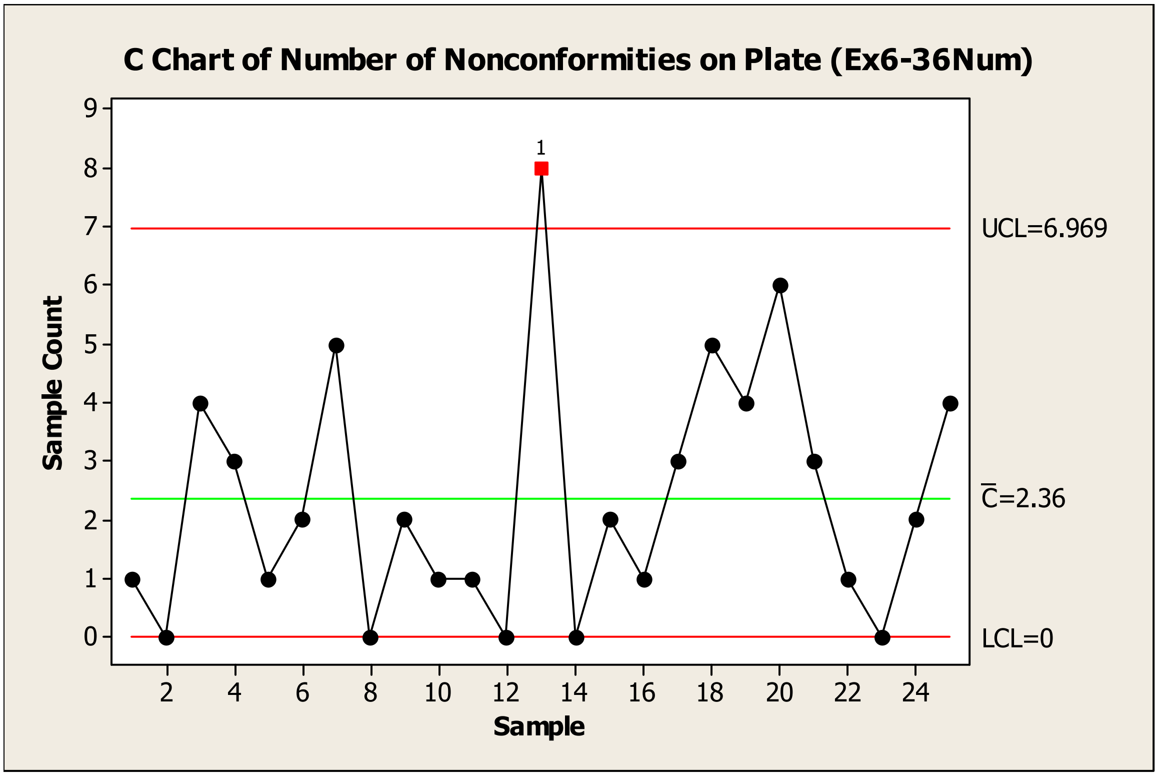
6-36.

$$\text{CL} = \bar{c} = 2.36$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 2.36 + 3\sqrt{2.36} = 6.97$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 2.36 - 3\sqrt{2.36} \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > C



Test Results for C Chart of Ex6-36Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  13
```

No. The plate process does not seem to be in statistical control.

6-37.

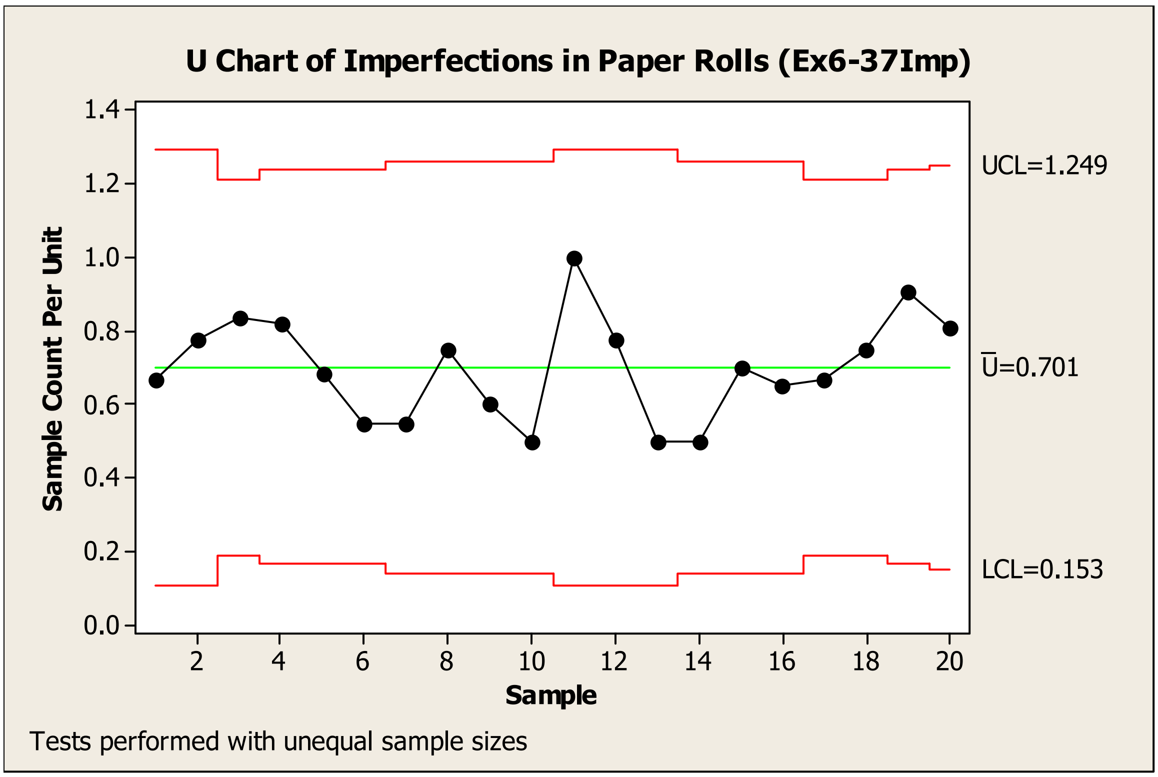
$$\text{CL} = \bar{u} = 0.7007$$

$$\text{UCL}_i = \bar{u} + 3\sqrt{\bar{u}/n_i} = 0.7007 + 3\sqrt{0.7007/n_i}$$

$$\text{LCL}_i = \bar{u} - 3\sqrt{\bar{u}/n_i} = 0.7007 - 3\sqrt{0.7007/n_i}$$

n_i	[LCL$_i$, UCL$_i$]
18	[0.1088, 1.2926]
20	[0.1392, 1.2622]
21	[0.1527, 1.2487]
22	[0.1653, 1.2361]
24	[0.1881, 1.2133]

MTB > Stat > Control Charts > Attributes Charts > U



6-38.

$$\text{CL} = \bar{u} = 0.7007; \quad \bar{n} = 20.55$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/\bar{n}} = 0.7007 + 3\sqrt{0.7007/20.55} = 1.2547$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/\bar{n}} = 0.7007 - 3\sqrt{0.7007/20.55} = 0.1467$$

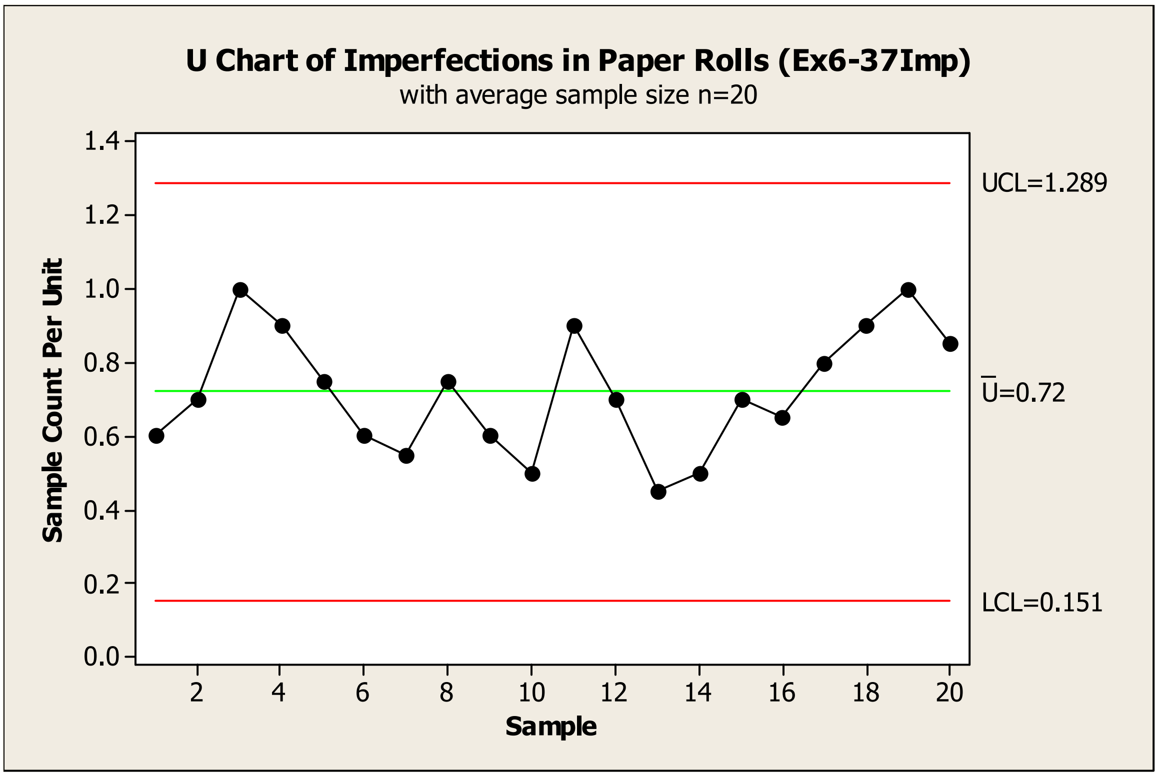
MTB > Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex6-37Rol

```
Variable    N    Mean
Ex6-37Rol  20  20.550
```

Average sample size is 20.55, however MINITAB accepts only integer values for *n*. Use a sample size of *n* = 20, and carefully examine points near the control limits.

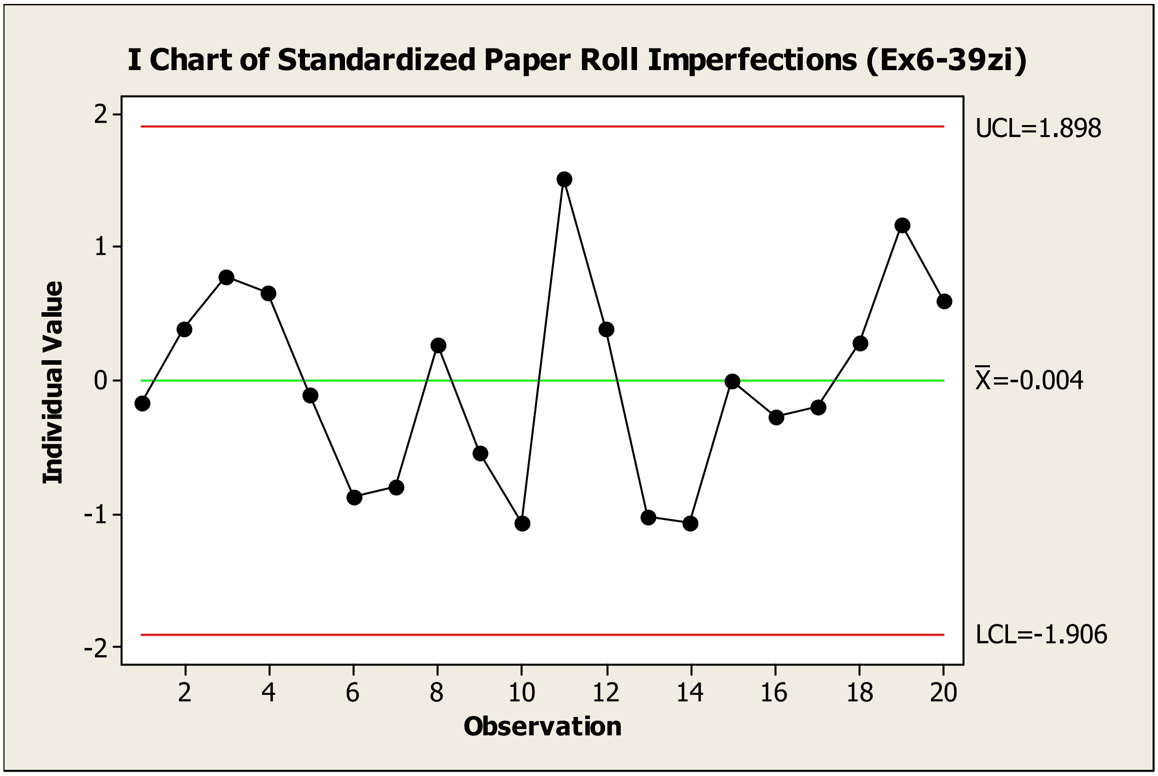
MTB > Stat > Control Charts > Attributes Charts > U



6-39.

$$z_i = (u_i - \bar{u}) \Big/ \sqrt{\bar{u}/n_i} = (u_i - 0.7007) \Big/ \sqrt{0.7007/n_i}$$

MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



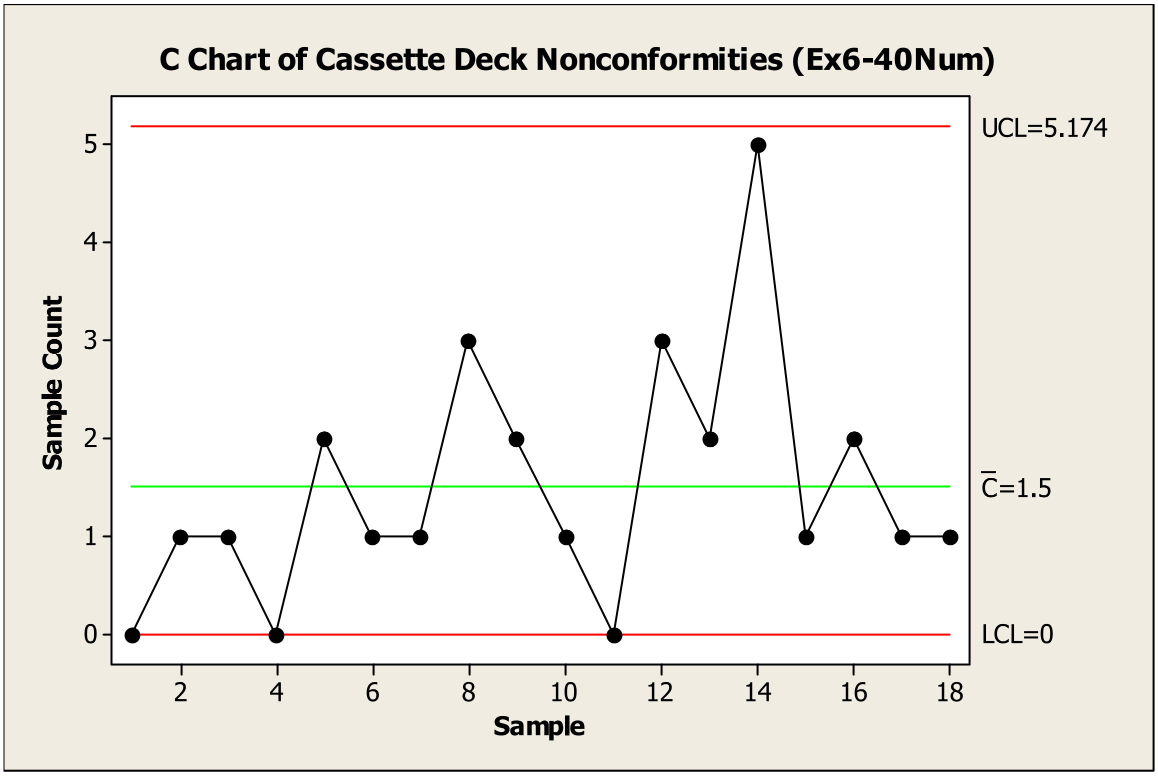
6-40.
c chart based on # of nonconformities per cassette deck

$\text{CL} = \bar{c} = 1.5$

$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 1.5 + 3\sqrt{1.5} = 5.17$

$\text{LCL} \Rightarrow 0$

MTB > Stat > Control Charts > Attributes Charts > C

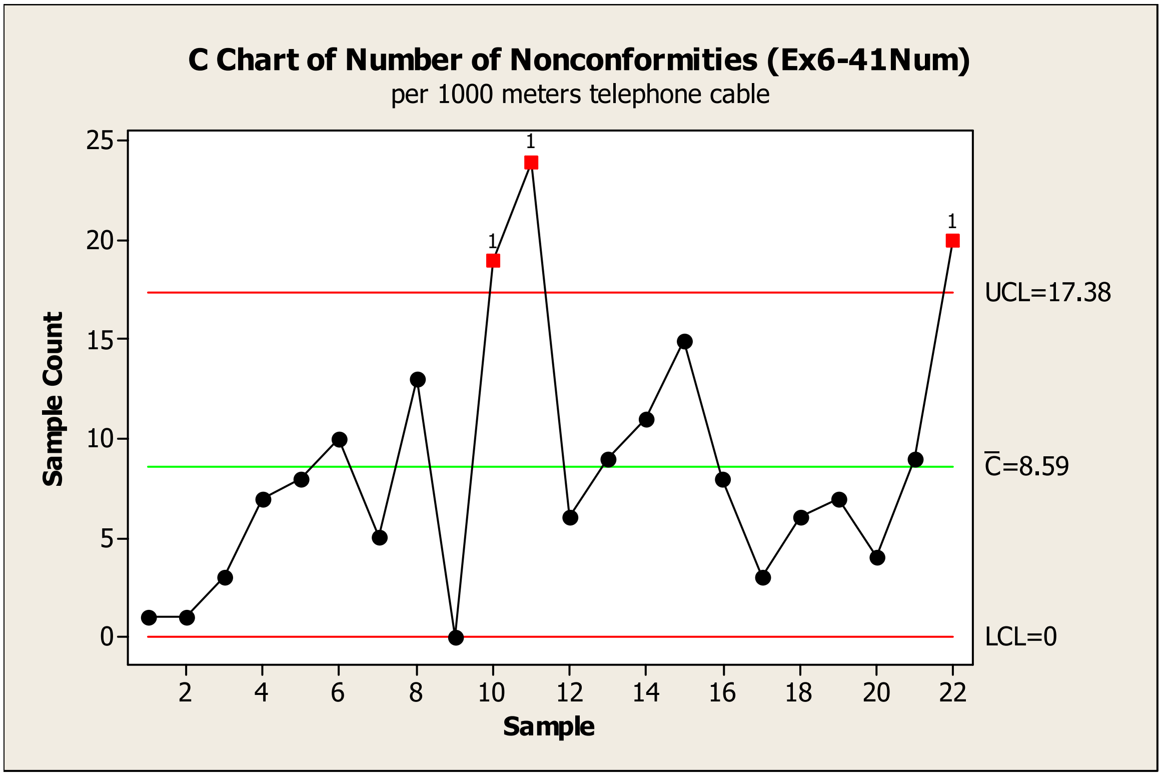


Process is in statistical control. Use these limits to control future production.

6-41.

$\text{CL} = \bar{c} = 8.59; \quad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 8.59 + 3\sqrt{8.59} = 17.384; \quad \text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 8.59 - 3\sqrt{8.59} \Rightarrow 0$

MTB > Stat > Control Charts > Attributes Charts > C



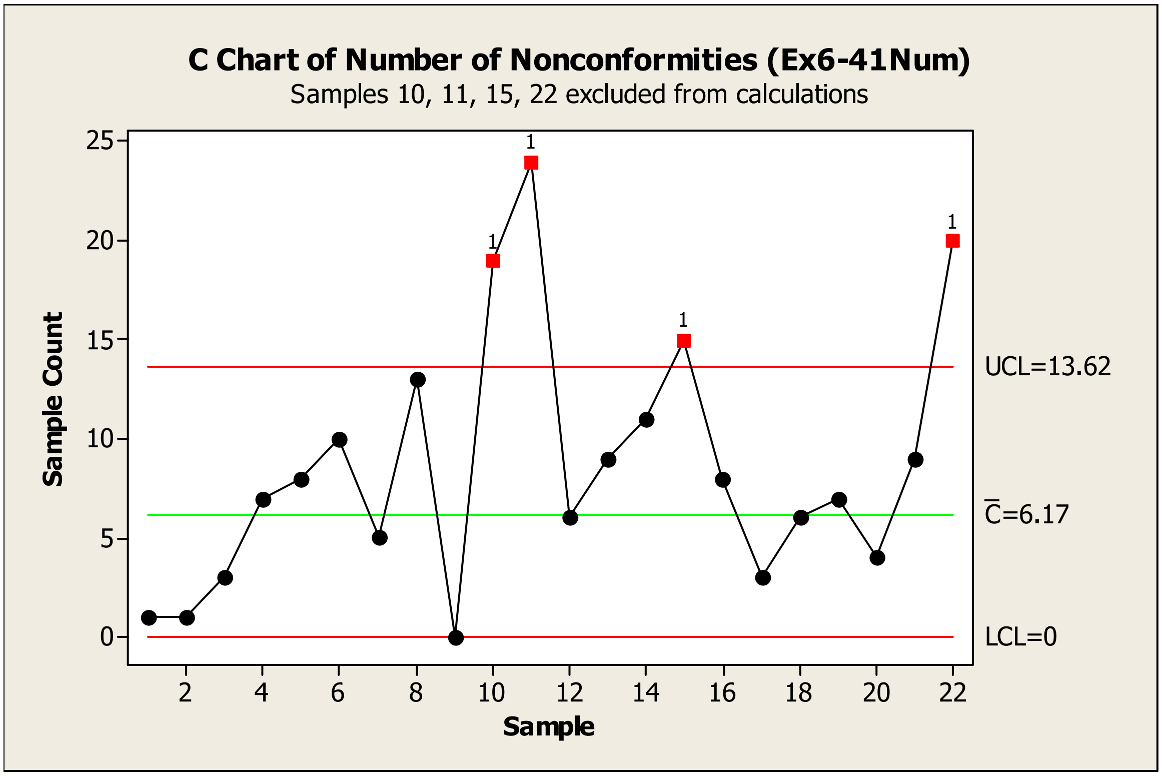
Test Results for C Chart of Ex6-41Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  10, 11, 22
```

6-41 continued

Process is not in statistical control; three subgroups exceed the UCL. Exclude subgroups 10, 11 and 22, then re-calculate the control limits. Subgroup 15 will then be out of control and should also be excluded.

$\text{CL} = \bar{c} = 6.17; \quad \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 6.17 + 3\sqrt{6.17} = 13.62; \quad \text{LCL} \Rightarrow 0$



Test Results for C Chart of Ex6-41Num

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  10, 11, 15, 22
```

6-42.
(a)
The new inspection unit is n = 4 cassette decks. A c chart of the total number of nonconformities per inspection unit is appropriate.

$$\text{CL} = n\bar{c} = 4(1.5) = 6$$

$$\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 6 + 3\sqrt{6} = 13.35$$

$$\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 6 - 3\sqrt{6} \Rightarrow 0$$

(b)
The sample is n =1 new inspection units. A u chart of average nonconformities per inspection unit is appropriate.

$$\text{CL} = \bar{u} = \frac{\text{total nonconformities}}{\text{total inspection units}} = \frac{27}{(18/4)} = 6.00$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 6 + 3\sqrt{6/1} = 13.35$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 6 - 3\sqrt{6/1} \Rightarrow 0$$

6-43.
(a)
The new inspection unit is n = 2500/1000 = 2.5 of the old unit. A c chart of the total number of nonconformities per inspection unit is appropriate.

$$\text{CL} = n\bar{c} = 2.5(6.17) = 15.43$$

$$\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 15.43 + 3\sqrt{15.43} = 27.21$$

$$\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 15.43 - 3\sqrt{15.43} = 3.65$$

The plot point, $\hat{c}$, is the total number of nonconformities found while inspecting a sample 2500m in length.

(b)
The sample is n =1 new inspection units. A u chart of average nonconformities per inspection unit is appropriate.

$$\text{CL} = \bar{u} = \frac{\text{total nonconformities}}{\text{total inspection units}} = \frac{111}{(18 \times 1000)/2500} = 15.42$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 15.42 + 3\sqrt{15.42/1} = 27.20$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 15.42 - 3\sqrt{15.42/1} = 3.64$$

The plot point, $\hat{u}$, is the average number of nonconformities found in 2500m, and since n = 1, this is the same as the total number of nonconformities.

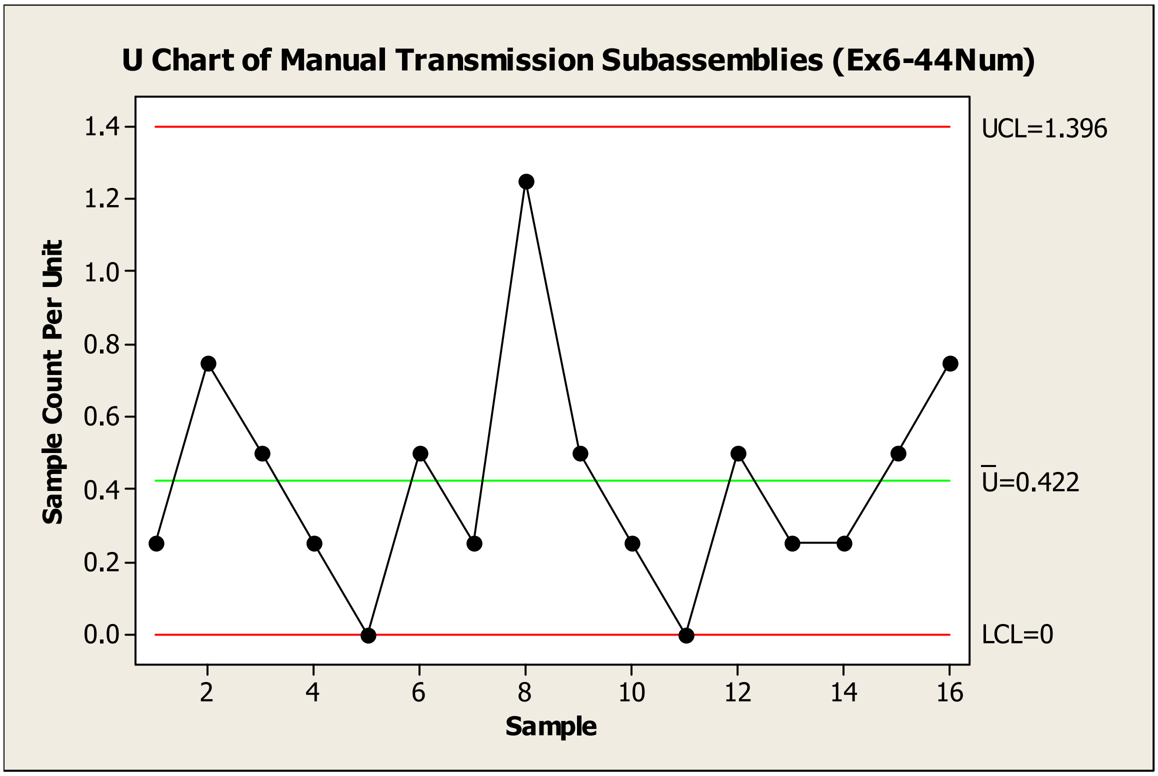
6-44.
(a)
A u chart of average number of nonconformities per unit is appropriate, with $n = 4$ transmissions in each inspection.

$$\text{CL} = \bar{u} = \sum u_i / m = \left(\sum x_i / n\right) / m = (27/4)/16 = 6.75/16 = 0.422$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 0.422 + 3\sqrt{0.422/4} = 1.396$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 0.422 - 3\sqrt{0.422/4} = -0.211 \Rightarrow 0$$

MTB > Stat > Control Charts > Attributes Charts > U



(b)
The process is in statistical control.

(c)
The new sample is $n = 8/4 = 2$ inspection units. However, since this chart was established for ***average*** nonconformities per unit, the same control limits may be used for future production with the new sample size. (If this was a c chart for ***total*** nonconformities in the sample, the control limits would need revision.)

6-45.
(a)

$$\text{CL} = \bar{c} = 4$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 4 + 3\sqrt{4} = 10$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 4 - 3\sqrt{4} \Rightarrow 0$$

(b)

$$c = 4; \quad n = 4$$

$$\text{CL} = \bar{u} = c / n = 4 / 4 = 1$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 1 + 3\sqrt{1/4} = 2.5$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 1 - 3\sqrt{1/4} \Rightarrow 0$$

6-46.
Use the cumulative Poisson tables.

$$\bar{c} = 16$$

$$\Pr\{x \le 21 \mid c = 16\} = 0.9108; \text{UCL} = 21$$

$$\Pr\{x \le 10 \mid c = 16\} = 0.0774; \text{LCL} = 10$$

6-47.
(a)

$$\text{CL} = \bar{c} = 9$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 9 + 3\sqrt{9} = 18$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 9 - 3\sqrt{9} = 0$$

(b)

$$c = 16; \quad n = 4$$

$$\text{CL} = \bar{u} = c / n = 16 / 4 = 4$$

$$\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 4 + 3\sqrt{4/4} = 7$$

$$\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 4 - 3\sqrt{4/4} = 1$$

6-48.
u chart with $u = 6.0$ and $n = 3$. $c = u \times n = 18$. Find limits such that $\Pr\{D \le \text{UCL}\} = 0.980$ and $\Pr\{D < \text{LCL}\} = 0.020$. From the cumulative Poisson tables:

x	**Pr{*D* ≤ *x* | *c* = 18}**
9	0.015
10	0.030
26	0.972
27	0.983

UCL = x/n = 27/3 = 9, and LCL = x/n = 9/3 = 3. As a comparison, the normal distribution gives:

$$\text{UCL} = \bar{u} + z_{0.980}\sqrt{\bar{u}/n} = 6 + 2.054\sqrt{6/3} = 8.905$$
$$\text{LCL} = \bar{u} + z_{0.020}\sqrt{\bar{u}/n} = 6 - 2.054\sqrt{6/3} = 3.095$$

6-49.
Using the cumulative Poisson distribution:

x	**Pr{*D* ≤ *x* | *c* = 7.6}**
2	0.019
3	0.055
12	0.954
13	0.976

for the c chart, UCL = 13 and LCL = 2. As a comparison, the normal distribution gives

$$\text{UCL} = \bar{c} + z_{0.975}\sqrt{\bar{c}} = 7.6 + 1.96\sqrt{7.6} = 13.00$$
$$\text{LCL} = \bar{c} - z_{0.025}\sqrt{\bar{c}} = 7.6 - 1.96\sqrt{7.6} = 2.20$$

6-50.
Using the cumulative Poisson distribution with $c = u\,n = 1.4(10) = 14$:

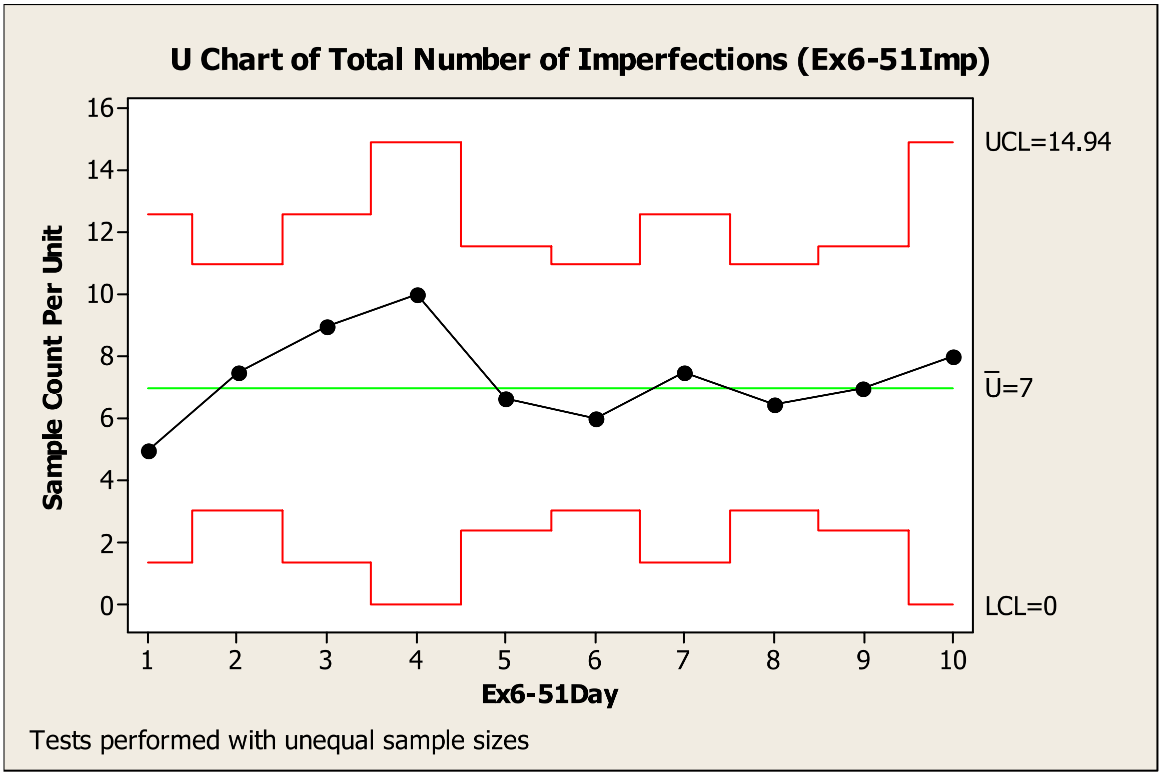
x	**Pr{*D* ≤ *x* | *c* = 14}**
7	0.032
8	0.062
19	0.923
20	0.952

UCL = x/n = 20/10 = 2.00, and LCL = x/n = 7/10 = 0.70. As a comparison, the normal distribution gives:

$$\text{UCL} = \bar{u} + z_{0.95}\sqrt{\bar{u}/n} = 1.4 + 1.645\sqrt{1.4/10} = 2.016$$
$$\text{LCL} = \bar{u} + z_{0.05}\sqrt{\bar{u}/n} = 1.4 - 1.645\sqrt{1.4/10} = 0.784$$

6-51.
u chart with control limits based on each sample size:
$\bar{u} = 7;\quad \text{UCL}_i = 7 + 3\sqrt{7/n_i};\quad \text{LCL}_i = 7 - 3\sqrt{7/n_i}$

MTB > Stat > Control Charts > Attributes Charts > U



The process is in statistical control.

6-52.
(a)
From the cumulative Poisson table, $\Pr\{x \le 6 \mid c = 2.0\} = 0.995$. So set UCL = 6.0.

(b)
Pr{two consecutive out-of-control points} = (0.005)(0.005) = 0.00003

6-53.
A c chart with one inspection unit equal to 50 manufacturing units is appropriate.
$\bar{c} = 850/100 = 8.5$. From the cumulative Poisson distribution:

x	$\Pr\{D \le x \mid c = 8.5\}$
3	0.030
13	0.949
14	0.973

LCL = 3 and UCL = 13. For comparison, the normal distribution gives

$$\text{UCL} = \bar{c} + z_{0.97}\sqrt{\bar{c}} = 8.5 + 1.88\sqrt{8.5} = 13.98$$

$$\text{LCL} = \bar{c} + z_{0.03}\sqrt{\bar{c}} = 8.5 - 1.88\sqrt{8.5} = 3.02$$

6-54.
(a)
Plot the number of nonconformities per water heater on a c chart.

$$\text{CL} = \bar{c} = \sum D/m = 924/176 = 5.25$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 5.25 + 3\sqrt{5.25} = 12.12$$

$$\text{LCL} \Rightarrow 0$$

Plot the results after inspection of each water heater, approximately 8/day.

(b)
Let new inspection unit n = 2 water heaters

$$\text{CL} = n\bar{c} = 2(5.25) = 10.5$$

$$\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 10.5 + 3\sqrt{10.5} = 20.22$$

$$\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 10.5 - 3\sqrt{10.5} = 0.78$$

(c)

$$\begin{aligned}\Pr\{\text{type I error}\} &= \Pr\{D < \text{LCL} \mid c\} + \Pr\{D > \text{UCL} \mid c\} \\ &= \Pr\{D < 0.78 \mid 10.5\} + \left[1 - \Pr\{D \le 20.22 \mid 10.5\}\right] \\ &= \text{POI}(0,10.5) + \left[1 - \text{POI}(20,10.5)\right] \\ &= 0.000 + \left[1 - 0.997\right] \\ &= 0.003\end{aligned}$$

6-55.
$\bar{u} = 4.0$ average number of nonconformities/unit. Desire α = 0.99. Use the cumulative Poisson distribution to determine the UCL:

MTB : worksheet Chap06.mtw

Ex6-55X	Ex6-55alpha
0	0.02
1	0.09
2	0.24
3	0.43
4	0.63
5	0.79
6	0.89
7	0.95
8	0.98
9	0.99
10	1.00
11	1.00

An UCL = 9 will give a probability of 0.99 of concluding the process is in control, when in fact it is.

6-56.
Use a c chart for nonconformities with an inspection unit n = 1 refrigerator.
$\sum D_i = 16$ in 30 refrigerators; $\bar{c} = 16/30 = 0.533$

(a)
3-sigma limits are $\bar{c} \pm 3\sqrt{\bar{c}} = 0.533 \pm 3\sqrt{0.533} = [0, 2.723]$

(b)
$$\begin{aligned}
\alpha &= \Pr\{D < \text{LCL} \mid c\} + \Pr\{D > \text{UCL} \mid c\} \\
&= \Pr\{D < 0 \mid 0.533\} + \left[1 - \Pr\{D \leq 2.72 \mid 0.533\}\right] \\
&= 0 + \left[1 - \text{POI}(2, 0.533)\right] \\
&= 1 - 0.983 \\
&= 0.017
\end{aligned}$$
where POI(·) is the cumulative Poisson distribution.

6-56 continued

(c)

$$\begin{aligned}\beta &= \Pr\{\text{not detecting shift}\}\\ &= \Pr\{D < \text{UCL} \mid c\} - \Pr\{D \le \text{LCL} \mid c\}\\ &= \Pr\{D < 2.72 \mid 2.0\} - \Pr\{D \le 0 \mid 2.0\}\\ &= \text{POI}(2,2) - \text{POI}(0,2)\\ &= 0.6767 - 0.1353\\ &= 0.5414\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(d)

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-0.541} = 2.18 \approx 2$$

6-57.

$\bar{c} = 0.533$

(a)

$$\bar{c} \pm 2\sqrt{\bar{c}} = 0.533 + 2\sqrt{0.533} = [0, 1.993]$$

(b)

$$\begin{aligned}\alpha &= \Pr\{D < \text{LCL} \mid \bar{c}\} + \Pr\{D > \text{UCL} \mid \bar{c}\}\\ &= \Pr\{D < 0 \mid 0.533\} + \left[1 - \Pr\{D \le 1.993 \mid 0.533\}\right]\\ &= 0 + \left[1 - \text{POI}(1, 0.533)\right]\\ &= 1 - 0.8996\\ &= 0.1004\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(c)

$$\begin{aligned}\beta &= \Pr\{D < \text{UCL} \mid c\} - \Pr\{D \le \text{LCL} \mid c\}\\ &= \Pr\{D < 1.993 \mid 2\} - \Pr\{D \le 0 \mid 2\}\\ &= \text{POI}(1,2) - \text{POI}(0,2)\\ &= 0.406 - 0.135\\ &= 0.271\end{aligned}$$

where POI(·) is the cumulative Poisson distribution.

(d)

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-0.271} = 1.372 \approx 2$$

6-58.
1 inspection unit = 10 radios, $\bar{u} = 0.5$ average nonconformities/radio
$\text{CL} = \bar{c} = \bar{u} \times n = 0.5(10) = 5$
$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 5 + 3\sqrt{5} = 11.708$
$\text{LCL} \Rightarrow 0$

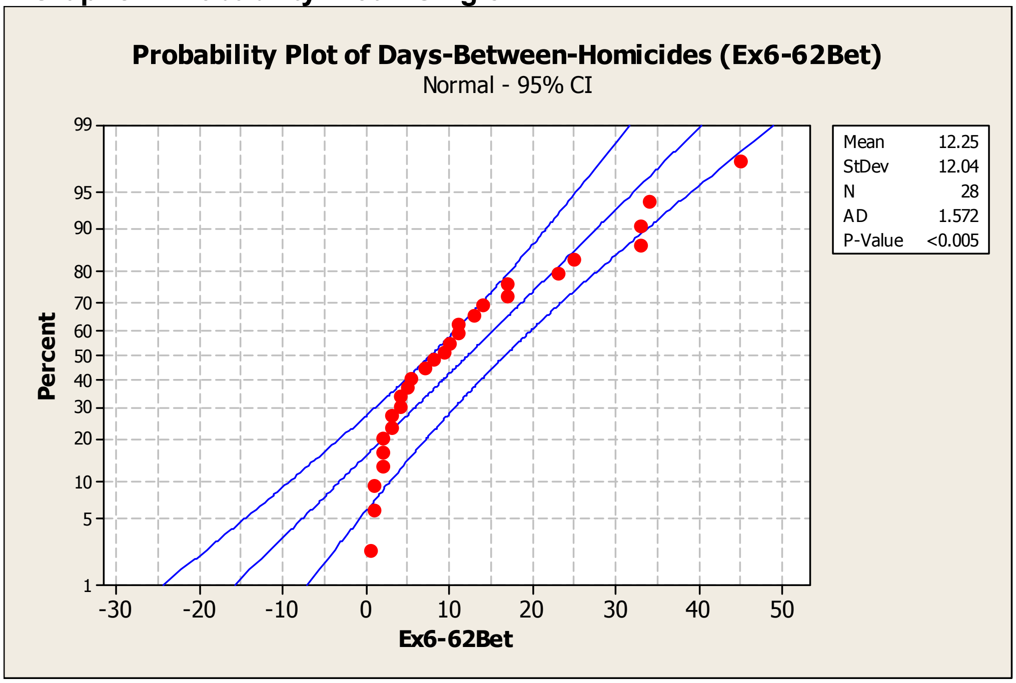
6-59.
$\bar{u}$ = average # nonconformities/calculator = 2
(a)
c chart with $\bar{c} = \bar{u} \times n = 2(2) = 4$ nonconformities/inspection unit
$\text{CL} = \bar{c} = 4$
$\text{UCL} = \bar{c} + k\sqrt{\bar{c}} = 4 + 3\sqrt{4} = 10$
$\text{LCL} = \bar{c} - k\sqrt{\bar{c}} = 4 - 3\sqrt{4} \Rightarrow 0$

(b)
Type I error =
$\alpha = \Pr\{D < \text{LCL} \mid \bar{c}\} + \Pr\{D > \text{UCL} \mid \bar{c}\}$
$= \Pr\{D < 0 \mid 4\} + \left[1 - \Pr\{D \le 10 \mid 4\}\right]$
$= 0 + \left[1 - \text{POI}(10, 4)\right]$
$= 1 - 0.997$
$= 0.003$
where POI(·) is the cumulative Poisson distribution.

6-60.
1 inspection unit = 6 clocks, $\bar{u} = 0.75$ nonconformities/clock
$\text{CL} = \bar{c} = \bar{u} \times n = 0.75(6) = 4.5$
$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 4.5 + 3\sqrt{4.5} = 10.86$
$\text{LCL} \Rightarrow 0$

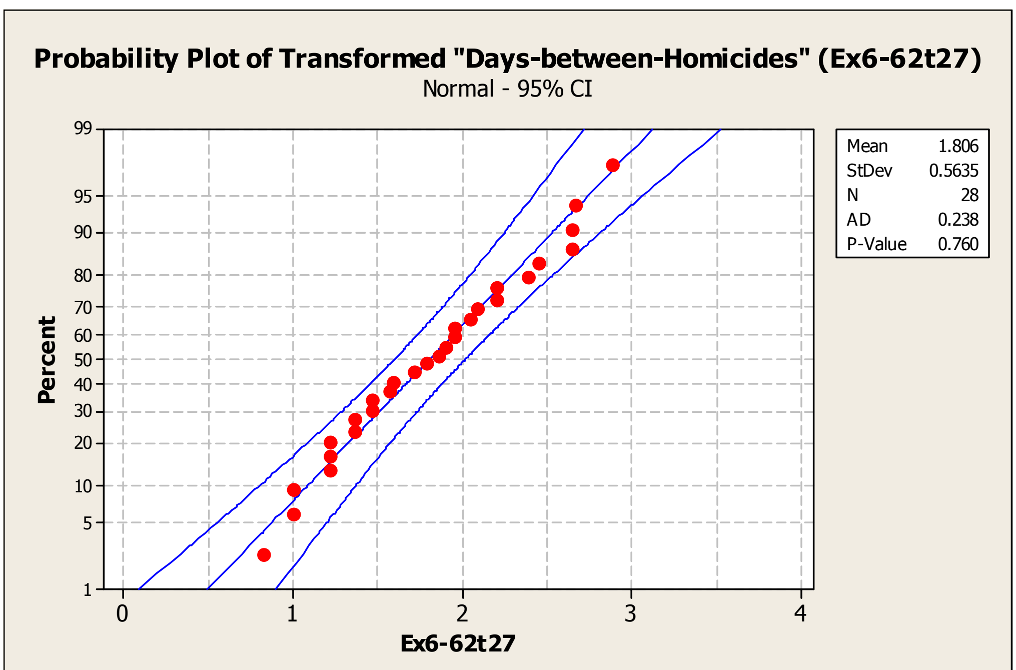
6-61.
c: nonconformities per unit; L: sigma control limits
$n\bar{c} - L\sqrt{n\bar{c}} > 0$
$n\bar{c} > L\sqrt{n\bar{c}}$
$n > L^2 / \bar{c}$

6-62.
(a)
MTB > Graphs > Probability Plot > Single



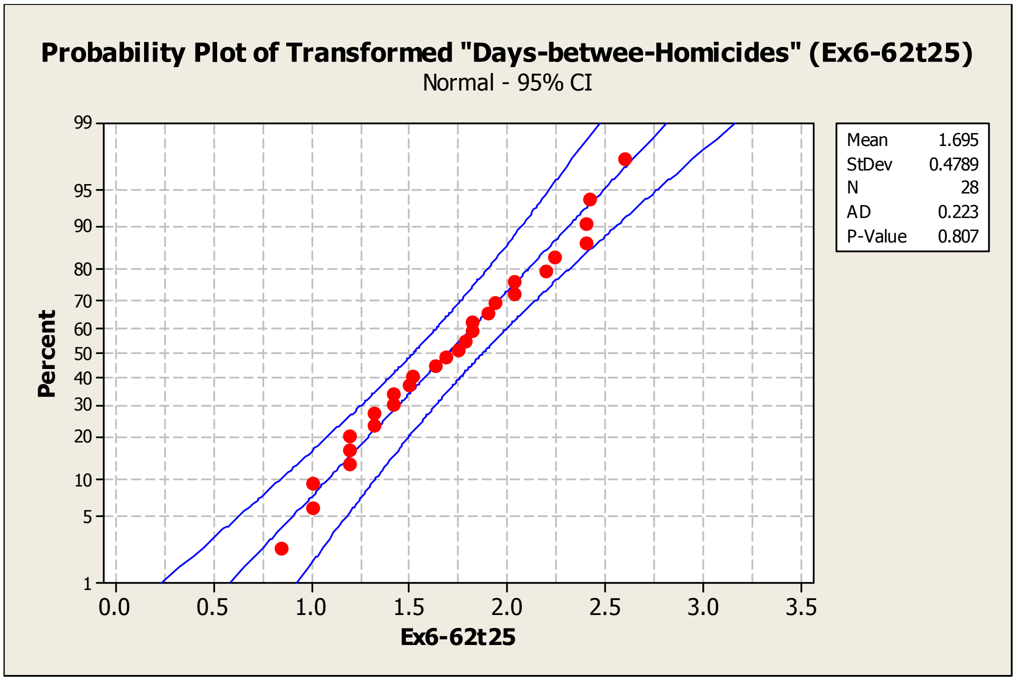
There is a huge curve in the plot points, indicating that the normal distribution assumption is not reasonable.

(b)



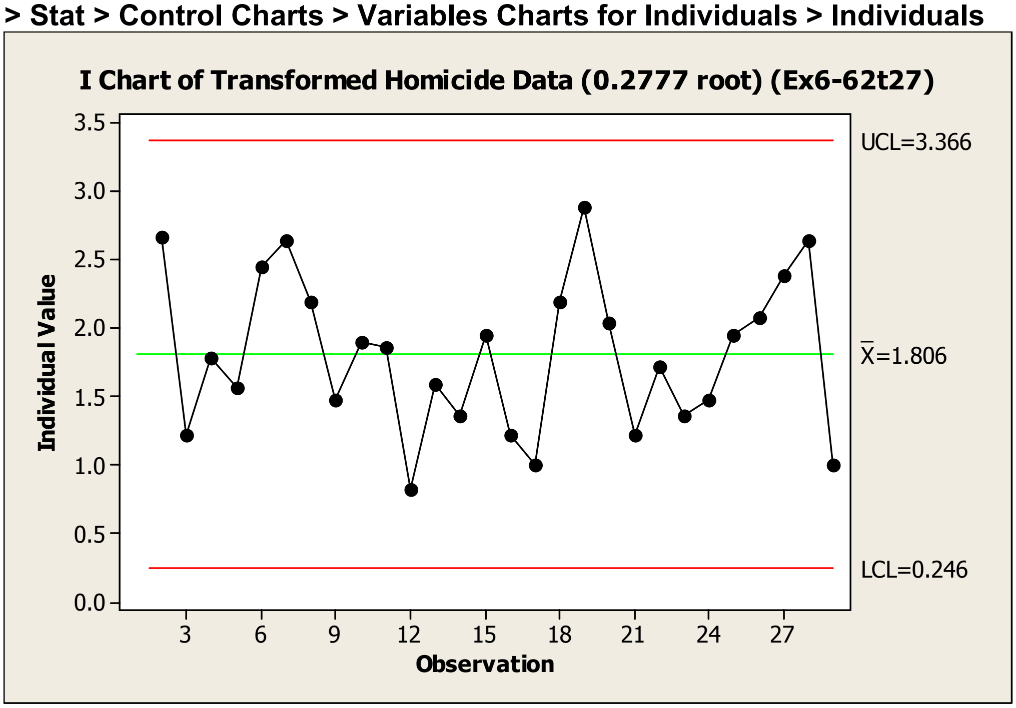
The 0.2777th root transformation makes the data more closely resemble a sample from a normal distribution.

6-62 continued
(c)

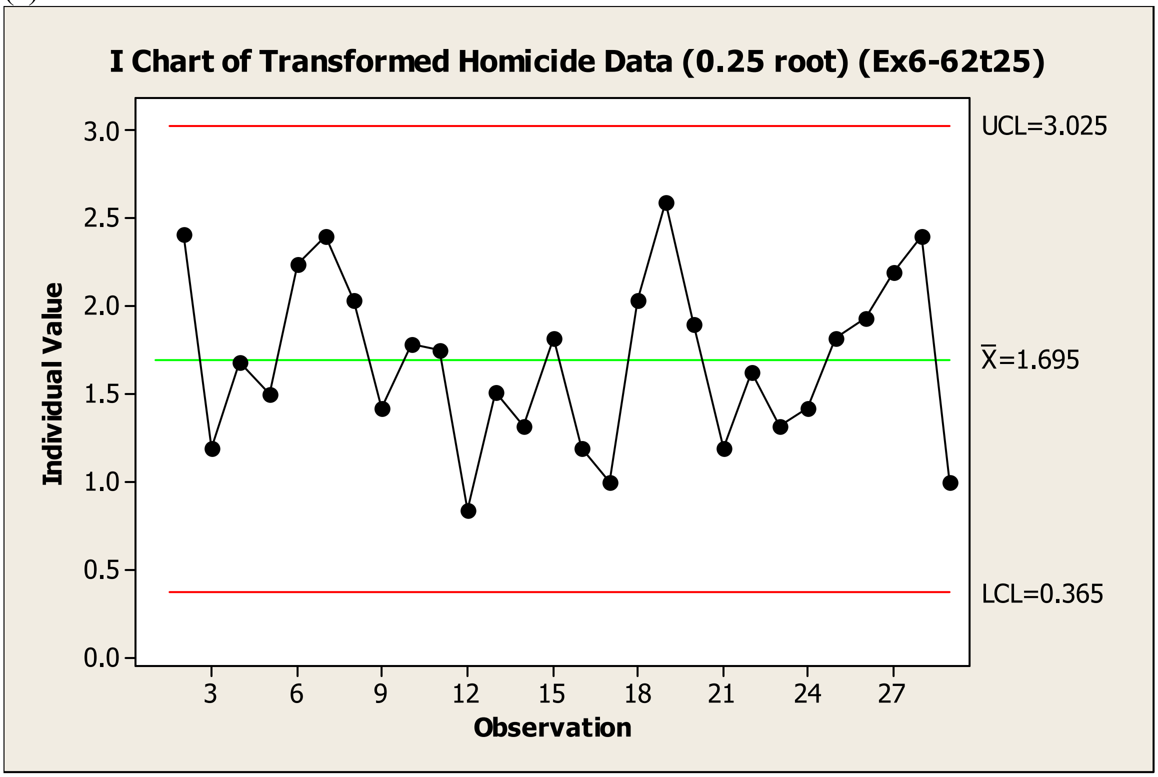


The 0.25th root transformation makes the data more closely resemble a sample from a normal distribution. It is not very different from the transformed data in (b).

(d)
MTB > Stat > Control Charts > Variables Charts for Individuals > Individuals



6-62 continued
(e)



Both Individuals charts are similar, with an identical pattern of points relative to the UCL, mean and LCL. There is no difference in interpretation.

(f)
The "process" is stable, meaning that the days-between-homicides is approximately constant. If a change is made, say in population, law, policy, workforce, etc., which affects the rate at which homicides occur, the mean time between may get longer (or shorter) with plot points above the upper (or below the lower) control limit.

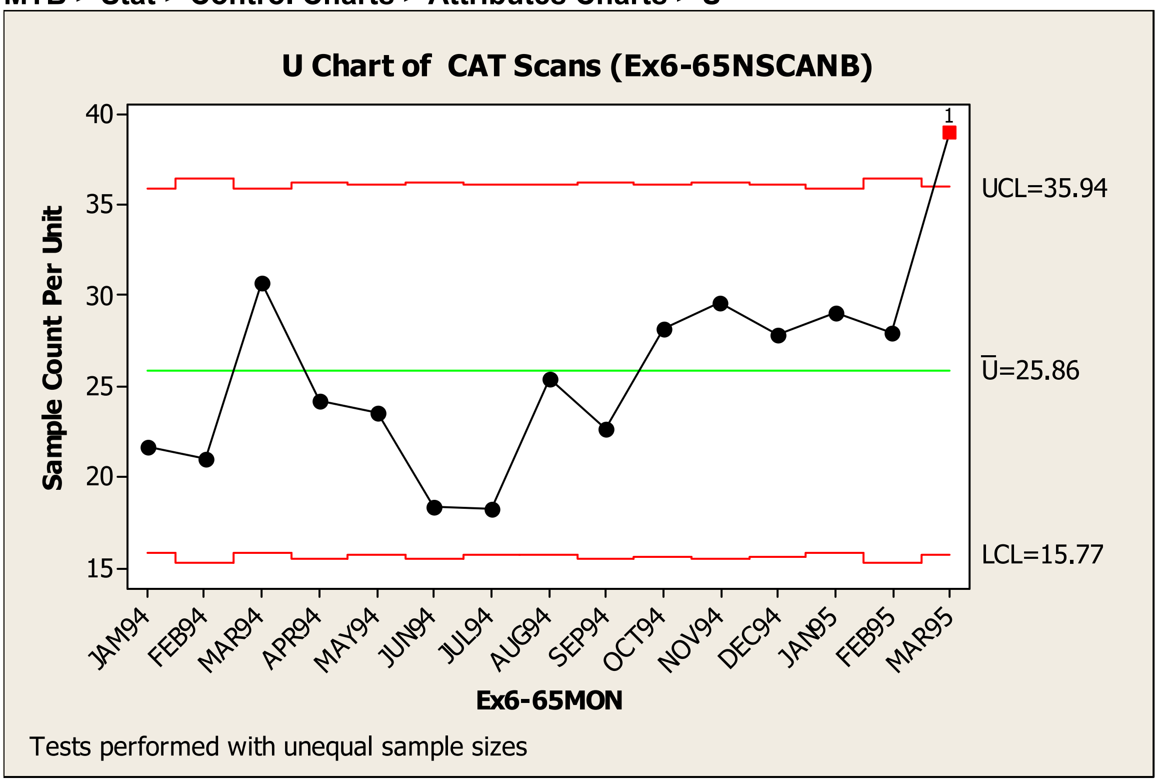
6-63.
There are endless possibilities for collection of attributes data from nonmanufacturing processes. Consider a product distribution center (or any warehouse) with processes for filling and shipping orders. One could track the number of orders filled incorrectly (wrong parts, too few/many parts, wrong part labeling,), packaged incorrectly (wrong material, wrong package labeling), invoiced incorrectly, etc. Or consider an accounting firm—errors in statements, errors in tax preparation, etc. (hopefully caught internally with a verification step).

6-64.
If time-between-events data (say failure time) is being sought for internally generated data, it can usually be obtained reliably and consistently. However, if you're looking for data on time-between-events that must be obtained from external sources (for example, time-to-field failures), it may be hard to determine with sufficient accuracy—both the "start" and the "end". Also, the conditions of use and the definition of "failure" may not be consistently applied.

There are ways to address these difficulties. Collection of "start" time data may be facilitated by serializing or date coding product.

6-65☺.
The variable NYRSB can be thought of as an "inspection unit", representing an identical "area of opportunity" for each "sample". The "process characteristic" to be controlled is the rate of CAT scans. A *u* chart which monitors the average number of CAT scans per NYRSB is appropriate.

MTB > Stat > Control Charts > Attributes Charts > U

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Test Results for U Chart of Ex6-65NSCANB
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  15
```
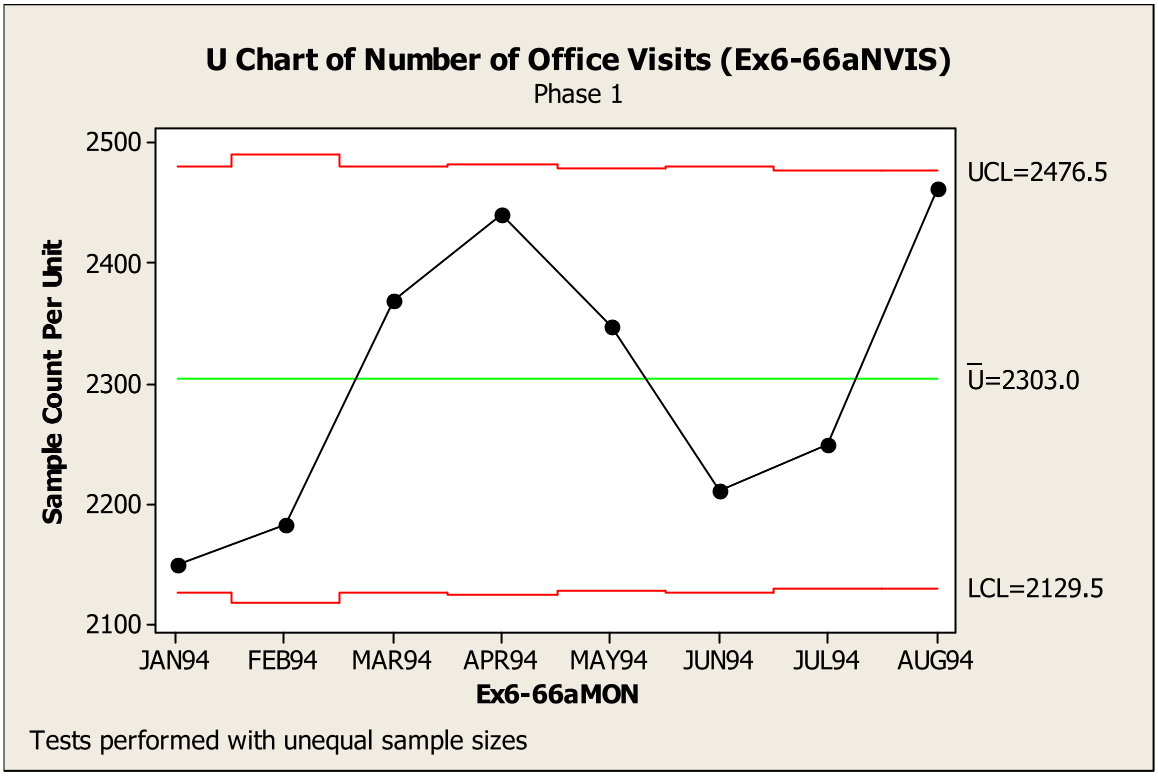

The rate of monthly CAT scans is out of control.

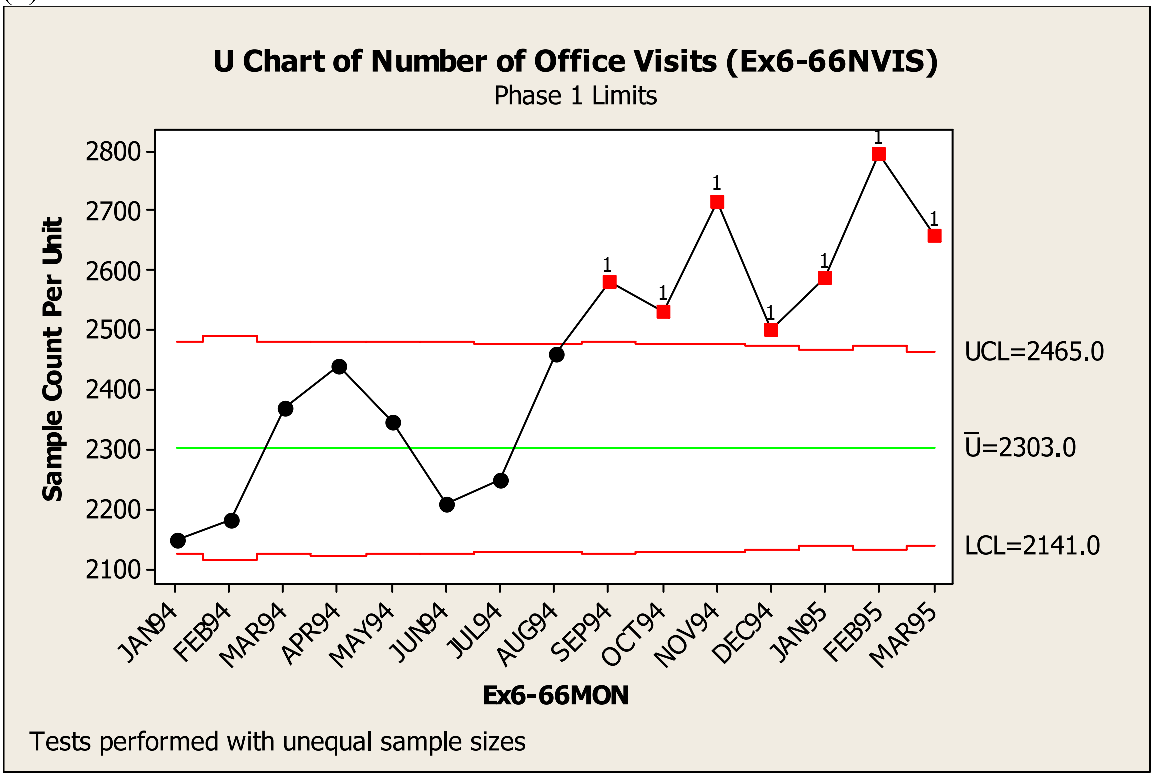
6-66☺.
The variable NYRSE can be thought of as an "inspection unit", representing an identical "area of opportunity" for each "sample". The "process characteristic" to be controlled is the rate of office visits. A *u* chart which monitors the average number of office visits per NYRSB is appropriate.

(a)
MTB > Stat > Control Charts > Attributes Charts > U



The chart is in statistical control

6-66 continued
(b)

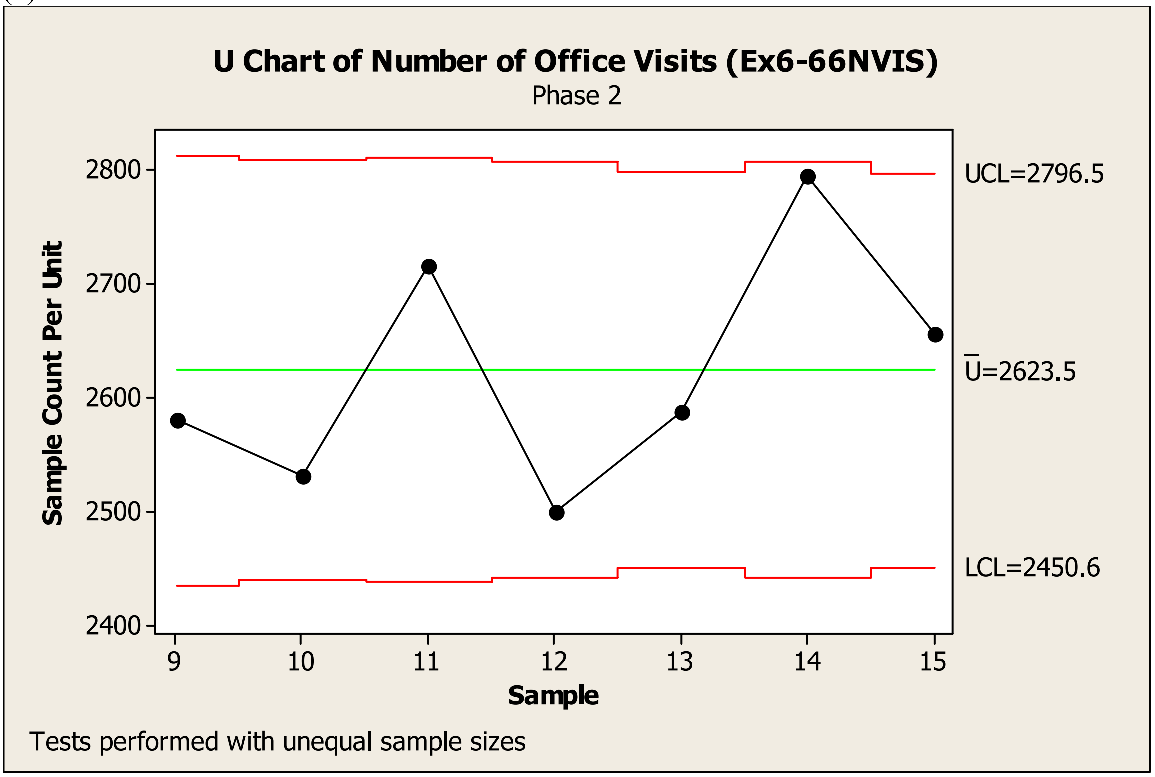


Test Results for U Chart of Ex6-66NVIS

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  9, 10, 11, 12, 13, 14, 15
```

The phase 2 data appears to have shifted up from phase 1. The 2nd phase is not in statistical control relative to the 1st phase.

6-66 continued
(c)



The Phase 2 data, separated from the Phase 1 data, are in statistical control.