Chapter 7 Exercise Solutions

Note: Several exercises in this chapter differ from those in the 4th edition. An "*" indicates that the description has changed. A second exercise number in parentheses indicates that the exercise number has changed. New exercises are denoted with an "☺".

7-1.

$$\hat{\mu} = \bar{\bar{x}} = 74.001; \quad \bar{R} = 0.023; \quad \hat{\sigma} = \bar{R}/d_2 = 0.023/2.326 = 0.010$$

$$\text{SL} = 74.000 \pm 0.035 = [73.965, 74.035]$$

$$\hat{C}_p \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{74.035 - 73.965}{6(0.010)} = 1.17$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{74.001 - 73.965}{3(0.010)} = 1.20$$

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{74.035 - 74.001}{3(0.010)} = 1.13$$

$$\hat{C}_{pk} = \min\left(\hat{C}_{pl}, \hat{C}_{pu}\right) = 1.13$$

7-2.

In Exercise 5-1, samples 12 and 15 are out of control, and the new process parameters are used in the process capability analysis.

$$n = 5; \quad \hat{\mu} = \bar{\bar{x}} = 33.65; \quad \bar{R} = 4.5; \quad \hat{\sigma} = \bar{R}/d_2 = 1.93$$

$$\text{USL} = 40; \quad \text{LSL} = 20$$

$$\hat{C}_p \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{40 - 20}{6(1.93)} = 1.73$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{33.65 - 20}{3(1.93)} = 2.36$$

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{40 - 33.65}{3(1.93)} = 1.10$$

$$\hat{C}_{pk} = \min\left(\hat{C}_{pl}, \hat{C}_{pu}\right) = 1.10$$

7-3.

$$\hat{\mu} = \overline{\overline{x}} = 10.375; \ \overline{R}_x = 6.25; \ \hat{\sigma}_x = \overline{R}/d_2 = 6.25/2.059 = 3.04$$

$$\text{USL}_x = [(350+5)-350]\times 10 = 50; \ \text{LSL}_x = [(350-5)-350]\times 10 = -50$$

$$x_i = (\text{obs}_i - 350)\times 10$$

$$\hat{C}_p = \frac{\text{USL}_x - \text{LSL}_x}{6\hat{\sigma}_x} = \frac{50-(-50)}{6(3.04)} = 5.48$$

The process produces product that uses approximately 18% of the total specification band.

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}}{3\hat{\sigma}_x} = \frac{50-10.375}{3(3.04)} = 4.34$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{10.375-(-50)}{3(3.04)} = 6.62$$

$$\hat{C}_{pk} = \min(\hat{C}_{pu}, \hat{C}_{pl}) = 4.34$$

This is an extremely capable process, with an estimated percent defective much less than 1 ppb. Note that the C_{pk} is less than C_p, indicating that the process is not centered and is not achieving potential capability. However, this PCR does not tell *where* the mean is located within the specification band.

$$V = \frac{T - \overline{\overline{x}}}{S} = \frac{0-10.375}{3.04} = -3.4128$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{5.48}{\sqrt{1+(-3.4128)^2}} = 1.54$$

Since C_{pm} is greater than 4/3, the mean μ lies within approximately the middle fourth of the specification band.

$$\hat{\xi} = \frac{\hat{\mu} - T}{\hat{\sigma}} = \frac{10.375-0}{3.04} = 3.41$$

$$\hat{C}_{pkm} = \frac{\hat{C}_{pk}}{\sqrt{1+\hat{\xi}^2}} = \frac{1.54}{\sqrt{1+3.41^2}} = 0.43$$

7-4.

$n = 5; \bar{\bar{x}} = 0.00109; \bar{R} = 0.00635; \hat{\sigma}_x = 0.00273$; tolerances: 0 ± 0.01

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{0.01 + 0.01}{6(0.00273)} = 1.22$$

The process produces product that uses approximately 82% of the total specification band.

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{0.01 - 0.00109}{3(0.00273)} = 1.09$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{0.00109 - (-0.01)}{3(0.00273)} = 1.35$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.09$$

This process is not considered capable, failing to meet the minimally acceptable definition of capable $C_{pk} \geq 1.33$

$$V = \frac{T - \bar{\bar{x}}}{S} = \frac{0 - 0.00109}{0.00273} = -0.399$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1 + V^2}} = \frac{1.22}{\sqrt{1 + (-0.399)^2}} = 1.13$$

Since C_{pm} is greater than 1, the mean μ lies within approximately the middle third of the specification band.

$$\hat{\xi} = \frac{\hat{\mu} - T}{\hat{\sigma}} = \frac{0.00109 - 0}{0.00273} = 0.399$$

$$\hat{C}_{pkm} = \frac{\hat{C}_{pk}}{\sqrt{1 + \hat{\xi}^2}} = \frac{1.09}{\sqrt{1 + 0.399^2}} = 1.01$$

7-5.
$\hat{\mu} = \overline{\overline{x}} = 100; \overline{s} = 1.05; \hat{\sigma}_x = \overline{s}/c_4 = 1.05/0.9400 = 1.117$

(a)
Potential: $\hat{C}_p = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \dfrac{(95+10)-(95-10)}{6(1.117)} = 2.98$

(b)

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{100-(95-10)}{3(1.117)} = 4.48$$

Actual: $\hat{C}_{pu} = \dfrac{\text{USL}_x - \hat{\mu}}{3\hat{\sigma}_x} = \dfrac{(95+10)-100}{3(1.117)} = 1.49$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.49$$

(c)

$$\begin{aligned}
\hat{p}_{\text{Actual}} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < \text{LSL}\} + \left[1 - \Pr\{x \le \text{USL}\}\right] \\
&= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + \left[1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\}\right] \\
&= \Pr\left\{z < \frac{85-100}{1.117}\right\} + \left[1 - \Pr\left\{z \le \frac{105-100}{1.117}\right\}\right] \\
&= \Phi(-13.429) + \left[1 - \Phi(4.476)\right] \\
&= 0.0000 + \left[1 - 0.999996\right] \\
&= 0.000004
\end{aligned}$$

$$\begin{aligned}
\hat{p}_{\text{Potential}} &= \Pr\left\{z < \frac{85-95}{1.117}\right\} + \left[1 - \Pr\left\{z \le \frac{105-95}{1.117}\right\}\right] \\
&= \Phi(-8.953) + \left[1 - \Phi(8.953)\right] \\
&= 0.000000 + \left[1 - 1.000000\right] \\
&= 0.000000
\end{aligned}$$

7-6☺.

$n = 4$; $\hat{\mu} = \overline{\overline{x}} = 199$; $\overline{R} = 3.5$; $\hat{\sigma}_x = \overline{R}/d_2 = 3.5/2.059 = 1.70$

USL = 200 + 8 = 208; LSL = 200 – 8 = 192

(a)

Potential: $\hat{C}_p = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \dfrac{208-192}{6(1.70)} = 1.57$

The process produces product that uses approximately 64% of the total specification band.

(b)

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{208-199}{3(1.70)} = 1.76$$

Actual: $\hat{C}_{pl} = \dfrac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \dfrac{199-192}{3(1.70)} = 1.37$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.37$$

(c)

The current fraction nonconforming is:

$$\begin{aligned}
\hat{p}_{\text{Actual}} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < \text{LSL}\} + \left[1 - \Pr\{x \le \text{USL}\}\right] \\
&= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + \left[1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\}\right] \\
&= \Pr\left\{z < \frac{192-199}{1.70}\right\} + \left[1 - \Pr\left\{z \le \frac{208-199}{1.70}\right\}\right] \\
&= \Phi(-4.1176) + \left[1 - \Phi(5.2941)\right] \\
&= 0.0000191 + \left[1 - 1\right] \\
&= 0.0000191
\end{aligned}$$

If the process mean could be centered at the specification target, the fraction nonconforming would be:

$$\begin{aligned}
\hat{p}_{\text{Potential}} &= 2 \times \Pr\left\{z < \frac{192-200}{1.70}\right\} \\
&= 2 \times 0.0000013 \\
&= 0.0000026
\end{aligned}$$

7-7☺.

$n = 2; \quad \hat{\mu} = \bar{\bar{x}} = 39.7; \quad \bar{R} = 2.5; \quad \hat{\sigma}_x = \bar{R}/d_2 = 2.5/1.128 = 2.216$

USL = 40 + 5 = 45; LSL = 40 – 5 = 35

(a)

Potential: $\hat{C}_p = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \dfrac{45-35}{6(2.216)} = 0.75$

(b)

$$\hat{C}_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{45-39.7}{3(2.216)} = 0.80$$

Actual: $\hat{C}_{pl} = \dfrac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \dfrac{39.7-35}{3(2.216)} = 0.71$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 0.71$$

(c)

$$V = \frac{\bar{x} - T}{s} = \frac{39.7-40}{2.216} = -0.135$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{0.75}{\sqrt{1+(-0.135)^2}} = 0.74$$

$$\hat{C}_{pkm} = \frac{\hat{C}_{pk}}{\sqrt{1+V^2}} = \frac{0.71}{\sqrt{1+(-0.135)^2}} = 0.70$$

The closeness of estimates for C_p, C_{pk}, C_{pm}, and C_{pkm} indicate that the process mean is very close to the specification target.

(d)

The current fraction nonconforming is:

$$\begin{aligned}
\hat{p}_{\text{Actual}} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < \text{LSL}\} + \left[1 - \Pr\{x \le \text{USL}\}\right] \\
&= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + \left[1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\}\right] \\
&= \Pr\left\{z < \frac{35-39.7}{2.216}\right\} + \left[1 - \Pr\left\{z \le \frac{45-39.7}{2.216}\right\}\right] \\
&= \Phi(-2.12094) + \left[1 - \Phi(2.39170)\right] \\
&= 0.0169634 + \left[1 - 0.991615\right] \\
&= 0.025348
\end{aligned}$$

7-7 (d) continued
If the process mean could be centered at the specification target, the fraction nonconforming would be:

$$\begin{aligned}\hat{p}_{\text{Potential}} &= 2 \times \Pr\left\{z < \frac{35-40}{2.216}\right\} \\ &= 2 \times \Pr\{z < -2.26\} \\ &= 2 \times 0.01191 \\ &= 0.02382\end{aligned}$$

7-8 (7-6).

$\hat{\mu} = 75; \overline{S} = 2; \hat{\sigma} = \hat{S}/c_4 = 2/0.9400 = 2.13$

(a)

Potential: $\hat{C}_p = \dfrac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \dfrac{2(8)}{6(2.13)} = 1.25$

(b)

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{75 - (80-8)}{3(2.13)} = 0.47$$

Actual: $\hat{C}_{pu} = \dfrac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \dfrac{80+8-75}{3(2.13)} = 2.03$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 0.47$$

(c) Let $\hat{\mu} = 80$

$$\begin{aligned}\hat{p}_{\text{Potential}} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\ &= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + 1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\} \\ &= \Pr\left\{z < \frac{72-80}{2.13}\right\} + 1 - \Pr\left\{z \le \frac{88-80}{2.13}\right\} \\ &= \Phi(-3.756) + 1 - \Phi(3.756) \\ &= 0.000086 + 1 - 0.999914 \\ &= 0.000172\end{aligned}$$

7-9 (7-7).
Assume $n = 5$

<u>Process A</u>

$$\hat{\mu} = \overline{\overline{x}}_A = 100; \ \overline{s}_A = 3; \ \hat{\sigma}_A = \overline{s}_A / c_4 = 3/0.9400 = 3.191$$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{(100+10)-(100-10)}{6(3.191)} = 1.045$$

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}}{3\hat{\sigma}_x} = \frac{(100+10)-100}{3(3.191)} = 1.045$$

$$\hat{C}_{pl} = \frac{\hat{\mu} - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{100-(100-10)}{3(3.191)} = 1.045$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.045$$

$$V = \frac{\overline{x} - T}{s} = \frac{100-100}{3.191} = 0$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{1.045}{\sqrt{1+(0)^2}} = 1.045$$

$$\begin{aligned}
\hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < \text{LSL}\} + 1 - \Pr\{x \le \text{USL}\} \\
&= \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} + 1 - \Pr\left\{z \le \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\} \\
&= \Pr\left\{z < \frac{90-100}{3.191}\right\} + 1 - \Pr\left\{z \le \frac{110-100}{3.191}\right\} \\
&= \Phi(-3.13) + 1 - \Phi(3.13) \\
&= 0.00087 + 1 - 0.99913 \\
&= 0.00174
\end{aligned}$$

<u>Process B</u>

$$\hat{\mu} = \overline{\overline{x}}_B = 105; \ \overline{s}_B = 1; \ \hat{\sigma}_B = \overline{s}_B / c_4 = 1/0.9400 = 1.064$$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{(100+10)-(100-10)}{6(1.064)} = 3.133$$

$$\hat{C}_{pl} = \frac{\hat{\mu}_x - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{105-(100-10)}{3(1.064)} = 4.699$$

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}_x}{3\hat{\sigma}_x} = \frac{(100+10)-105}{3(1.064)} = 1.566$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 1.566$$

7-9 continued

$$V = \frac{\overline{x} - T}{s} = \frac{100 - 105}{1.064} = -4.699$$

$$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+V^2}} = \frac{3.133}{\sqrt{1+(-4.699)^2}} = 0.652$$

$$\hat{p} = \Pr\left\{z < \frac{90-105}{1.064}\right\} + 1 - \Pr\left\{z \le \frac{110-105}{1.064}\right\}$$
$$= \Phi(-14.098) + 1 - \Phi(4.699)$$
$$= 0.000000 + 1 - 0.999999$$
$$= 0.000001$$

Prefer to use Process B with estimated process fallout of 0.000001 instead of Process A with estimated fallout 0.001726.

7-10 (7-8).

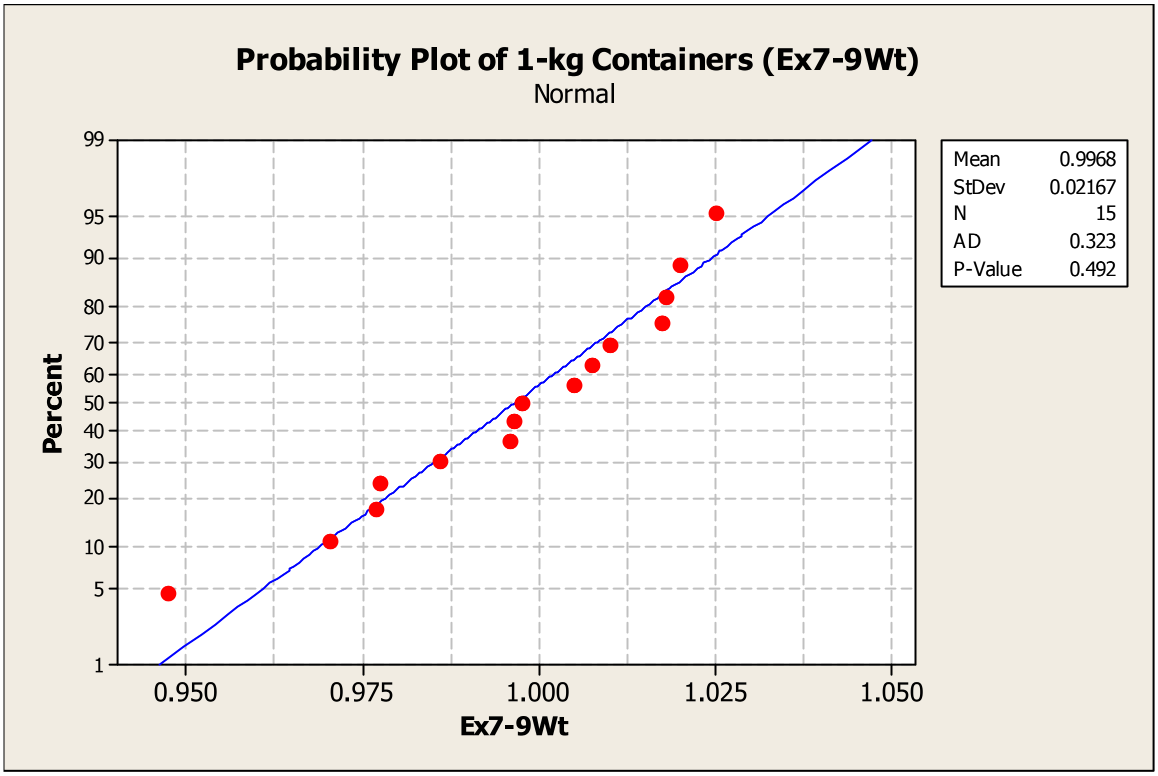
Process A: $\hat{\mu}_A = 20(100) = 2000;\ \hat{\sigma}_A = \sqrt{20\hat{\sigma}^2} = \sqrt{20(3.191)^2} = 14.271$

Process B: $\hat{\mu}_B = 20(105) = 2100;\ \hat{\sigma}_B = \sqrt{20\hat{\sigma}^2} = \sqrt{20(1.064)^2} = 4.758$

Process B will result in fewer defective assemblies. For the parts $\left(\hat{C}_{pk,A} = 1.045\right) < \left(1.566 = \hat{C}_{pk,B}\right)$ indicates that more parts from Process B are within specification than from Process A.

7-11 (7-9).

MTB > Stat > Basic Statistics > Normality Test



A normal probability plot of the 1-kg container weights shows the distribution is close to normal.

$$\bar{x} \approx p_{50} = 0.9975; \quad p_{84} = 1.0200$$

$$\hat{\sigma} = p_{84} - p_{50} = 1.0200 - 0.9975 = 0.0225$$

$$6\hat{\sigma} = 6(0.0225) = 0.1350$$

7-12☺.

LSL = 0.985 kg

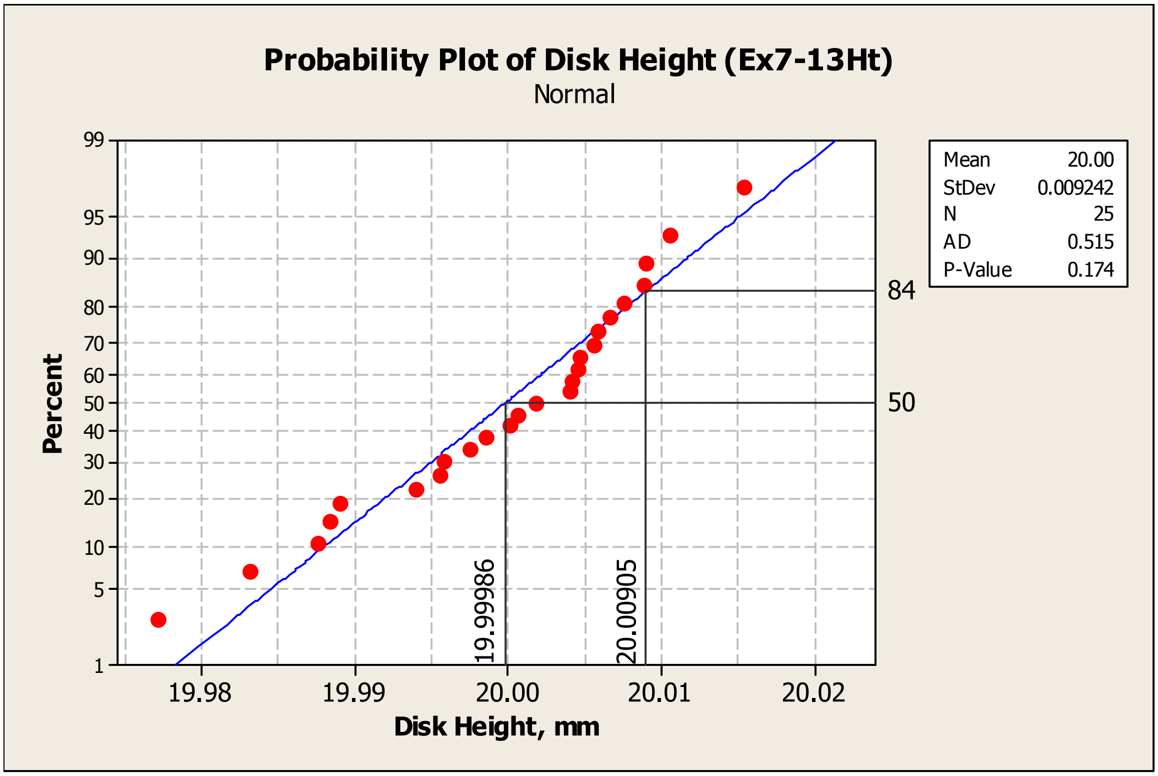
$$C_{pl} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{0.9975 - 0.985}{3(0.0225)} = 0.19$$

$$\hat{p} = \Pr\left\{z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right\} = \Pr\left\{z < \frac{0.985 - 0.9975}{0.0225}\right\} = \Phi(-0.556) = 0.289105$$

7-13☺.

MTB > Stat > Basic Statistics > Normality Test

(Add percentile lines at Y values 50 and 84 to estimate μ and σ.)



A normal probability plot of computer disk heights shows the distribution is close to normal.

$$\bar{x} \approx p_{50} = 19.99986$$

$$p_{84} = 20.00905$$

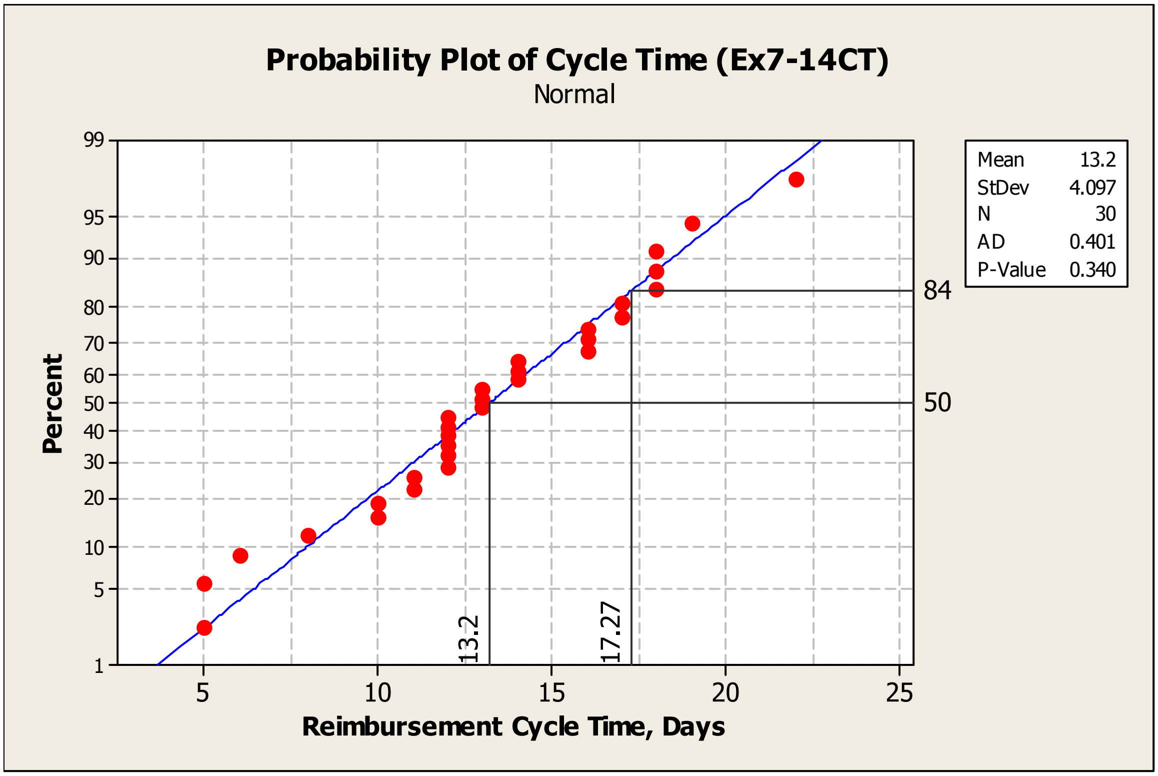
$$\hat{\sigma} = p_{84} - p_{50} = 20.00905 - 19.99986 = 0.00919$$

$$6\hat{\sigma} = 6(0.00919) = 0.05514$$

7-14☺.

MTB > Stat > Basic Statistics > Normality Test

(Add percentile lines at Y values 50 and 84 to estimate μ and σ.)



A normal probability plot of reimbursement cycle times shows the distribution is close to normal.

$$\bar{x} \approx p_{50} = 13.2$$

$$p_{84} = 17.27$$

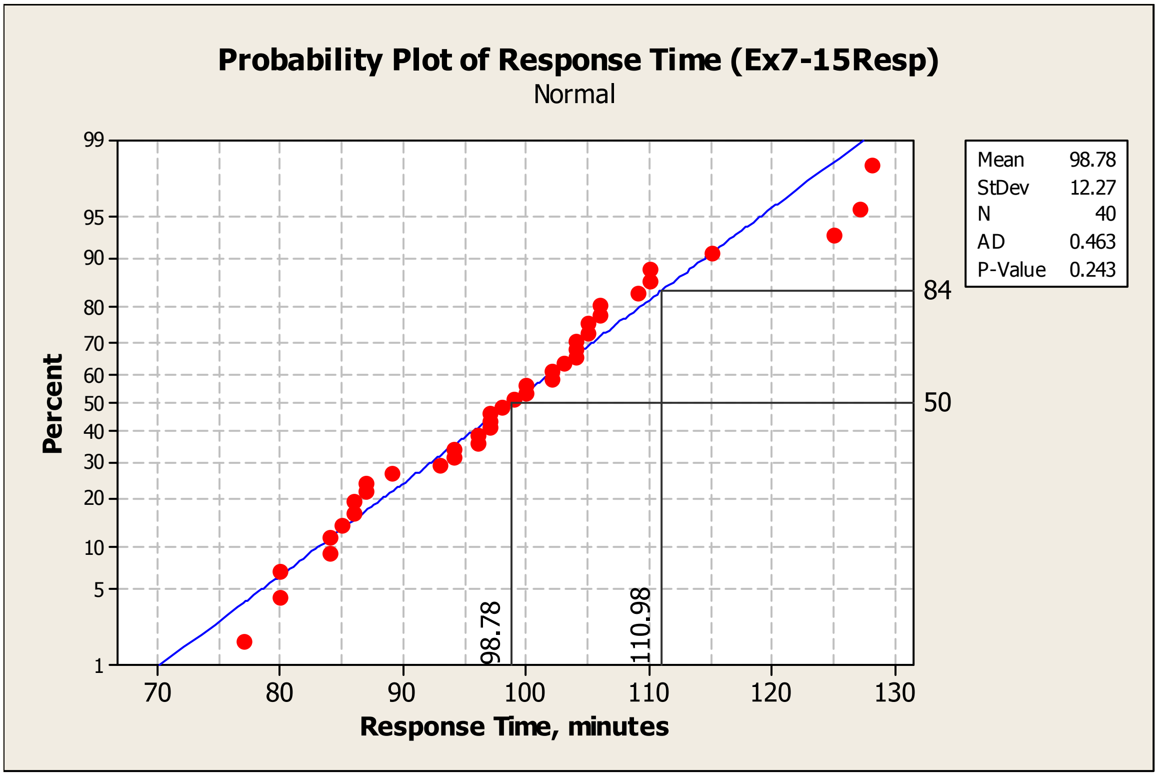
$$\hat{\sigma} = p_{84} - p_{50} = 17.27 - 13.2 = 4.07$$

$$6\hat{\sigma} = 6(4.07) = 24.42$$

7-15☺.

MTB > Stat > Basic Statistics > Normality Test

(Add percentile lines at Y values 50 and 84 to estimate μ and σ.)



A normal probability plot of response times shows the distribution is close to normal.

(a)

$$\bar{x} \approx p_{50} = 98.78$$

$$p_{84} = 110.98$$

$$\hat{\sigma} = p_{84} - p_{50} = 110.98 - 98.78 = 12.2$$

$$6\hat{\sigma} = 6(12.2) = 73.2$$

(b)

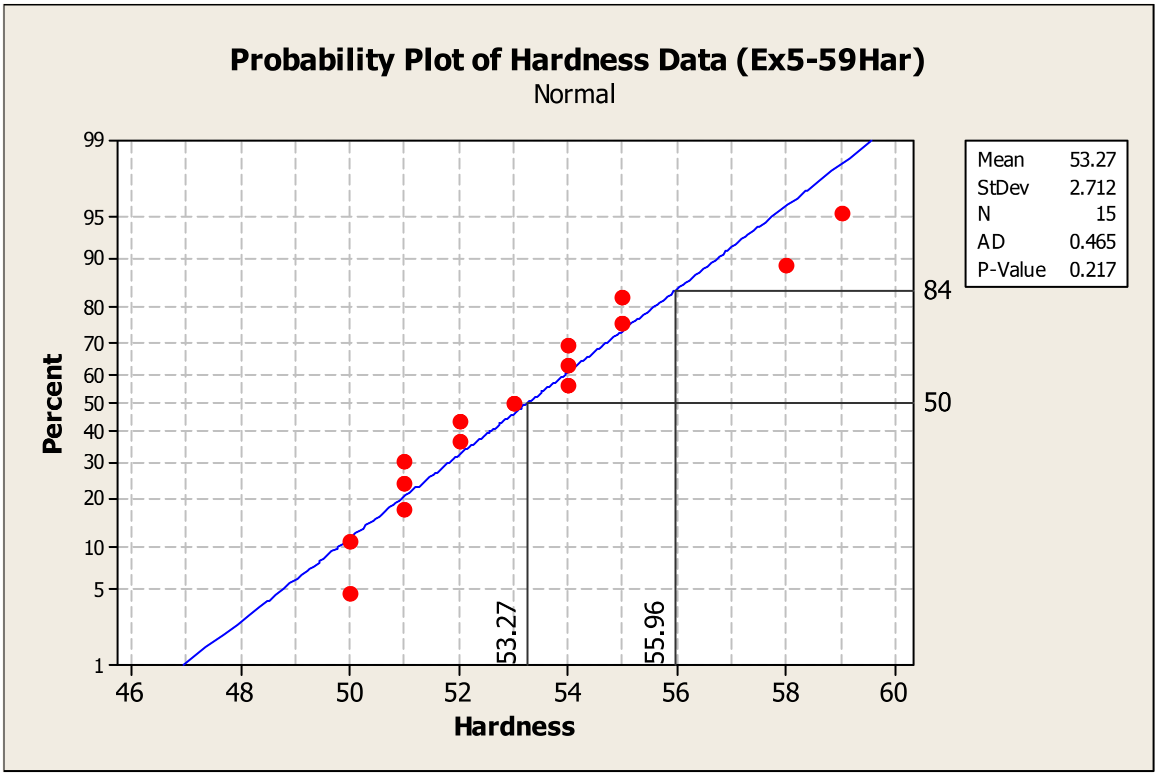
USL = 2 hrs = 120 mins

$$C_{pu} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{120 - 98.78}{3(12.2)} = 0.58$$

$$\hat{p} = \Pr\left\{z > \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\} = 1 - \Pr\left\{z < \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right\} = 1 - \Pr\left\{z < \frac{120 - 98.78}{12.2}\right\}$$

$$= 1 - \Phi(1.739) = 1 - 0.958983 = 0.041017$$

7-16 (7-10).
MTB > Stat > Basic Statistics > Normality Test
(Add percentile lines at Y values 50 and 84 to estimate μ and σ.)



A normal probability plot of hardness data shows the distribution is close to normal.

$\bar{x} \approx p_{50} = 53.27$

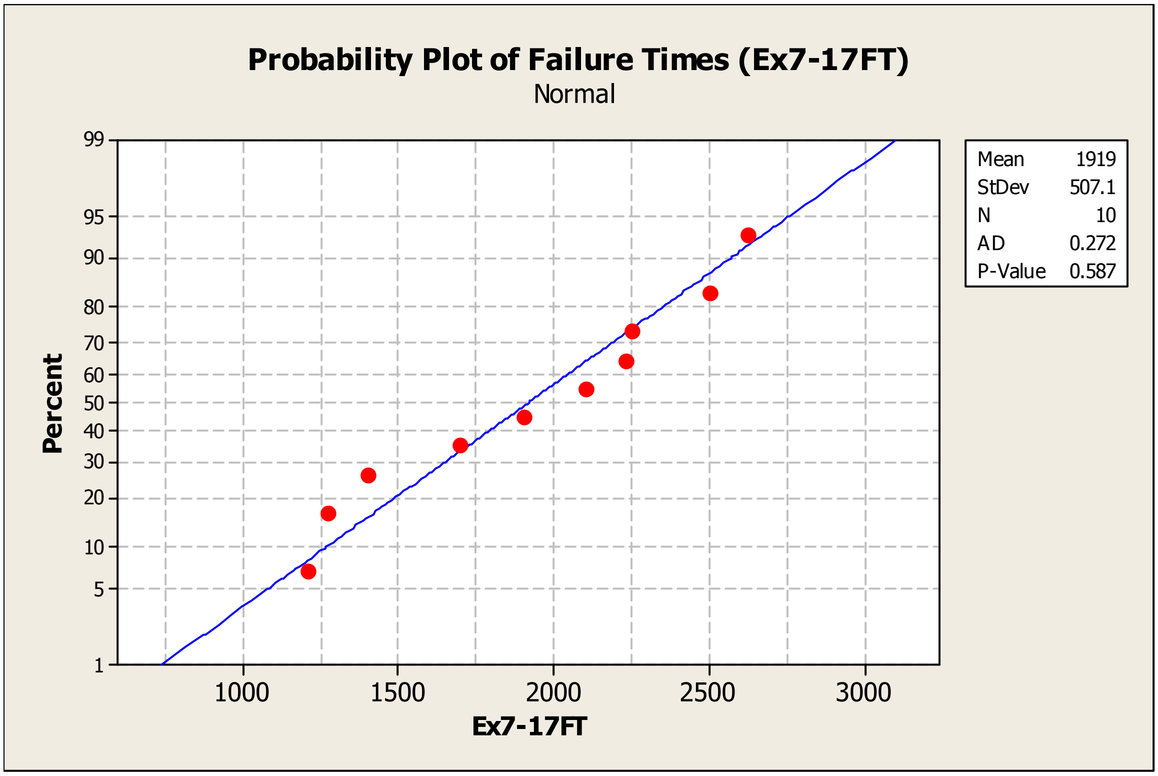
$p_{84} = 55.96$

$\hat{\sigma} = p_{84} - p_{50} = 55.96 - 53.27 = 2.69$

$6\hat{\sigma} = 6(2.69) = 16.14$

7-17 (7-11).

MTB > Stat > Basic Statistics > Normality Test



The plot shows that the data is not normally distributed; so it is not appropriate to estimate capability.

7-18 (7-12).
LSL = 75; USL = 85; n = 25; S = 1.5

(a)
$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{85-75}{6(1.5)} = 1.11$$

(b)
$$\alpha = 0.05$$
$$\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,24} = 12.40$$
$$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,24} = 39.36$$
$$\hat{C}_p\sqrt{\frac{\chi^2_{1-\alpha/2,n-1}}{n-1}} \le C_p \le \hat{C}_p\sqrt{\frac{\chi^2_{\alpha/2,n-1}}{n-1}}$$
$$1.11\sqrt{\frac{12.40}{25-1}} \le C_p \le 1.11\sqrt{\frac{39.36}{25-1}}$$
$$0.80 \le C_p \le 1.42$$

This confidence interval is wide enough that the process may either be capable (ppm = 27) or far from it (ppm ≈ 16,395).

7-19 (7-13).
$$n = 50$$
$$\hat{C}_p = 1.52$$
$$1-\alpha = 0.95$$
$$\chi^2_{1-\alpha,n-1} = \chi^2_{0.95,49} = 33.9303$$
$$\hat{C}_p\sqrt{\frac{\chi^2_{1-\alpha,n-1}}{n-1}} \le C_p$$
$$1.52\sqrt{\frac{33.9303}{49}} = 1.26 \le C_p$$

The company cannot demonstrate that the PCR exceeds 1.33 at a 95% confidence level.
$$1.52\sqrt{\frac{\chi^2_{1-\alpha,49}}{49}} = 1.33$$
$$\chi^2_{1-\alpha,49} = 49\left(\frac{1.33}{1.52}\right)^2 = 37.52$$
$$1-\alpha = 0.88$$
$$\alpha = 0.12$$

7-20 (7-14).

$n = 30;\ \bar{x} = 97;\ S = 1.6;\ \text{USL} = 100;\ \text{LSL} = 90$

(a)

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}_x}{3\hat{\sigma}_x} = \frac{100-97}{3(1.6)} = 0.63$$

$$\hat{C}_{pl} = \frac{\hat{\mu}_x - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{97-90}{3(1.6)} = 1.46$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 0.63$$

(b)

$\alpha = 0.05$

$z_{\alpha/2} = z_{0.025} = 1.960$

$$\hat{C}_{pk}\left[1 - z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right] \le C_{pk} \le \hat{C}_{pk}\left[1 + z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right]$$

$$0.63\left[1 - 1.96\sqrt{\frac{1}{9(30)(0.63)^2} + \frac{1}{2(30-1)}}\right] \le C_{pk} \le 0.63\left[1 + 1.96\sqrt{\frac{1}{9(30)(0.63)^2} + \frac{1}{2(30-1)}}\right]$$

$$0.4287 \le C_{pk} \le 0.8313$$

7-21 (7-15).

USL = 2350; LSL = 2100; nominal = 2225; $\bar{x} = 2275$; $s = 60$; $n = 50$

(a)

$$\hat{C}_{pu} = \frac{\text{USL}_x - \hat{\mu}_x}{3\hat{\sigma}_x} = \frac{2350 - 2275}{3(60)} = 0.42$$

$$\hat{C}_{pl} = \frac{\hat{\mu}_x - \text{LSL}_x}{3\hat{\sigma}_x} = \frac{2275 - 2100}{3(60)} = 0.97$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = 0.42$$

(b)

$\alpha = 0.05$; $z_{\alpha/2} = z_{0.025} = 1.960$

$$\hat{C}_{pk}\left[1 - z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right] \le C_{pk} \le \hat{C}_{pk}\left[1 + z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right]$$

$$0.42\left[1 - 1.96\sqrt{\frac{1}{9(50)(0.42)^2} + \frac{1}{2(50-1)}}\right] \le C_{pk} \le 0.42\left[1 + 1.96\sqrt{\frac{1}{9(50)(0.42)^2} + \frac{1}{2(50-1)}}\right]$$

$$0.2957 \le C_{pk} \le 0.5443$$

7-22 (7-16).

from Ex. 7-20, $\hat{C}_{pk} = 0.63$; $z_{\alpha/2} = 1.96$; $n = 30$

$$\hat{C}_{pk}\left[1 - z_{\alpha/2}\sqrt{\frac{1}{2(n-1)}}\right] \le C_{pk} \le \hat{C}_{pk}\left[1 + z_{\alpha/2}\sqrt{\frac{1}{2(n-1)}}\right]$$

$$0.63\left[1 - 1.96\sqrt{\frac{1}{2(30-1)}}\right] \le C_{pk} \le 0.63\left[1 + 1.96\sqrt{\frac{1}{2(30-1)}}\right]$$

$$0.47 \le C_{pk} \le 0.79$$

The approximation yields a narrower confidence interval, but it is not too far off.

7-23 (7-17).

$\sigma_{OI} = 0$; $\hat{\sigma}_I = 3$; $\hat{\sigma}_{\text{Total}} = 5$

$$\hat{\sigma}_{\text{Total}}^2 = \hat{\sigma}_{\text{Meas}}^2 + \hat{\sigma}_{\text{Process}}^2$$

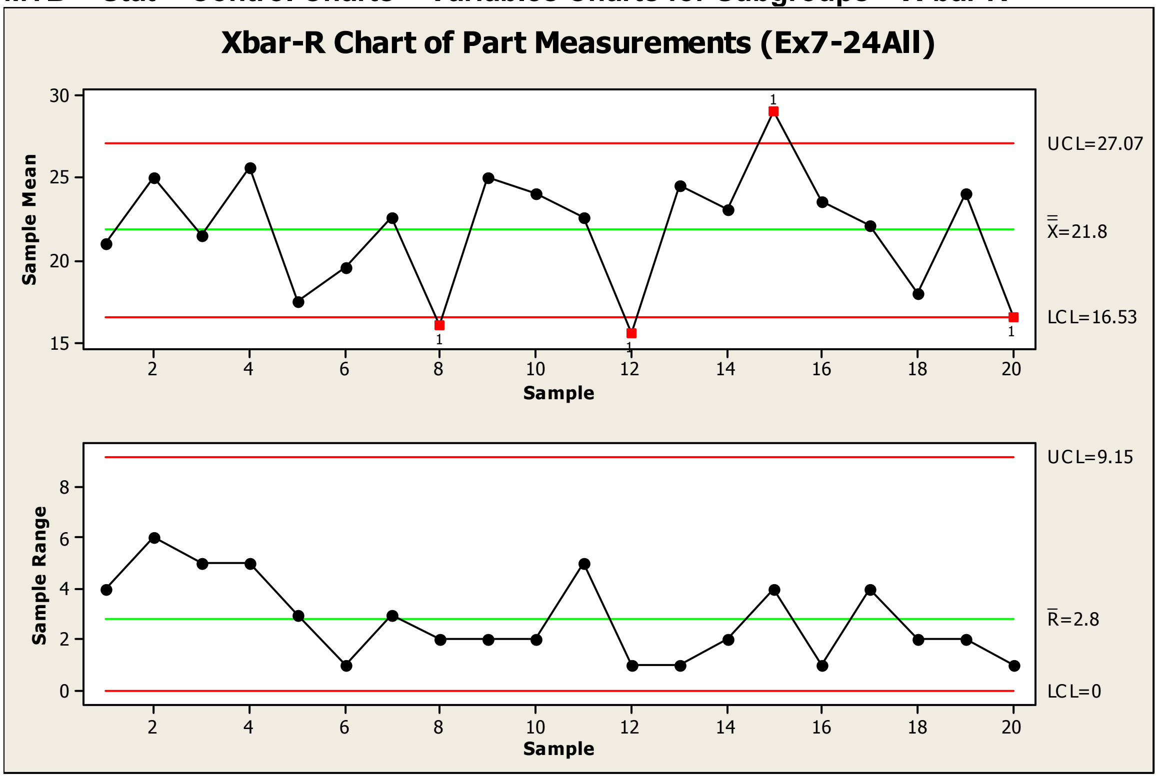
$$\hat{\sigma}_{\text{Process}} = \sqrt{\hat{\sigma}_{\text{Total}}^2 - \hat{\sigma}_{\text{Meas}}^2} = \sqrt{5^2 - 3^2} = 4$$

7-24 (7-18).

(a)

$n = 2; \bar{\bar{x}} = 21.8; \bar{R} = 2.8; \hat{\sigma}_{\text{Gauge}} = 2.482$

MTB > Stat > Control Charts > Variables Charts for Subgroups > X-bar R



Test Results for Xbar Chart of Ex7-24All

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8, 12, 15, 20
```

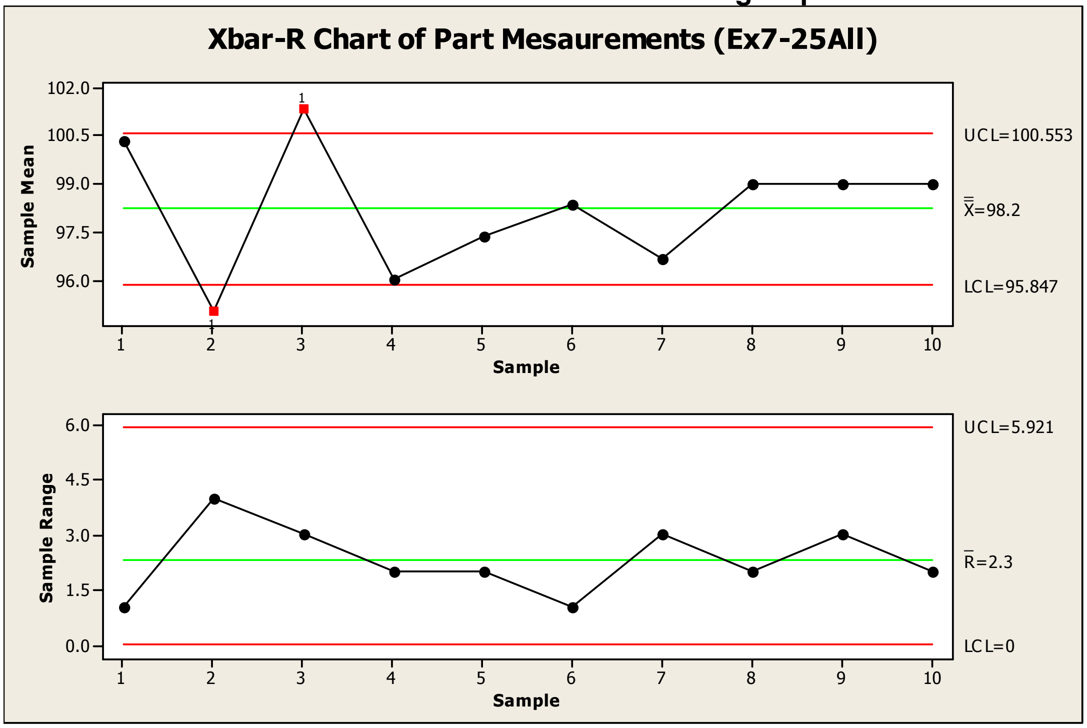
The R chart is in control, and the $\bar{x}$ chart has a few out-of-control parts. The new gauge is more repeatable than the old one.

(b) specs: 25 ± 15

$$\frac{P}{T} = \frac{6\hat{\sigma}_{\text{Gauge}}}{\text{USL} - \text{LSL}} \times 100 = \frac{6(2.482)}{2(15)} \times 100 = 49.6\%$$

7-25 (7-19).

MTB > Stat > Control Charts > Variables Charts for Subgroups > X-bar R




```
Test Results for Xbar Chart of Ex7-25All
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  2, 3
```

The $\bar{x}$ chart has a couple out-of-control points, and the R chart is in control. This indicates that the operator is not having difficulty making consistent measurements.

(b)

$$\bar{\bar{x}} = 98.2;\ \bar{R} = 2.3;\ \hat{\sigma}_{\text{Gauge}} = \bar{R}/d_2 = 2.3/1.693 = 1.359$$

$$\hat{\sigma}^2_{\text{Total}} = 4.717$$

$$\hat{\sigma}^2_{\text{Product}} = \hat{\sigma}^2_{\text{Total}} - \hat{\sigma}^2_{\text{Gauge}} = 4.717 - 1.359^2 = 2.872$$

$$\hat{\sigma}_{\text{Product}} = 1.695$$

(c)

$$\frac{\hat{\sigma}_{\text{Gauge}}}{\hat{\sigma}_{\text{Total}}} \times 100 = \frac{1.359}{\sqrt{4.717}} \times 100 = 62.5\%$$

(d)

USL = 100 + 15 = 115; LSL = 100 – 15 = 85

$$\frac{P}{T} = \frac{6\hat{\sigma}_{\text{Gauge}}}{\text{USL} - \text{LSL}} = \frac{6(1.359)}{115 - 85} = 0.272$$

7-26 (7-20).

(a)

Excel : workbook Chap07.xls : worksheet Ex7-26

$\overline{\overline{x}}_1 = 50.03;\ \overline{R}_1 = 1.70;\ \overline{\overline{x}}_2 = 49.87;\ \overline{R}_2 = 2.30$

$\overline{\overline{R}} = 2.00$

$n = 3$ repeat measurements

$d_2 = 1.693$

$\hat{\sigma}_{\text{Repeatability}} = \overline{\overline{R}}/d_2 = 2.00/1.693 = 1.181$

$R_{\overline{\overline{x}}} = 0.17$

$n = 2$ operators

$d_2 = 1.128$

$\hat{\sigma}_{\text{Reproducibility}} = R_{\overline{\overline{x}}}/d_2 = 0.17/1.128 = 0.151$

(b)

$\hat{\sigma}^2_{\text{Measurement Error}} = \hat{\sigma}^2_{\text{Repeatability}} + \hat{\sigma}^2_{\text{Reproducibility}} = 1.181^2 + 0.151^2 = 1.418$

$\hat{\sigma}_{\text{Measurement Error}} = 1.191$

(c) specs: 50 ± 10

$$\frac{P}{T} = \frac{6\hat{\sigma}_{\text{Gauge}}}{\text{USL} - \text{LSL}} \times 100 = \frac{6(1.191)}{60 - 40} \times 100 = 35.7\%$$

7-27 (7-21).

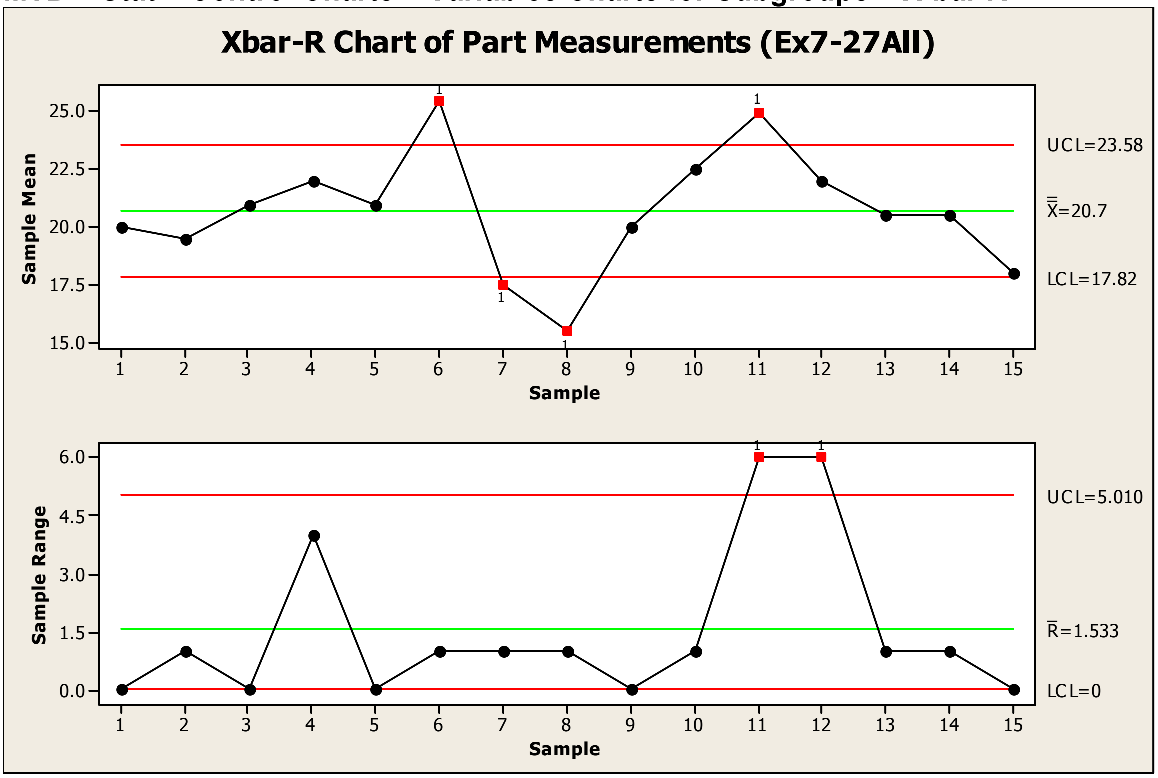
(a)

$\hat{\sigma}_{\text{Gauge}} = \overline{R}/d_2 = 1.533/1.128 = 1.359$

Gauge capability: $6\hat{\sigma} = 8.154$

(b)

MTB > Stat > Control Charts > Variables Charts for Subgroups > X-bar R



Test Results for R Chart of Ex7-27All

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  11, 12
```

Out-of-control points on *R* chart indicate operator difficulty with using gage.

7-28☺.

MTB > Stat > ANOVA > Balanced ANOVA
In Results, select "Display expected mean squares and variance components"

ANOVA: Ex7-28Reading versus Ex7-28Part, Ex7-28Op

```
Factor      Type    Levels
Ex7-28Part  random      20
Ex7-28Op    random       3

Factor      Values
Ex7-28Part   1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16, 17,
18, 19, 20
Ex7-28Op    1, 2, 3

Analysis of Variance for Ex7-28Reading
Source                DF        SS      MS      F      P
Ex7-28Part            19  1185.425  62.391  87.65  0.000
Ex7-28Op               2     2.617   1.308   1.84  0.173
Ex7-28Part*Ex7-28Op   38    27.050   0.712   0.72  0.861
Error                 60    59.500   0.992
Total                119  1274.592

S = 0.995825   R-Sq = 95.33%   R-Sq(adj) = 90.74%

                                            Expected Mean Square
                         Variance   Error   for Each Term (using
   Source                component   term   unrestricted model)
1  Ex7-28Part              10.2798      3   (4) + 2 (3) + 6 (1)
2  Ex7-28Op                 0.0149      3   (4) + 2 (3) + 40 (2)
3  Ex7-28Part*Ex7-28Op     -0.1399      4   (4) + 2 (3)
4  Error                    0.9917          (4)
```

$$\hat{\sigma}^2_{\text{Repeatability}} = MS_{\text{Error}} = 0.992$$

$$\hat{\sigma}^2_{\text{Part}\times\text{Operator}} = \frac{MS_{\text{P}\times\text{O}} - MS_{\text{E}}}{n} = \frac{0.712 - 0.992}{2} = -0.1400 \Rightarrow 0$$

$$\hat{\sigma}^2_{\text{Operator}} = \frac{MS_{\text{O}} - MS_{\text{P}\times\text{O}}}{pn} = \frac{1.308 - 0.712}{20(2)} = 0.0149$$

$$\hat{\sigma}^2_{\text{Part}} = \frac{MS_{\text{P}} - MS_{\text{P}\times\text{O}}}{on} = \frac{62.391 - 0.712}{3(2)} = 10.2798$$

The manual calculations match the MINITAB results. Note the Part × Operator variance component is negative. Since the Part × Operator term is not significant ($\alpha = 0.10$), we can fit a reduced model without that term. For the reduced model:

ANOVA: Ex7-28Reading versus Ex7-28Part, Ex7-28Op

…

```
                                  Expected
                                  Mean Square
                                  for Each
                                  Term (using
                 Variance  Error  unrestricted
   Source       component   term  model)
1  Ex7-28Part     10.2513      3  (3) + 6 (1)
2  Ex7-28Op        0.0106      3  (3) + 40 (2)
3  Error           0.8832         (3)
```

(a)

$$\hat{\sigma}^2_{\text{Reproducibility}} = \hat{\sigma}^2_{\text{Operator}} = 0.0106$$

$$\hat{\sigma}^2_{\text{Repeatability}} = \hat{\sigma}^2_{\text{Error}} = 0.8832$$

(b)

$$\hat{\sigma}^2_{\text{Gauge}} = \hat{\sigma}^2_{\text{Reproducibility}} + \hat{\sigma}^2_{\text{Repeatability}} = 0.0106 + 0.8832 = 0.8938$$

$$\hat{\sigma}_{\text{Gauge}} = 0.9454$$

(c)

$$\widehat{P/T} = \frac{6 \times \hat{\sigma}_{\text{Gauge}}}{\text{USL-LSL}} = \frac{6 \times 0.9454}{60 - 6} = 0.1050$$

This gauge is borderline capable since the estimate of P/T ratio just exceeds 0.10.

Estimates of variance components, reproducibility, repeatability, and total gauge variability may also be found using:

MTB > Stat > Quality Tools > Gage Study > Gage R&R Study (Crossed)

Gage R&R Study - ANOVA Method

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
Ex7-28Part	19	1185.43	62.3908	87.6470	0.000
Ex7-28Op	2	2.62	1.3083	1.8380	0.173
Ex7-28Part * Ex7-28Op	38	27.05	0.7118	0.7178	0.861
Repeatability	60	59.50	0.9917		
Total	119	1274.59			

Two-Way ANOVA Table Without Interaction

Source	DF	SS	MS	F	P
Ex7-28Part	19	1185.43	62.3908	70.6447	0.000
Ex7-28Op	2	2.62	1.3083	1.4814	0.232
Repeatability	98	86.55	0.8832		
Total	119	1274.59			

Gage R&R

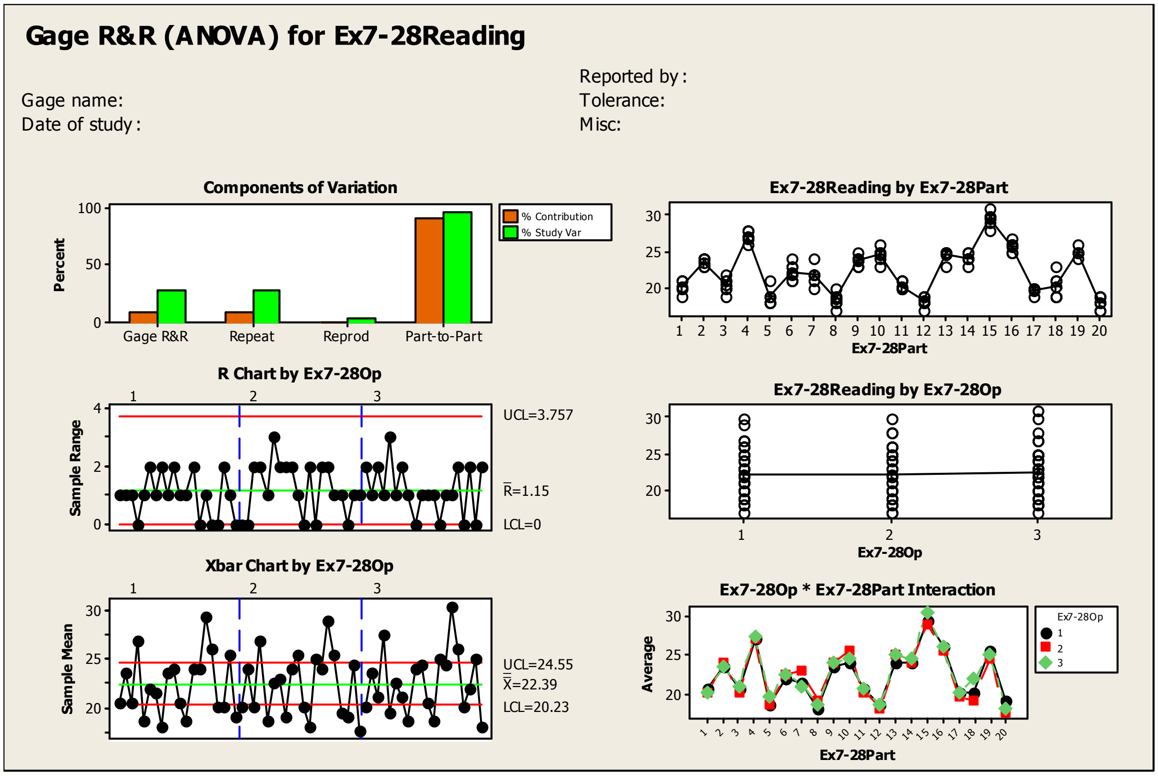
Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.8938	8.02
Repeatability	0.8832	7.92
Reproducibility	0.0106	0.10
Ex7-28Op	0.0106	0.10
Part-To-Part	10.2513	91.98
Total Variation	11.1451	100.00

Source	StdDev (SD)	Study Var (6 * SD)	%Study Var (%SV)
Total Gage R&R	0.94541	5.6724	28.32
Repeatability	0.93977	5.6386	28.15
Reproducibility	0.10310	0.6186	3.09
Ex7-28Op	0.10310	0.6186	3.09
Part-To-Part	3.20176	19.2106	95.91
Total Variation	3.33842	20.0305	100.00

Number of Distinct Categories = 4

7-28 continued

Visual representations of variability and stability are also provided:



7-29☺.

$$\hat{\sigma}^2_{\text{Part}} = 10.2513; \hat{\sigma}^2_{\text{Total}} = 11.1451$$

$$\hat{\rho}_P = \frac{\hat{\sigma}^2_{\text{Part}}}{\hat{\sigma}^2_{\text{Total}}} = \frac{10.2513}{11.1451} = 0.9198$$

$$\widehat{SNR} = \sqrt{\frac{2\hat{\rho}_P}{1-\hat{\rho}_P}} = \sqrt{\frac{2(0.9198)}{1-0.9198}} = 4.79$$

$$\widehat{DR} = \frac{1+\hat{\rho}_P}{1-\hat{\rho}_P} = \frac{1+0.9198}{1-0.9198} = 23.94$$

SNR = 4.79 indicates that fewer than five distinct levels can be reliably obtained from the measurements. This is near the AIAG-recommended value of five levels or more, but larger than a value of two (or less) that indicates inadequate gauge capability. (Also note that the MINITAB Gage R&R output indicates "`Number of Distinct Categories = 4`"; this is also the number of distinct categories of parts that the gauge is able to distinguish)

DR = 23.94, exceeding the minimum recommendation of four. By this measure, the gauge is capable.

7-30 (7-22).

$$\mu = \mu_1 + \mu_2 + \mu_3 = 100 + 75 + 75 = 250$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^3} = \sqrt{4^2 + 4^2 + 2^2} = 6$$

$$\begin{aligned}
\Pr\{x > 262\} &= 1 - \Pr\{x \le 262\} \\
&= 1 - \Pr\left\{z \le \frac{262-\mu}{\sigma}\right\} \\
&= 1 - \Pr\left\{z \le \frac{262-250}{6}\right\} \\
&= 1 - \Phi(2.000) \\
&= 1 - 0.9772 \\
&= 0.0228
\end{aligned}$$

7-31 (7-23).

$x_1 \sim N(20, 0.3^2);\ x_2 \sim N(19.6, 0.4^2)$

Nonconformities will occur if $y = x_1 - x_2 < 0.1$ or $y = x_1 - x_2 > 0.9$

$\mu_y = \mu_1 - \mu_2 = 20 - 19.6 = 0.4$

$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 = 0.3^2 + 0.4^2 = 0.25$

$\sigma_y = 0.50$

$$\begin{aligned}
\Pr\{\text{Nonconformities}\} &= \Pr\{y < \text{LSL}\} + \Pr\{y > \text{USL}\} \\
&= \Pr\{y < 0.1\} + \Pr\{y > 0.9\} \\
&= \Pr\{y < 0.1\} + 1 - \Pr\{y \le 0.9\} \\
&= \Phi\left(\frac{0.1 - 0.4}{\sqrt{0.25}}\right) + 1 - \Phi\left(\frac{0.9 - 0.4}{\sqrt{0.25}}\right) \\
&= \Phi(-0.6) + 1 - \Phi(1.00) \\
&= 0.2743 + 1 - 0.8413 \\
&= 0.4330
\end{aligned}$$

7-32 (7-24).

$$\begin{aligned}
\text{Volume} &= L \times H \times W \\
&\cong \mu_L \mu_H \mu_W + (L - \mu_L)\mu_H \mu_W + (H - \mu_H)\mu_L \mu_W + (W - \mu_W)\mu_L \mu_H
\end{aligned}$$

$\hat{\mu}_{\text{Volume}} \cong \mu_L \mu_H \mu_W = 6.0(3.0)(4.0) = 72.0$

$$\begin{aligned}
\sigma^2_{\text{Volume}} &\cong \mu_L^2 \sigma_H^2 \sigma_W^2 + \mu_H^2 \sigma_L^2 \sigma_W^2 + \mu_W^2 \sigma_L^2 \sigma_H^2 \\
&= 6.0^2(0.01)(0.01) + 3.0^2(0.01)(0.01) + 4.0^2(0.01)(0.01) \\
&= 0.0061
\end{aligned}$$

7-33 (7-25).

$$\begin{aligned}
\text{Weight} &= d \times W \times L \times T \\
&\cong d\left[\mu_W \mu_L \mu_T + (W - \mu_W)\mu_L \mu_T + (L - \mu_L)\mu_W \mu_T + (T - \mu_T)\mu_W \mu_L\right]
\end{aligned}$$

$\hat{\mu}_{\text{Weight}} \cong d[\mu_W \mu_L \mu_T] = 0.08(10)(20)(3) = 48$

$$\begin{aligned}
\hat{\sigma}^2_{\text{Weight}} &\cong d^2\left[\hat{\mu}_W^2 \hat{\sigma}_L^2 \hat{\sigma}_T^2 + \hat{\mu}_L^2 \hat{\sigma}_W^2 \hat{\sigma}_T^2 + \hat{\mu}_T^2 \hat{\sigma}_W^2 \hat{\sigma}_L^2\right] \\
&= 0.08^2\left[10^2(0.3^2)(0.1^2) + 20^2(0.2^2)(0.1^2) + 3^2(0.2^2)(0.3^2)\right] = 0.00181
\end{aligned}$$

$\hat{\sigma}_{\text{Weight}} \cong 0.04252$

7-34 (7-26).

$s = (3+0.05x)^2$ and $f(x) = \frac{1}{26}(5x-2);\ 2 \le x \le 4$

$$E(x) = \mu_x = \int x f(x)dx = \int_2^4 x\left[\frac{1}{26}(5x-2)\right]dx = \frac{1}{26}\left(\frac{5}{3}x^3\Big|_2^4 - x^2\Big|_2^4\right) = 3.1282$$

$$E\left(x^2\right) = \int x^2 f(x)dx = \int_2^4 x^2\left[\frac{1}{26}(5x-2)\right]dx = \frac{1}{26}\left(\frac{5}{4}x^4\Big|_2^4 - \frac{2}{3}x^3\Big|_2^4\right) = 10.1026$$

$$\sigma_x^2 = E(x^2) - \left[E(x)\right]^2 = 10.1026 - (3.1282)^2 = 0.3170$$

$$\mu_s \cong g(x) = \left[3+0.05(\mu_x)\right]^2 = \left[3+0.05(3.1282)\right]^2 = 9.9629$$

$$\sigma_s^2 \cong \left[\frac{\partial g(x)}{\partial x}\right]^2\Bigg|_{\mu_x} \sigma_x^2$$

$$= \left[\frac{\partial(3+0.05x)^2}{\partial x}\right]^2\Bigg|_{\mu_x} \sigma_x^2$$

$$= 2(3+0.05\mu_x)(0.05)\sigma_x^2$$

$$= 2\left[3+0.05(3.1282)\right](0.05)(0.3170)$$

$$= 0.1001$$

7-35 (7-27).

$$I = E/(R_1+R_2)$$

$$\mu_I \cong \mu_E/(\mu_{R_1}+\mu_{R_2})$$

$$\sigma_I^2 \cong \frac{\sigma_E^2}{(\mu_{R_1}+\mu_{R_2})} + \frac{\mu_E}{(\mu_{R_1}+\mu_{R_2})^2}\left(\sigma_{R_1}^2+\sigma_{R_2}^2\right)$$

7-36 (7-28).

$x_1 \sim N(\mu_1, 0.400^2); x_2 \sim N(\mu_2, 0.300^2)$

$\mu_y = \mu_1 - \mu_2$

$\sigma_y = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{0.400^2 + 0.300^2} = 0.5$

$$\Pr\{y < 0.09\} = 0.006$$

$$\Pr\left\{z < \frac{0.09 - \mu_y}{\sigma_y}\right\} = \Phi^{-1}(0.006)$$

$$\frac{0.09 - \mu_y}{0.5} = -2.512$$

$\mu_y = -[0.5(-2.512) - 0.09] = 1.346$

7-37 (7-29).

$\text{ID} \sim N(2.010, 0.002^2)$ and $\text{OD} \sim N(2.004, 0.001^2)$

Interference occurs if $y = \text{ID} - \text{OD} < 0$

$\mu_y = \mu_{\text{ID}} - \mu_{\text{OD}} = 2.010 - 2.004 = 0.006$

$\sigma_y^2 = \sigma_{\text{ID}}^2 + \sigma_{\text{OD}}^2 = 0.002^2 + 0.001^2 = 0.000005$

$\sigma_y = 0.002236$

$$\begin{aligned}
\Pr\{\text{positive clearance}\} &= 1 - \Pr\{\text{interference}\} \\
&= 1 - \Pr\{y < 0\} \\
&= 1 - \Phi\left(\frac{0 - 0.006}{\sqrt{0.000005}}\right) \\
&= 1 - \Phi(-2.683) \\
&= 1 - 0.0036 \\
&= 0.9964
\end{aligned}$$

7-38 (7-30).

$\alpha = 0.01$

$\gamma = 0.80$

$\chi^2_{1-\gamma,4} = \chi^2_{0.20,4} = 5.989$

$$n \cong \frac{1}{2} + \left(\frac{2-\alpha}{\alpha}\right)\frac{\chi^2_{1-\gamma,4}}{4} = \frac{1}{2} + \left(\frac{2-0.01}{0.01}\right)\frac{5.989}{4} = 299$$

7-39 (7-31).
$n = 10$; $x \sim N(300, 10^2)$; $\alpha = 0.10$; $\gamma = 0.95$; one-sided
From Appendix VIII: $K = 2.355$
$\text{UTL} = \bar{x} + KS = 300 + 2.355(10) = 323.55$

7-40 (7-32).
$n = 25$; $x \sim N(85, 1^2)$; $\alpha = 0.10$; $\gamma = 0.95$; one-sided
From Appendix VIII: $K = 1.838$
$\bar{x} - KS = 85 - 1.838(1) = 83.162$

7-41 (7-33).
$n = 20$; $x \sim N(350, 10^2)$; $\alpha = 0.05$; $\gamma = 0.90$; one-sided
From Appendix VIII: $K = 2.208$
$\text{UTL} = \bar{x} + KS = 350 + 2.208(10) = 372.08$

7-42 (7-34).

$$\alpha = 0.05$$

$$\gamma = 0.90$$

$$\chi^2_{1-\gamma,4} = \chi^2_{0.10,4} = 7.779$$

$$n \cong \frac{1}{2} + \left(\frac{2-\alpha}{\alpha}\right)\frac{\chi^2_{1-\gamma,4}}{4} = \frac{1}{2} + \left(\frac{2-0.05}{0.05}\right)\frac{7.779}{4} = 77$$

After the data are collected, a natural tolerance interval would be the smallest to largest observations.

7-43 (7-35).
$x \sim N\left(0.1264, 0.0003^2\right)$

(a)
$\alpha = 0.05$; $\gamma = 0.95$; and two-sided
From Appendix VII: $K = 2.445$
TI on $x : \bar{x} \pm KS = 0.1264 \pm 2.445(0.0003) = [0.1257, 0.1271]$

(b)
$\alpha = 0.05; t_{\alpha/2,n-1} = t_{0.025,39} = 2.023$
CI on $\bar{x} : \bar{x} \pm t_{\alpha/2,n-1} S/\sqrt{n} = 0.1264 \pm 2.023\left(0.0003/\sqrt{40}\right) = [0.1263, 0.1265]$

Part (a) is a tolerance interval on individual thickness observations; part (b) is a confidence interval on mean thickness. In part (a), the interval relates to individual observations (random variables), while in part (b) the interval refers to a parameter of a distribution (an unknown constant).

7-44 (7-36).
$\alpha = 0.05; \gamma = 0.95$

$$n = \frac{\log(1-\gamma)}{\log(1-\alpha)} = \frac{\log(1-0.95)}{\log(1-0.05)} = 59$$

The largest observation would be the nonparametric upper tolerance limit.