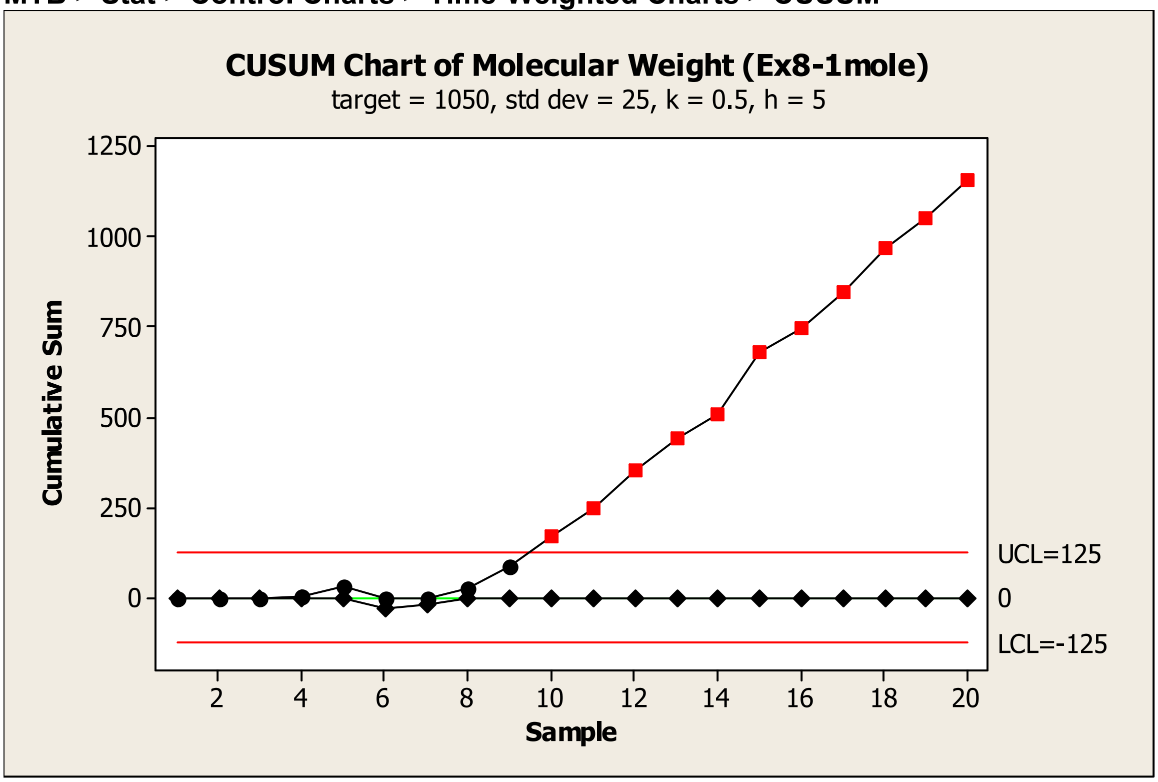
Chapter 8 Exercise Solutions

Several exercises in this chapter differ from those in the 4th edition. An "*" following the exercise number indicates that the description has changed. New exercises are denoted with an "☺". A number in parentheses gives the exercise number from the 4th edition.

8-1.
$\mu_0 = 1050$; $\sigma = 25$; $\delta = 1\sigma$; $K = (\delta/2)\sigma = (1/2)25 = 12.5$; $H = 5\sigma = 5(25) = 125$

(a)
MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM

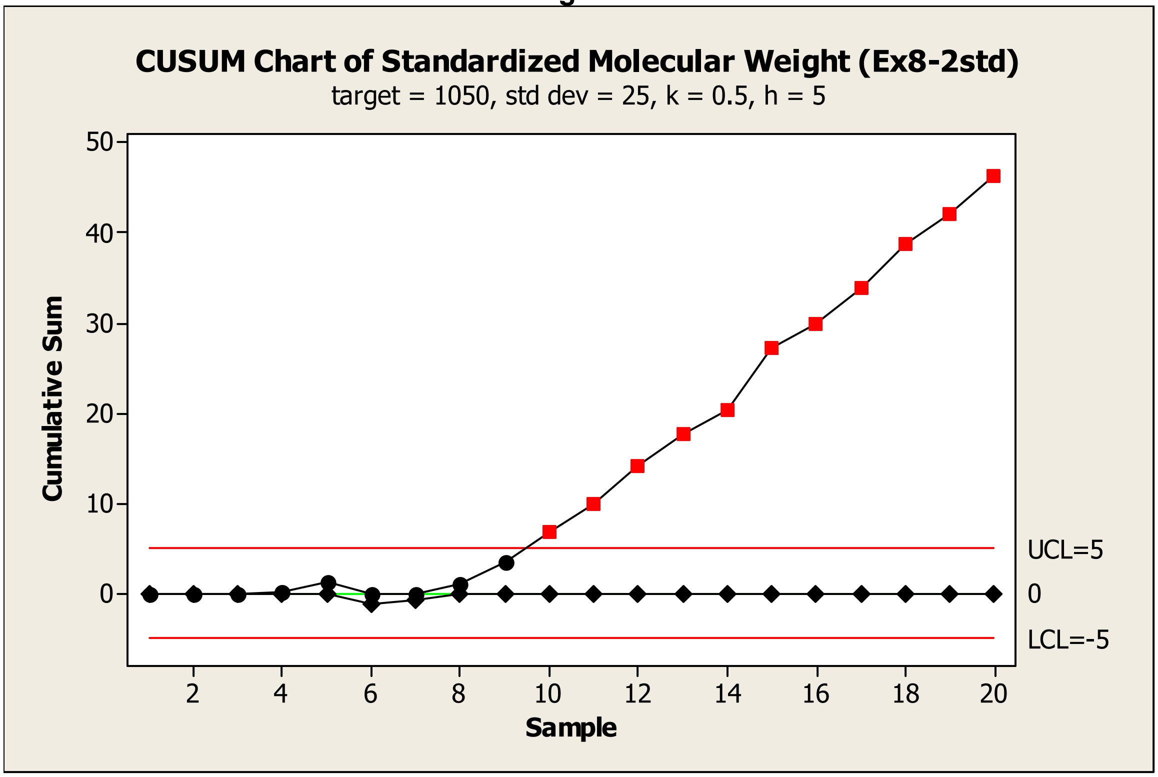


The process signals out of control at observation 10. The point at which the assignable cause occurred can be determined by counting the number of increasing plot points. The assignable cause occurred after observation 10 – 3 = 7.

(b)
$\hat{\sigma} = \overline{\text{MR2}}/d_2 = 38.8421/1.128 = 34.4345$
No. The estimate used for σ is much smaller than that from the data.

8-2.

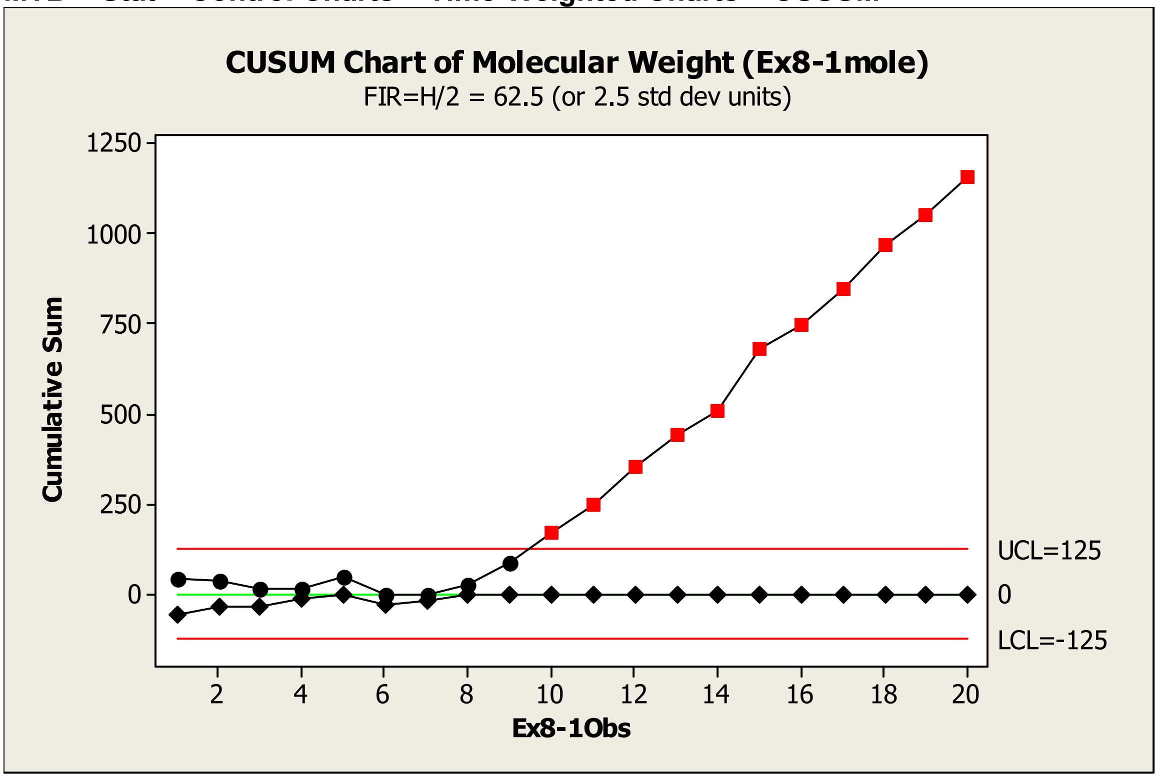
MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



The process signals out of control at observation 10. The assignable cause occurred after observation 10 – 3 = 7.

8-3.
(a)
$\mu_0 = 1050$, $\sigma = 25$, $k = 0.5$, $K = 12.5$, $h = 5$, $H/2 = 125/2 = 62.5$
FIR = $H/2 = 62.5$, in std dev units = $62.5/25 = 2.5$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM

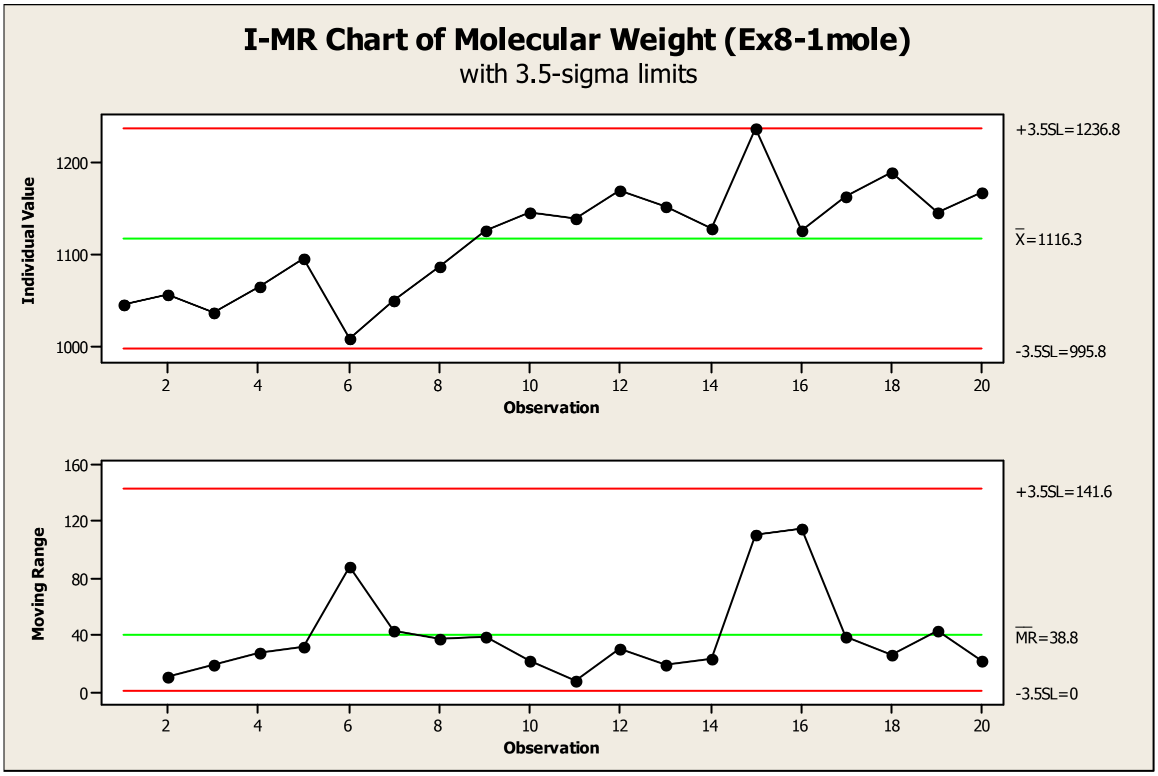


For example,

$$C_1^+ = \max\left[0, x_i - \left(\mu_0 - K\right) + C_0^+\right] = \max\left[0, 1045 - (1050 + 12.5) + 62.5\right] = 45$$

Using the tabular CUSUM, the process signals out of control at observation 10, the same as the CUSUM without a FIR feature.

8-3 continued
(b)
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



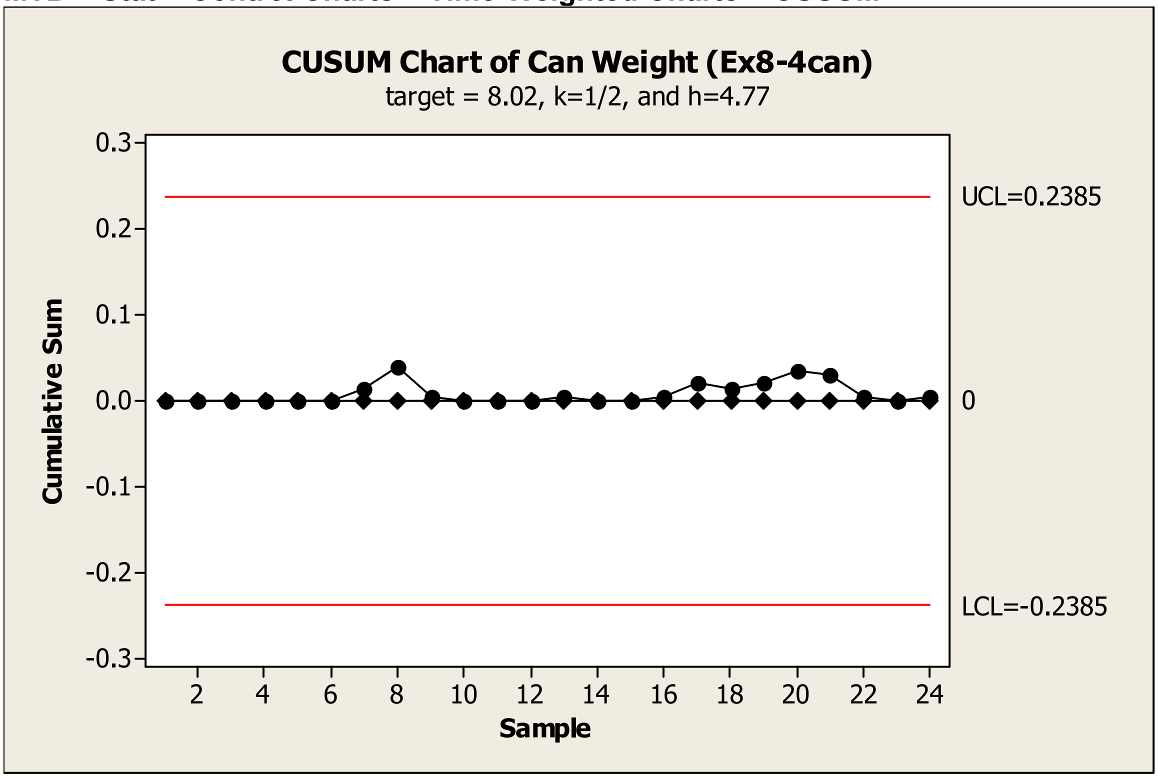
Using 3.5σ limits on the Individuals chart, there are no out-of-control signals. However there does appear to be a trend up from observations 6 through 12—this is the situation detected by the cumulative sum.

8-4.

$\mu_0 = 8.02$, $\sigma = 0.05$, $k = 0.5$, $h = 4.77$, $H = h\sigma = 4.77\ (0.05) = 0.2385$

(a)

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



There are no out-of-control signals.

(b)

$\hat{\sigma} = \overline{\text{MR2}}/1.128 = 0.0186957/1.128 = 0.0166$, so $\sigma = 0.05$ is probably not reasonable.

<u>In Exercise 8-4:</u>

$$\mu_0 = 8.02;\ \sigma = 0.05;\ k = 1/2;\ h = 4.77;\ b = h + 1.166 = 4.77 + 1.166 = 5.936$$

$$\delta^* = 0;\quad \Delta^+ = \delta^* - k = 0 - 0.5 = -0.5;\quad \Delta^- = -\delta^* - k = -0 - 0.5 = -0.5$$

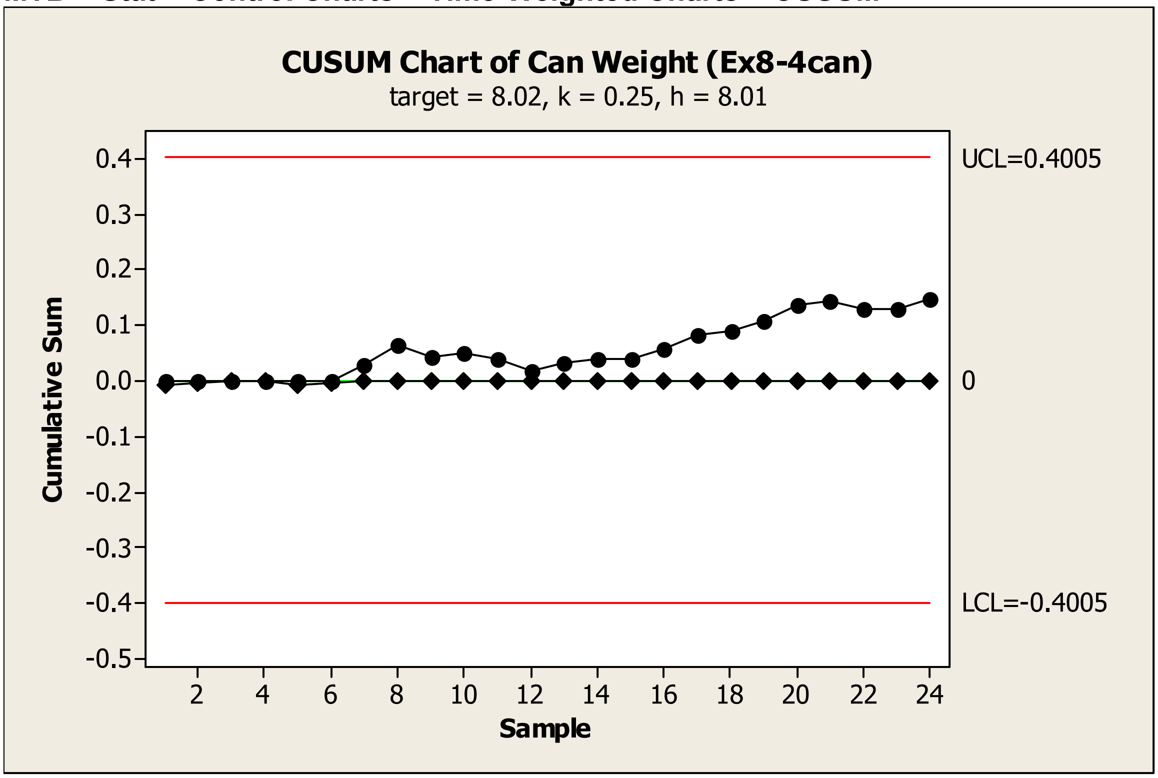
$$\text{ARL}_0^+ = \text{ARL}_0^- \cong \frac{\exp[-2(-0.5)(5.936)] + 2(-0.5)(5.936) - 1}{2(-0.5)^2} = 742.964$$

$$\frac{1}{\text{ARL}_0} = \frac{1}{\text{ARL}_0^+} + \frac{1}{\text{ARL}_0^-} = \frac{2}{742.964} = 0.0027$$

$$\text{ARL}_0 = 1/0.0027 = 371.48$$

8-5.
$\mu_0 = 8.02,\ \sigma = 0.05,\ k = 0.25,\ h = 8.01,\ H = h\sigma = 8.01\ (0.05) = 0.4005$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



There are no out-of-control signals.

In Exercise 8-5:

$$\mu_0 = 8.02;\ \sigma = 0.05;\ k = 0.25;\ h = 8.01;\ b = h + 1.166 = 8.01 + 1.166 = 9.176$$

$$\delta^* = 0;\quad \Delta^+ = \delta^* - k = 0 - 0.25 = -0.25;\quad \Delta^- = -\delta^* - k = -0 - 0.25 = -0.25$$

$$\text{ARL}_0^+ = \text{ARL}_0^- \cong \frac{\exp[-2(-0.25)(9.176)] + 2(-0.25)(9.176) - 1}{2(-0.25)^2} = 741.6771$$

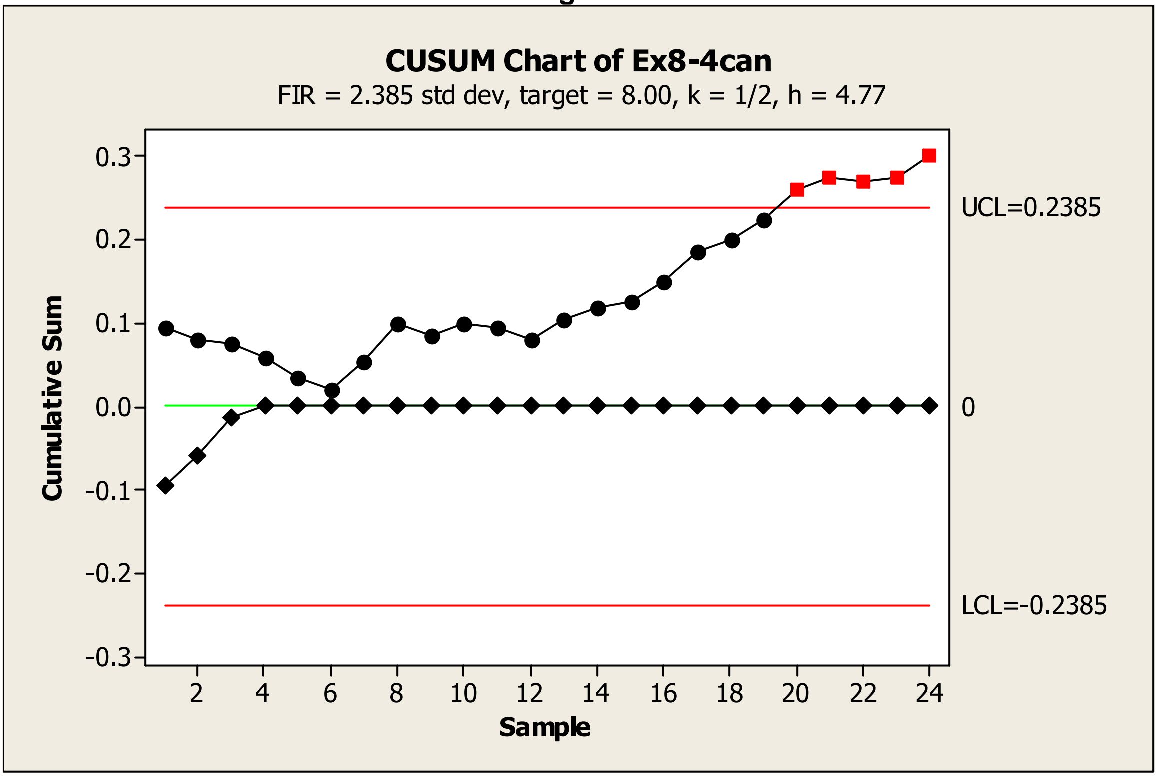
$$\frac{1}{\text{ARL}_0} = \frac{1}{\text{ARL}_0^+} + \frac{1}{\text{ARL}_0^-} = \frac{2}{741.6771} = 0.0027$$

$$\text{ARL}_0 = 1/0.0027 = 370.84$$

The theoretical performance of these two CUSUM schemes is the same for Exercises 8-4 and 8-5.

8-6.
$\mu_0 = 8.00$, $\sigma = 0.05$, $k = 0.5$, $h = 4.77$, $H = h\ \sigma = 4.77\ (0.05) = 0.2385$
FIR = $H/2$, FIR in # of standard deviations = $h/2 = 4.77/2 = 2.385$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



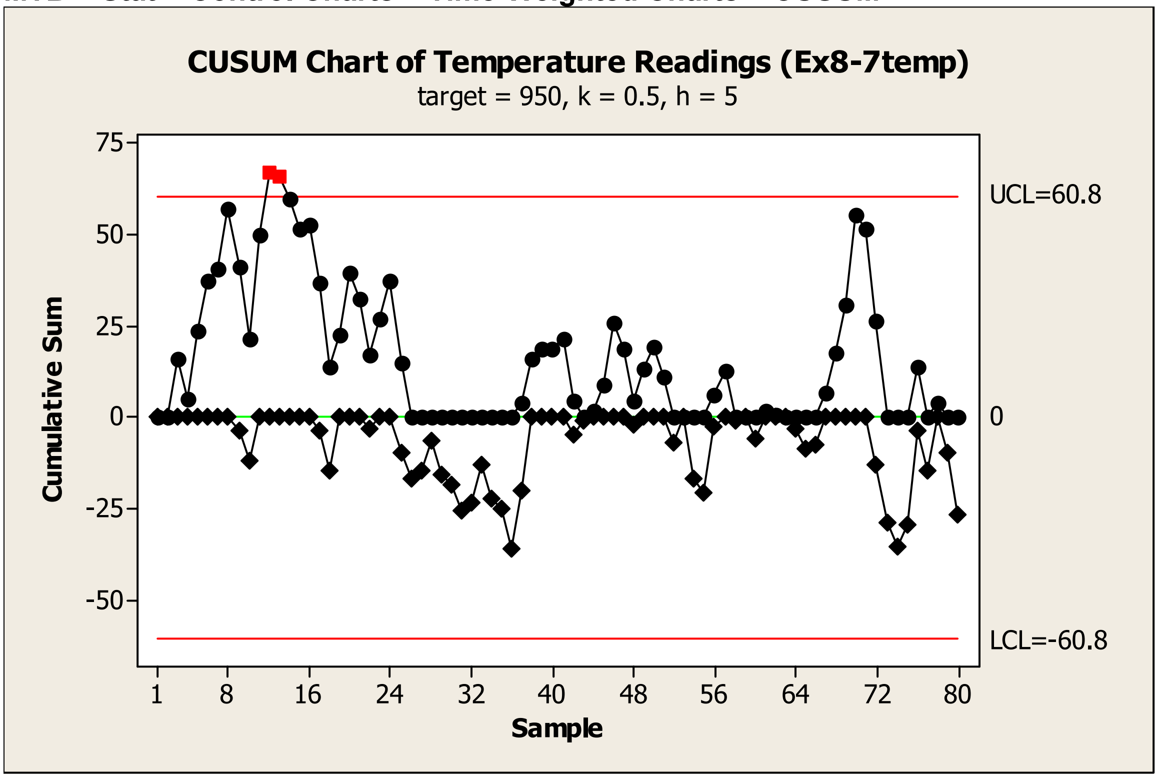
The process signals out of control at observation 20. Process was out of control at process start-up.

8-7.

(a) $\hat{\sigma} = \overline{\text{MR}2}/d_2 = 13.7215/1.128 = 12.16$

(b) $\mu_0 = 950; \hat{\sigma} = 12.16; k = 1/2; h = 5$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM

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Test Results for CUSUM Chart of Ex8-7temp
TEST. One point beyond control limits.
Test Failed at points:  12, 13
```

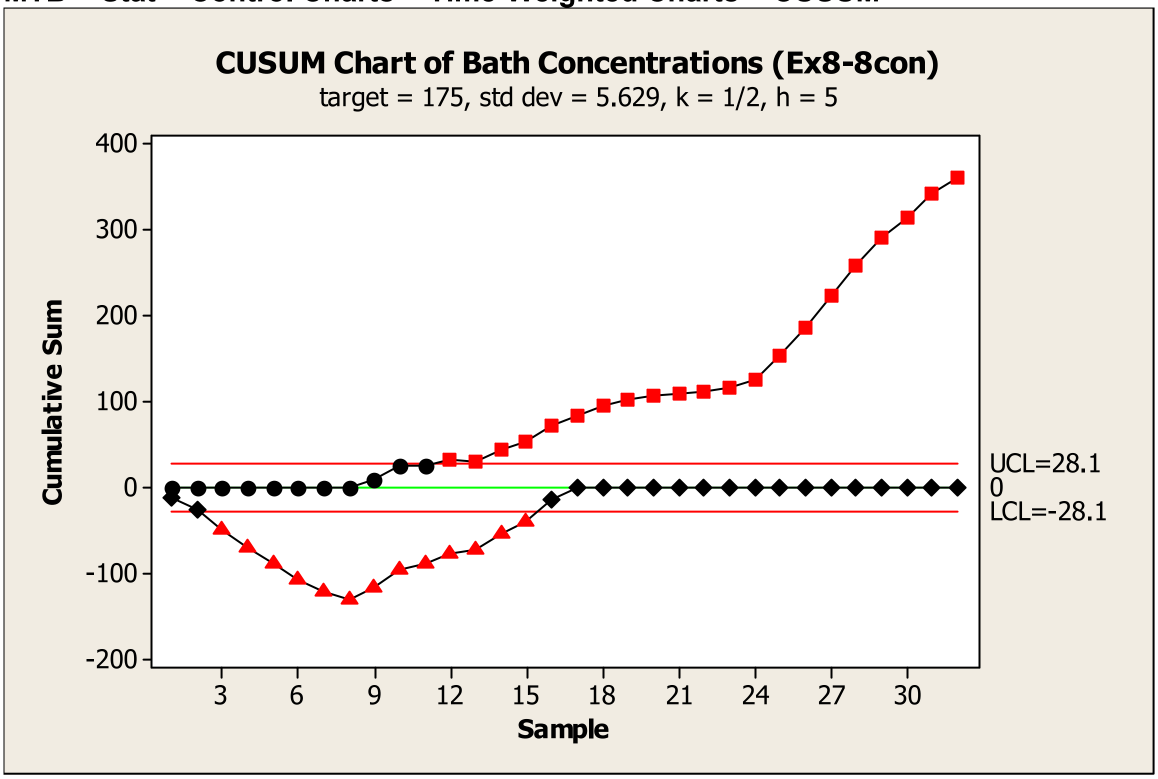
The process signals out of control at observation 12. The assignable cause occurred after observation 12 – 10 = 2.

8-8.

(a) $\hat{\sigma} = \overline{\text{MR2}}/d_2 = 6.35/1.128 = 5.629$ (from a Moving Range chart with CL = 6.35)

(b) $\mu_0 = 175; \hat{\sigma} = 5.629; k = 1/2; h = 5$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



Test Results for CUSUM Chart of Ex8-8con

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TEST. One point beyond control limits.
Test Failed at points:  12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25,
                        26, 27, 28, 29, 30, 31, 32
```

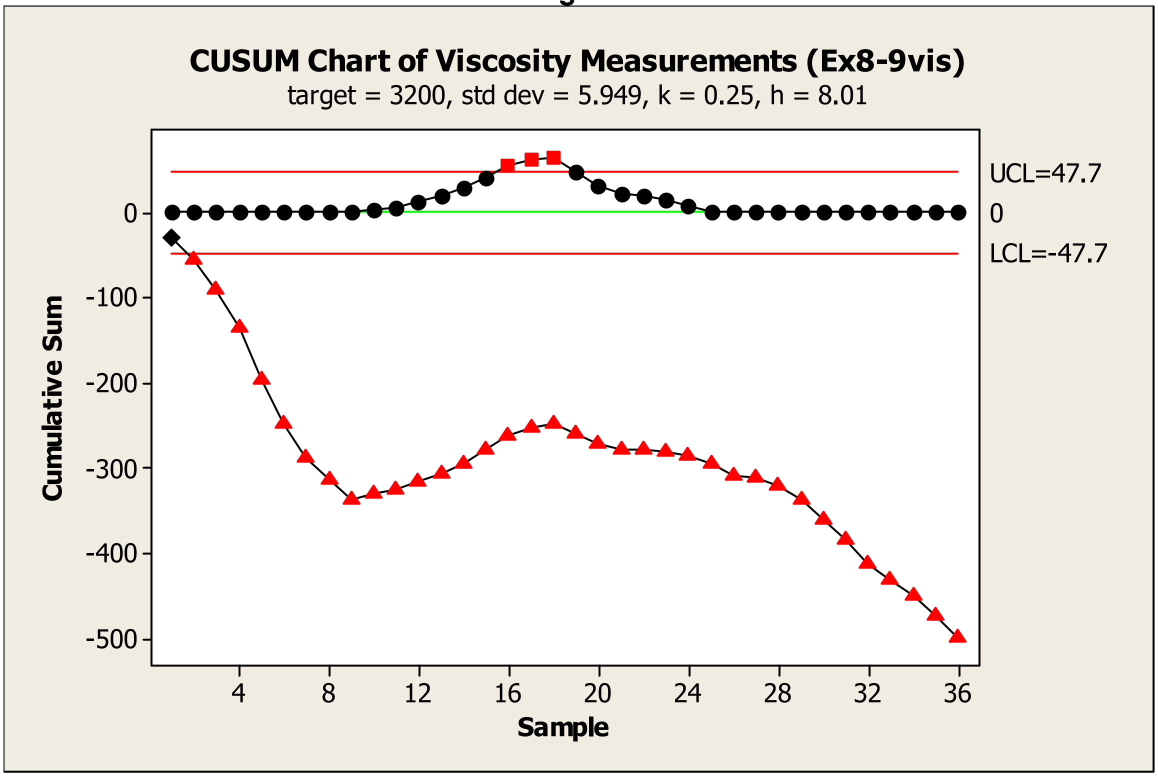
The process signals out of control on the lower side at sample 3 and on the upper side at sample 12. Assignable causes occurred after startup (sample 3 – 3 = 0) and after sample 8 (12 – 4).

8-9.

(a) $\hat{\sigma} = \overline{MR2}/d_2 = 6.71/1.128 = 5.949$ (from a Moving Range chart with CL = 6.71)

(b) $\mu_0 = 3200; \hat{\sigma} = 5.949; k = 0.25; h = 8.01$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM




```
Test Results for CUSUM Chart of Ex8-9vis
TEST. One point beyond control limits.
Test Failed at points:  16, 17, 18
```

The process signals out of control on the lower side at sample 2 and on the upper side at sample 16. Assignable causes occurred after startup (sample 2 – 2) and after sample 9 (16 – 7).

(c)

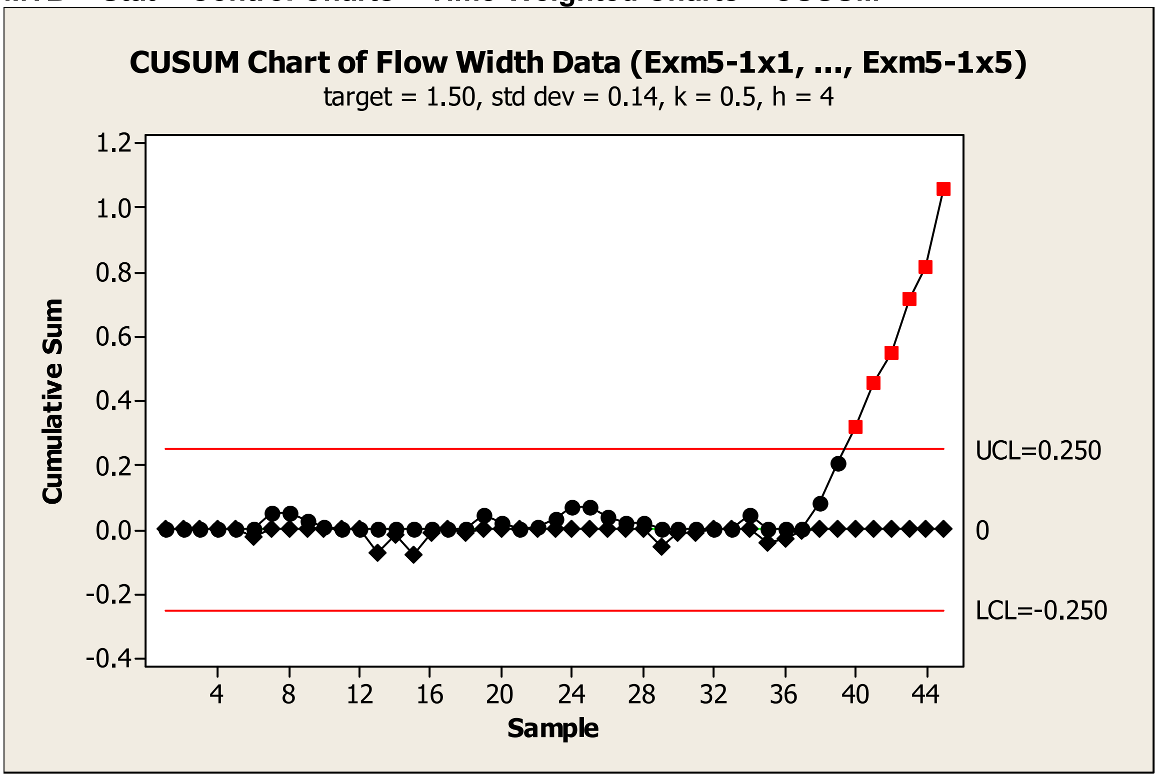
Selecting a smaller shift to detect, $k = 0.25$, should be balanced by a larger control limit, $h = 8.01$, to give longer in-control ARLs with shorter out-of-control ARLs.

8-10*.

$n = 5;\ \mu_0 = 1.50;\ \sigma = 0.14;\quad \sigma_{\bar{x}} = \sigma/\sqrt{n} = 0.14/\sqrt{5} = 0.0626$

$\delta = 1;\ k = \delta/2 = 0.5;\ h = 4;\quad K = k\sigma_{\bar{x}} = 0.0313;\ H = h\sigma_{\bar{x}} = 0.2504$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



Test Results for CUSUM Chart of Exm5-1x1, ..., Exm5-1x5

```
TEST. One point beyond control limits.
Test Failed at points:  40, 41, 42, 43, 44, 45
```

The CUSUM chart signals out of control at sample 40, and remains above the upper limit. The $\bar{x}$-R chart shown in Figure 5-4 signals out of control at sample 43. This CUSUM detects the shift in process mean earlier, at sample 40 versus sample 43.

8-11.

$$V_i = \left(\sqrt{|y_i|} - 0.822\right) \Big/ 0.349$$

Excel file: workbook Chap08.xls : worksheet Ex8-11

mu0 =	1050											
sigma =	25											
delta =	1 sigma											
k =	0.5											
h =	5											
				one-sided upper cusum					**one-sided lower cusum**			
Obs, i	**xi**	**yi**	**vi**	**Si+**	**N+**	**OOC?**	**When?**		**Si-**	**N-**	**OOC?**	**When?**
No FIR				0					0			
1	1045	-0.2	-1.07	0	0				0.57	1		
2	1055	0.2	-1.07	0	0				1.15	2		
3	1037	-0.52	-0.29	0	0				0.94	3		
4	1064	0.56	-0.21	0	0				0.65	4		
5	1095	1.8	1.49	0.989	1				0	0		
6	1008	-1.68	1.36	1.848	2				0	0		
7	1050	0	-2.36	0	0				1.86	1		
8	1087	1.48	1.13	0.631	1				0.22	2		
9	1125	3	2.61	2.738	2				0	0		
10	1146	3.84	3.26	5.498	3	OOC	7		0	0		
11	1139	3.56	3.05	8.049	4	OOC	7		0	0		
12	1169	4.76	3.90	11.44	5	OOC	7		0	0		
13	1151	4.04	3.40	14.35	6	OOC	7		0	0		
14	1128	3.12	2.71	16.55	7	OOC	7		0	0		
15	1238	7.52	5.50	21.56	8	OOC	7		0	0		
16	1125	3	2.61	23.66	9	OOC	7		0	0		
17	1163	4.52	3.74	26.9	10	OOC	7		0	0		
18	1188	5.52	4.38	30.78	11	OOC	7		0	0		
19	1146	3.84	3.26	33.54	12	OOC	7		0	0		
20	1167	4.68	3.84	36.88	13	OOC	7		0	0		

The process is out of control after observation 10 – 3 = 7. Process variability is increasing.

8-12.

$$V_i = \left(\sqrt{|y_i|} - 0.822\right) \Big/ 0.349$$

Excel file : workbook Chap08.xls : worksheet Ex8-12

mu0 =	175											
sigma =	5.6294	(from Exercise 8-8)										
delta =	1 sigma											
k =	0.5											
h =	5											
				one-sided upper cusum					**one-sided lower cusum**			
i	**xi**	**yi**	**vi**	**Si+**	**N+**	**OOC?**	**When?**		**Si-**	**N-**	**OOC?**	**When?**
No FIR				0					0			
1	160	-2.6646	2.32	1.822	1				0	0		
2	158	-3.0199	2.62	**3.946**	**2**				0	0		
3	150	-4.4410	3.68	7.129	3	OOC	0		0	0		
4	151	-4.2633	3.56	10.19	4	OOC	0		0	0		
5	153	-3.9081	3.31	13	5	OOC	0		0	0		
6	154	-3.7304	3.18	15.68	6	OOC	0		0	0		
7	158	-3.0199	2.62	17.8	7	OOC	0		0	0		
8	162	-2.3093	2.00	19.3	8	OOC	0		0	0		
9	186	1.9540	1.65	20.45	9	OOC	0		0	0		
10	195	3.5528	3.05	23	10	OOC	0		0	0		
11	179	0.7106	0.06	22.56	11	OOC	0		0	0		
12	184	1.5987	1.27	23.32	12	OOC	0		0	1		
13	175	0.0000	-2.36	20.47	13	OOC	0		1.86	0		
14	192	3.0199	2.62	22.59	14	OOC	0		0	0		
15	186	1.9540	1.65	23.74	15	OOC	0		0	0		
16	197	3.9081	3.31	26.55	16	OOC	0		0	0		
17	190	2.6646	2.32	28.37	17	OOC	0		0	0		
18	189	2.4869	2.16	30.04	18	OOC	0		0	0		
19	185	1.7764	1.46	31	19	OOC	0		0	0		
20	182	1.2435	0.84	31.34	20	OOC	0		0	0		
…												

The process was last in control at period 2 – 2 = 0. Process variability has been increasing since start-up.

8-13.
Standardized, two-sided cusum with $k = 0.2$ and $h = 8$

In control ARL performance:

$\delta^* = 0$

$\Delta^+ = \delta^* - k = 0 - 0.2 = -0.2$

$\Delta^- = -\delta^* - k = -0 - 0.2 = -0.2$

$b = h + 1.166 = 8 + 1.166 = 9.166$

$$\text{ARL}_0^+ = \text{ARL}_0^- \cong \frac{\exp[-2(-0.2)(9.166)] + 2(-0.2)(9.166) - 1}{2(-0.2)^2} = 430.556$$

$$\frac{1}{\text{ARL}_0} = \frac{1}{\text{ARL}_0^+} + \frac{1}{\text{ARL}_0^-} = \frac{2}{430.556} = 0.005$$

$\text{ARL}_0 = 1/0.005 = 215.23$

Out of control ARL Performance:

$\delta^* = 0.5$

$\Delta^+ = \delta^* - k = 0.5 - 0.2 = 0.3$

$\Delta^- = -\delta^* - k = -0.5 - 0.2 = -0.7$

$b = h + 1.166 = 8 + 1.166 = 9.166$

$$\text{ARL}_1^+ = \frac{\exp[-2(0.3)(9.166)] + 2(0.3)(9.166) - 1}{2(0.3)^2} = 25.023$$

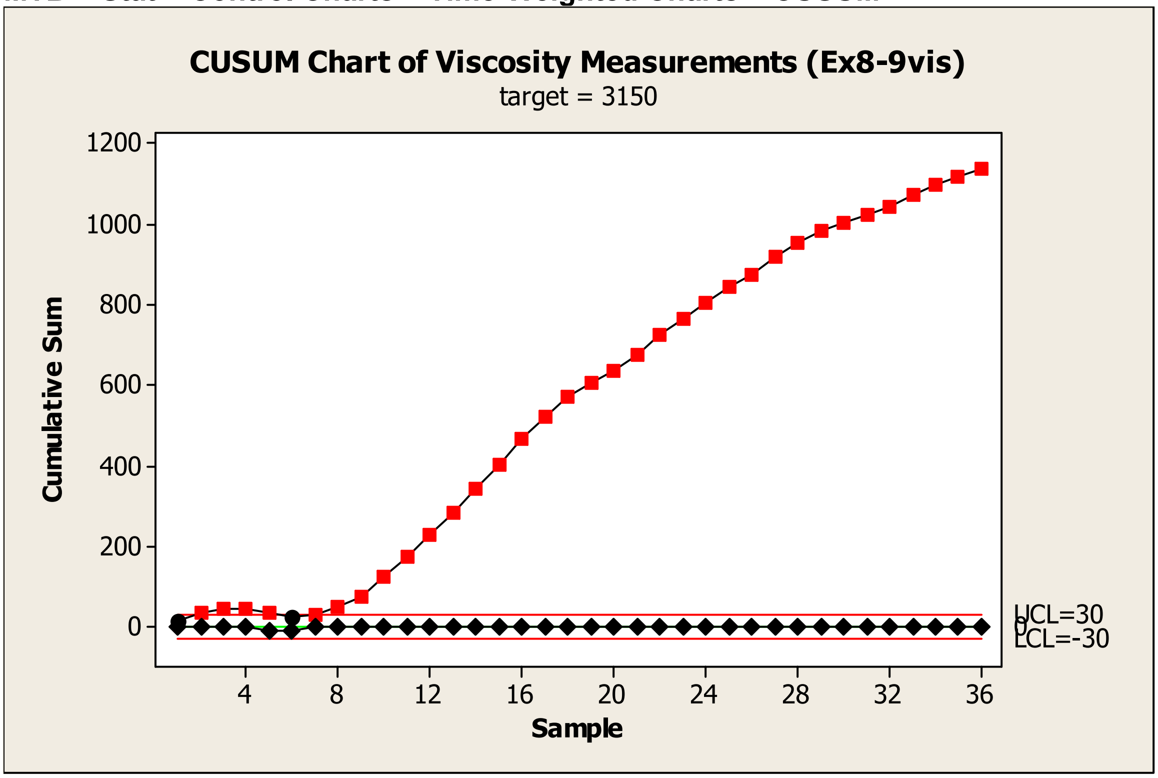
$$\text{ARL}_1^- = \frac{\exp[-2(-0.7)(9.166)] + 2(-0.7)(9.166) - 1}{2(-0.7)^2} = 381,767$$

$$\frac{1}{\text{ARL}_1} = \frac{1}{\text{ARL}_1^+} + \frac{1}{\text{ARL}_1^-} = \frac{1}{25.023} + \frac{1}{381,767} = 0.040$$

$\text{ARL}_1 = 1/0.040 = 25.02$

8-14.
$\mu_0 = 3150$, $s = 5.95238$, $k = 0.5$, $h = 5$
$K = ks = 0.5\ (5.95238) = 2.976$, $H = hs = 5(5.95238) = 29.762$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



MINITAB displays both the upper and lower sides of a CUSUM chart on the same graph; there is no option to display a single-sided chart. The upper CUSUM is used to detect upward shifts in the level of the process.

The process signals out of control on the upper side at sample 2. The assignable cause occurred at start-up (2 – 2).

8-15☺.

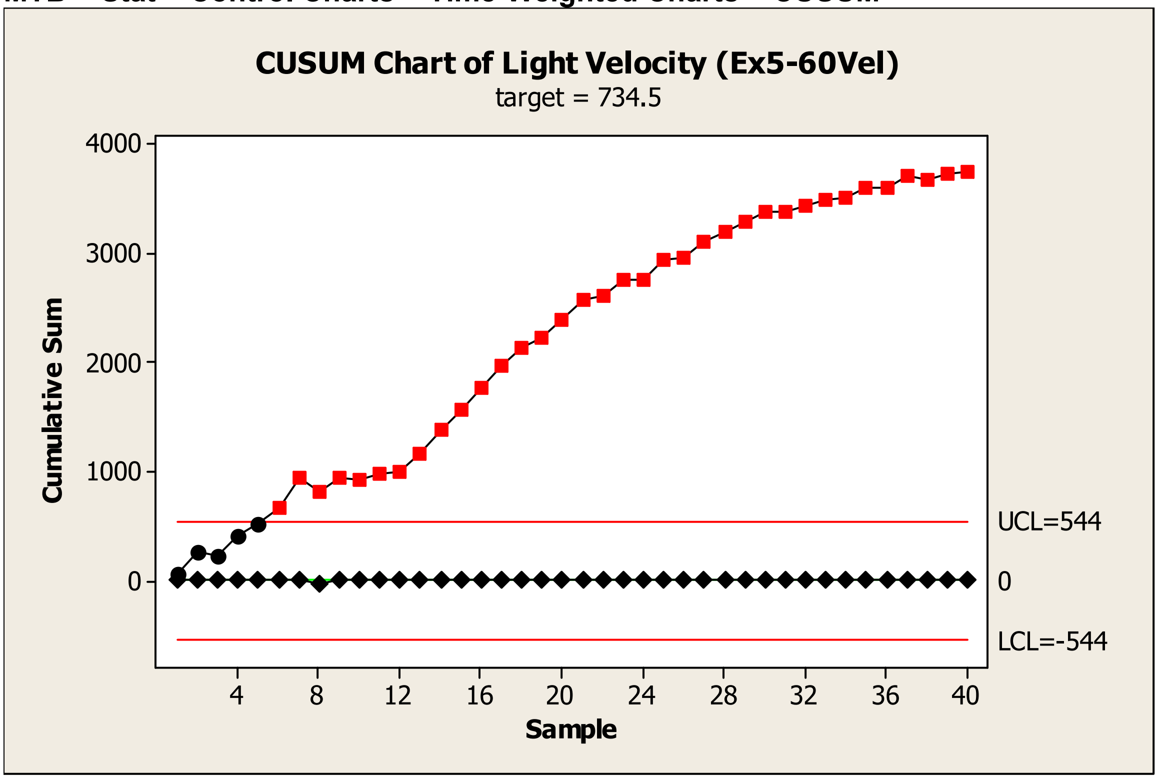
$\hat{\sigma} = \overline{\text{MR2}} / d_2 = 122.6 / 1.128 = 108.7$ (from a Moving Range chart with CL = 122.6)

$\mu_0 = 734.5;\ k = 0.5;\ h = 5$

$K = k\hat{\sigma} = 0.5(108.7) = 54.35$

$H = h\hat{\sigma} = 5(108.7) = 543.5$

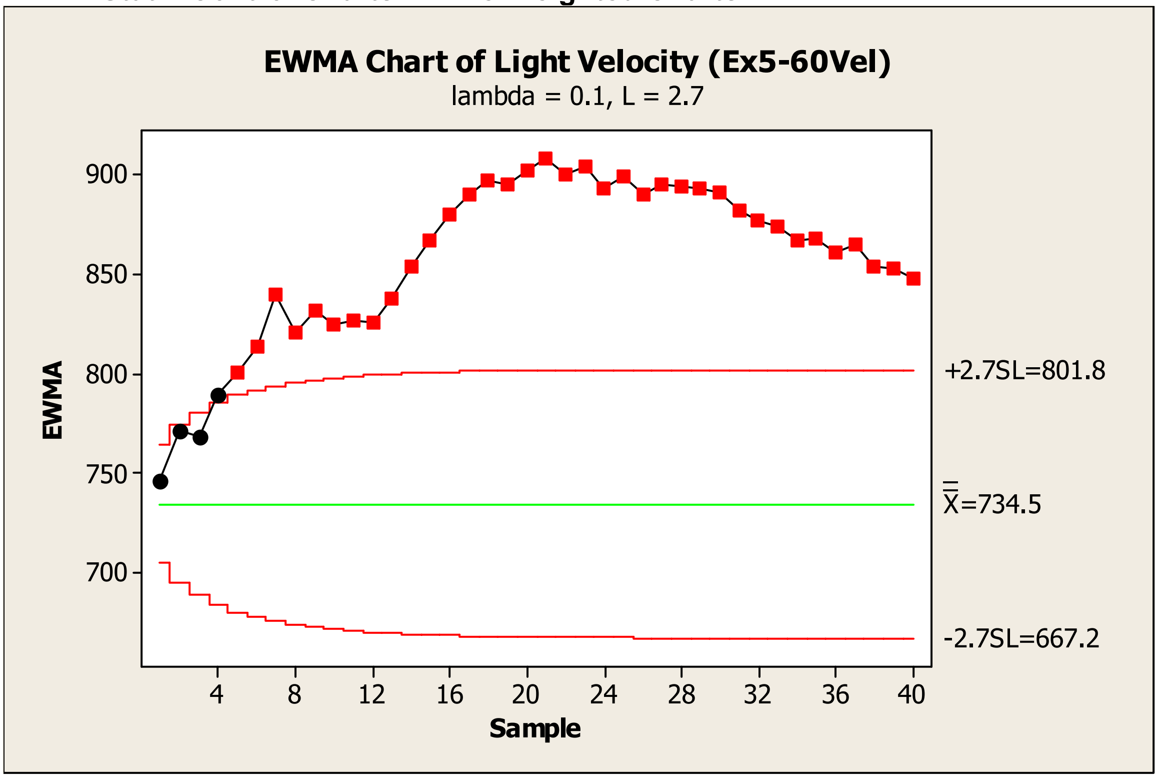
MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



The Individuals I-MR chart, with a centerline at $\bar{x} = 909$, displayed a distinct downward trend in measurements, starting at about sample 18. The CUSUM chart reflects a consistent run above the target value 734.5, from virtually the first sample. There is a distinct signal on both charts, of either a trend/drift or a shit in measurements. The out-of-control signals should lead us to investigate and determine the assignable cause.

8-16☺.
$\lambda = 0.1$; $L = 2.7$; CL $= \mu_0 = 734.5$; $\sigma = 108.7$

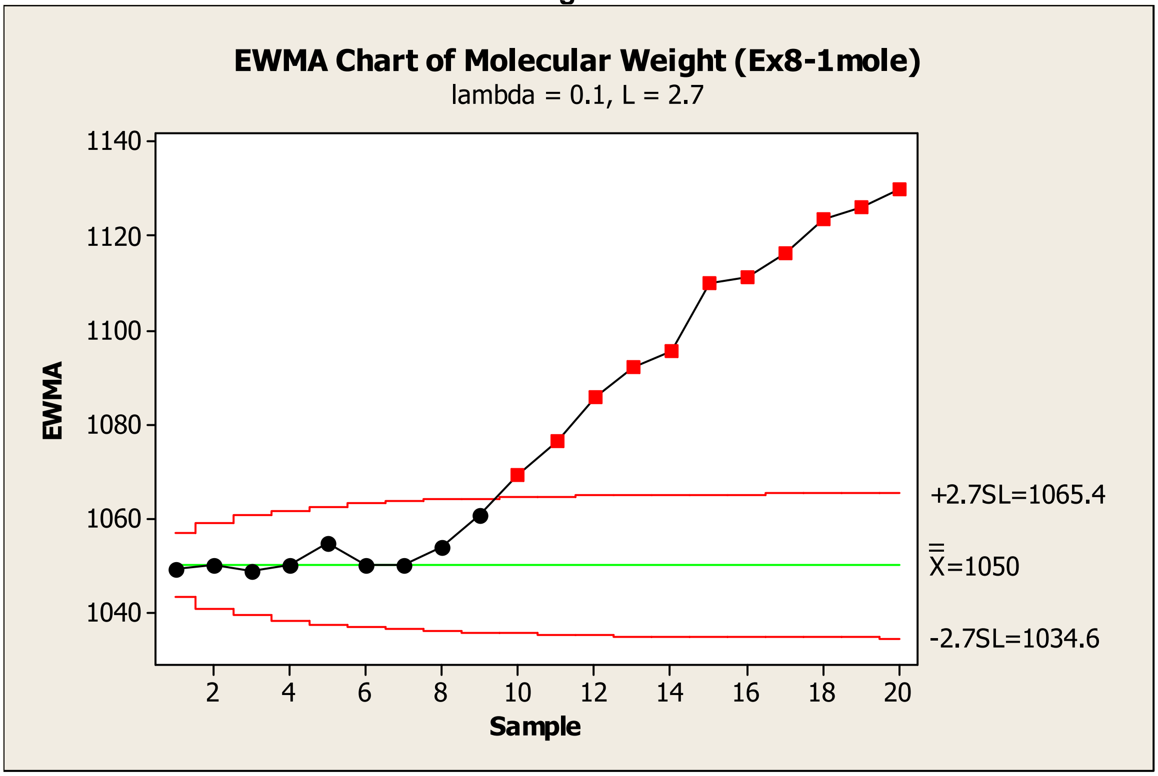
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



The EWMA chart reflects the consistent trend above the target value, 734.5, and also indicates the slight downward trend starting at about sample 22.

8-17 (8-15).
λ = 0.1, L = 2.7, σ = 25, CL = μ_0 = 1050, UCL = 1065.49, LCL = 1034.51

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



Process exceeds upper control limit at sample 10; the same as the CUSUM chart.

8-18 (8-16).
(a)
λ = 0.1, L = 3

$$\text{limits} = \mu_0 \pm L\sigma\sqrt{\lambda/(2-\lambda)} = 10 \pm 3(1)\sqrt{0.1/(2-0.1)} = [9.31, 10.69]$$

(b)
λ = 0.2, L = 3

$$\text{limits} = \mu_0 \pm L\sigma\sqrt{\lambda/(2-\lambda)} = 10 \pm 3(1)\sqrt{0.2/(2-0.2)} = [9, 11]$$

(c)
λ = 0.4, L = 3

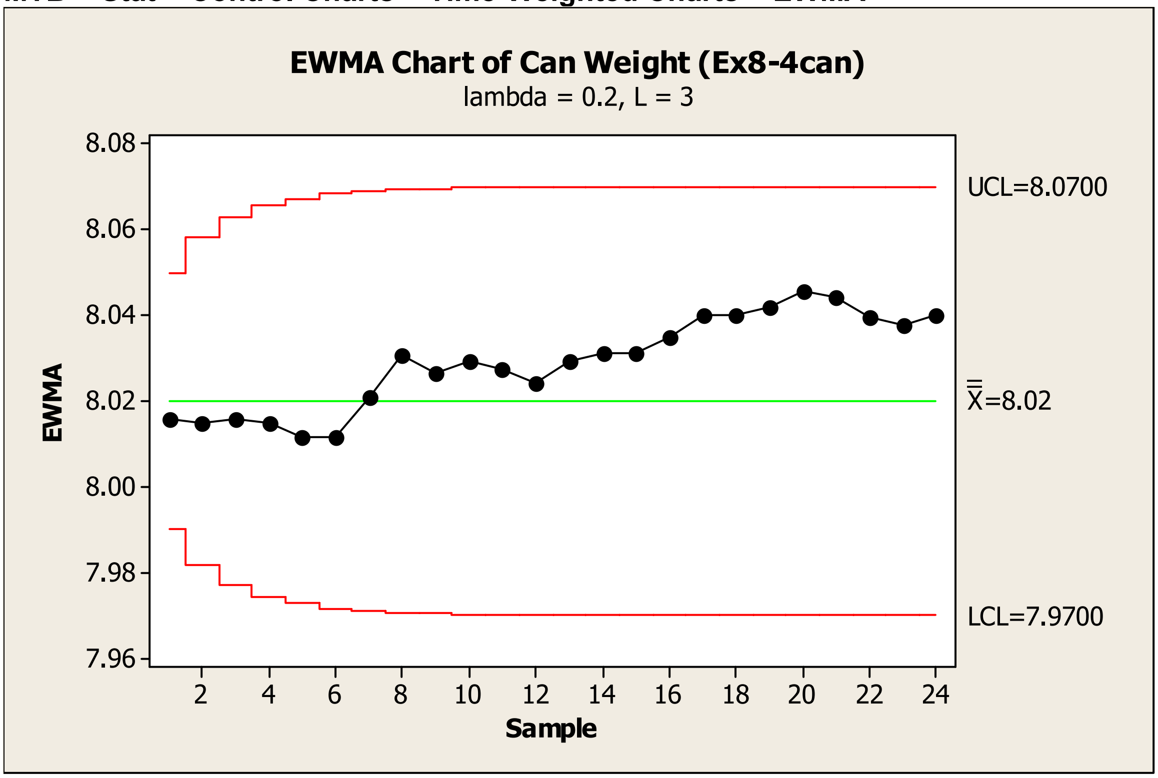
$$\text{limits} = \mu_0 \pm L\sigma\sqrt{\lambda/(2-\lambda)} = 10 \pm 3(1)\sqrt{0.4/(2-0.4)} = [8.5, 11.5]$$

As λ increases, the width of the control limits also increases.

8-19 (8-17).

$\lambda = 0.2$, $L = 3$. Assume $\sigma = 0.05$. CL $= \mu_0 = 8.02$, UCL $= 8.07$, LCL $= 7.97$

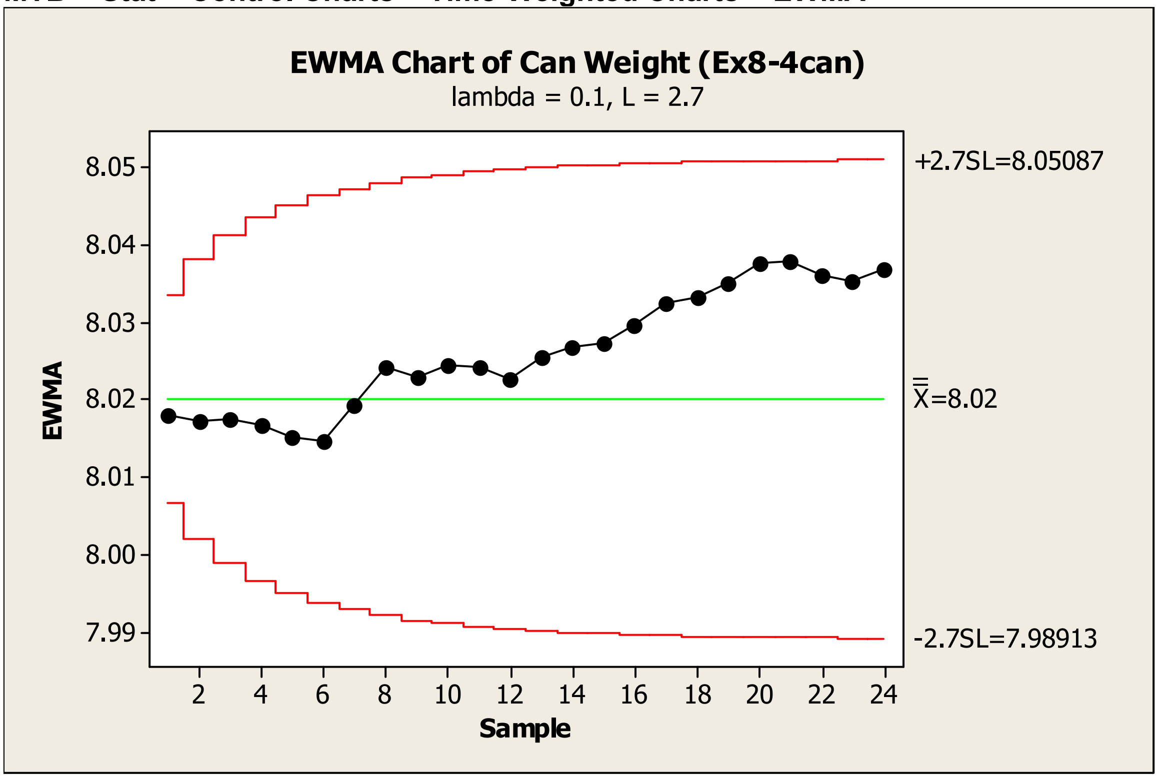
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



The process is in control.

8-20 (8-18).
$\lambda = 0.1$, $L = 2.7$. Assume $\sigma = 0.05$. CL = $\mu_0 = 8.02$, UCL = 8.05, LCL = 7.99

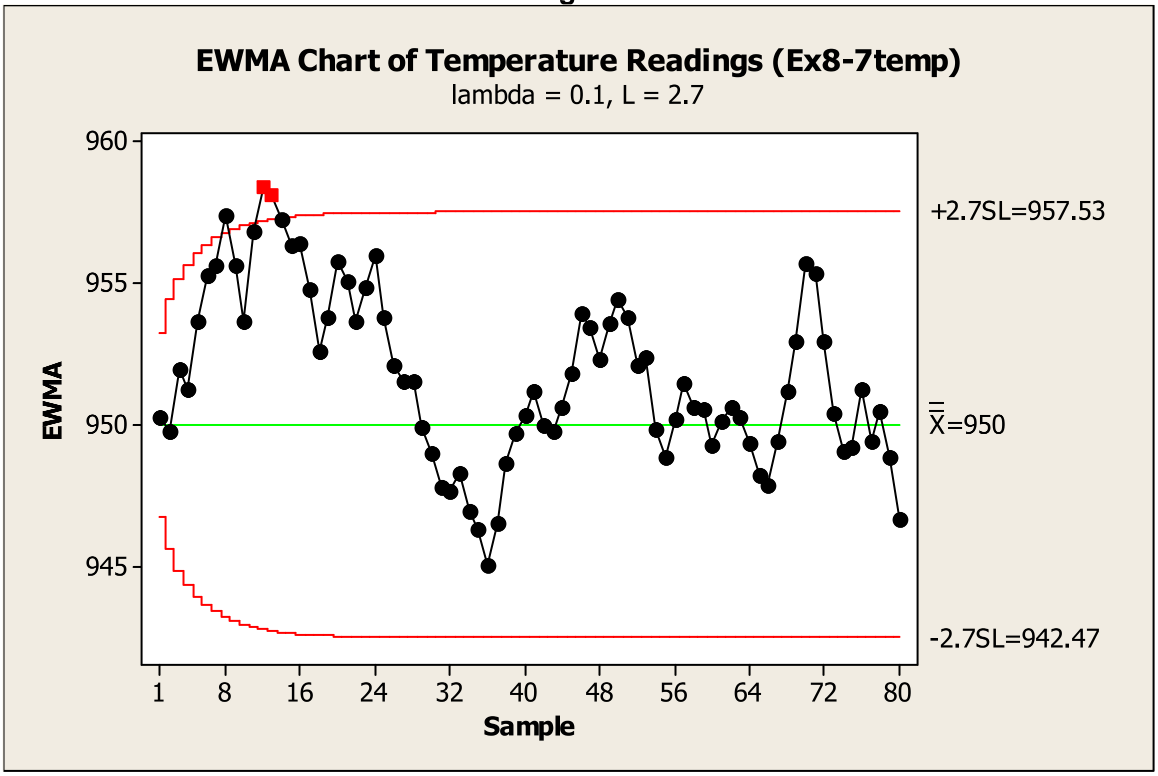
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



The process is in control. There is not much difference between the control charts.

8-21 (8-19).
$\lambda = 0.1$, $L = 2.7$, $\hat{\sigma} = 12.16$, CL = μ_0 = 950, UCL = 957.53, LCL = 942.47.

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA

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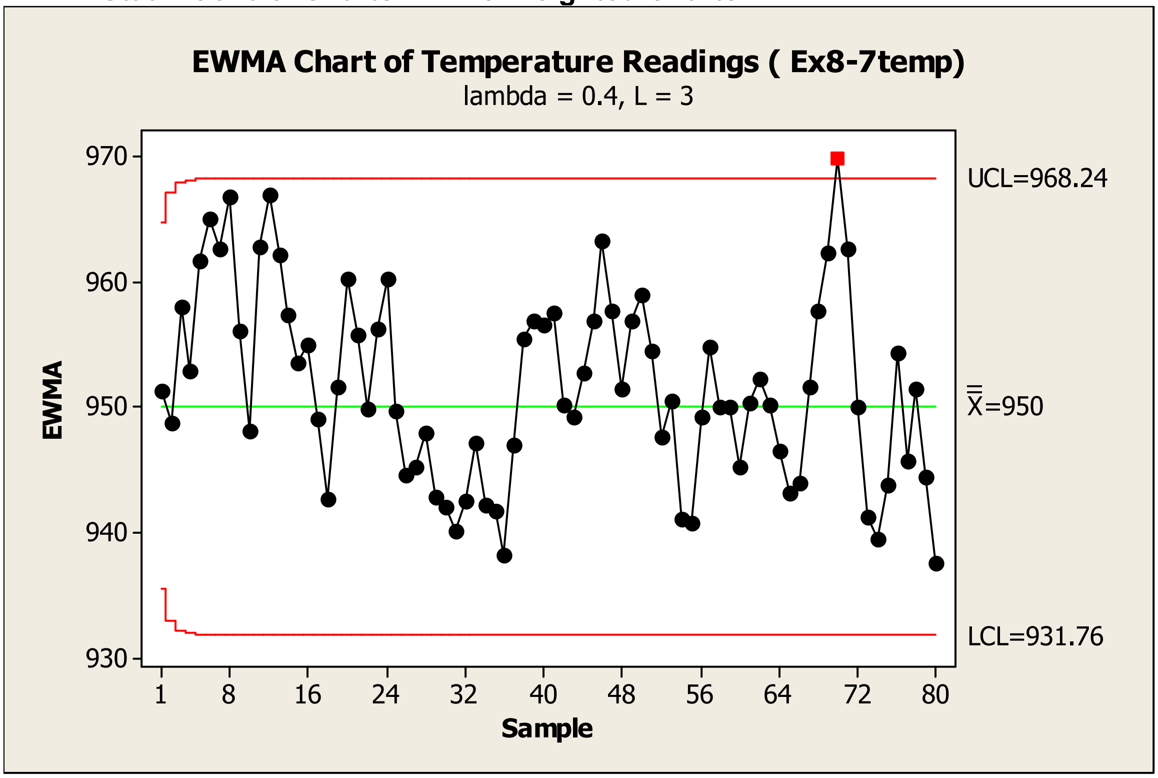
Test Results for EWMA Chart of Ex8-7temp
TEST. One point beyond control limits.
Test Failed at points:  12, 13
```

Process is out of control at samples 8 (beyond upper limit, but not flagged on chart), 12 and 13.

8-22 (8-20).
$\lambda = 0.4$, $L = 3$, $\hat{\sigma} = 12.16$, CL = μ_0 = 950, UCL = 968.24, LCL = 931.76.

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA

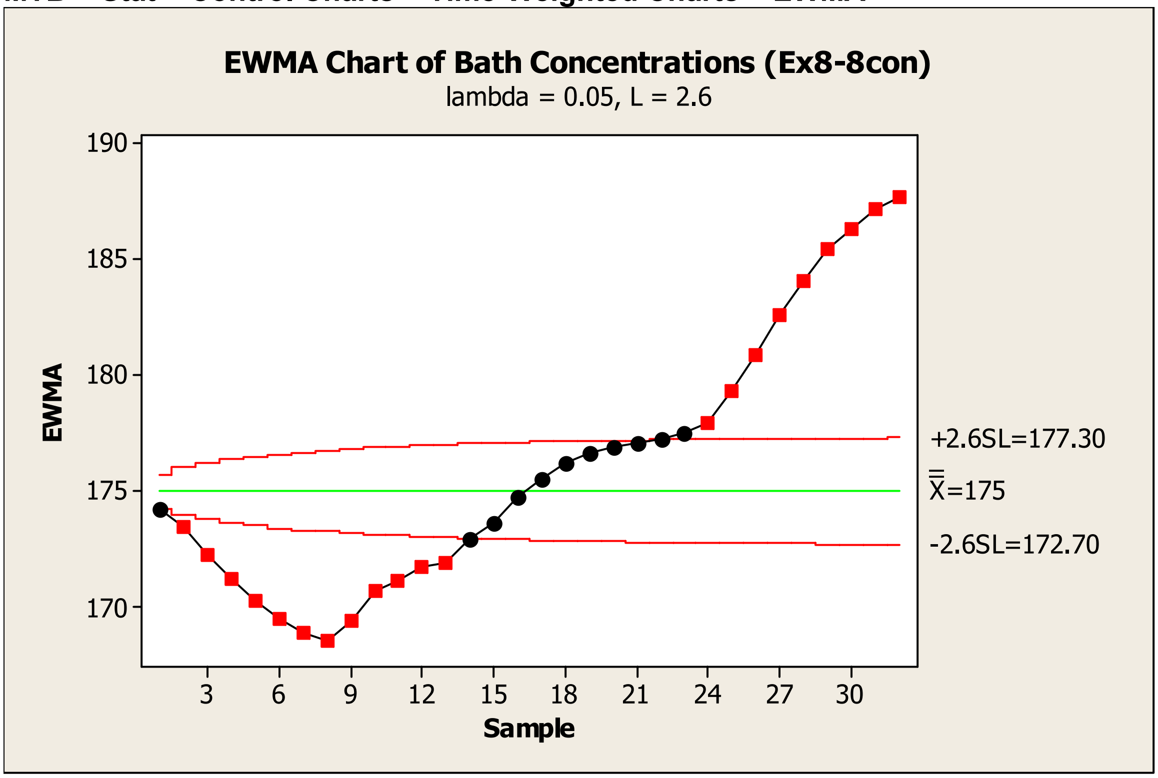


Test Results for EWMA Chart of Ex8-7temp
```
TEST. One point beyond control limits.
Test Failed at points:  70
```

With the larger λ, the process is out of control at observation 70, as compared to the chart in the Exercise 21 (with the smaller λ) which signaled out of control at earlier samples.

8-23 (8-21).
$\lambda = 0.05$, $L = 2.6$, $\hat{\sigma} = 5.634$, CL = μ_0 = 175, UCL = 177.30, LCL = 172.70.

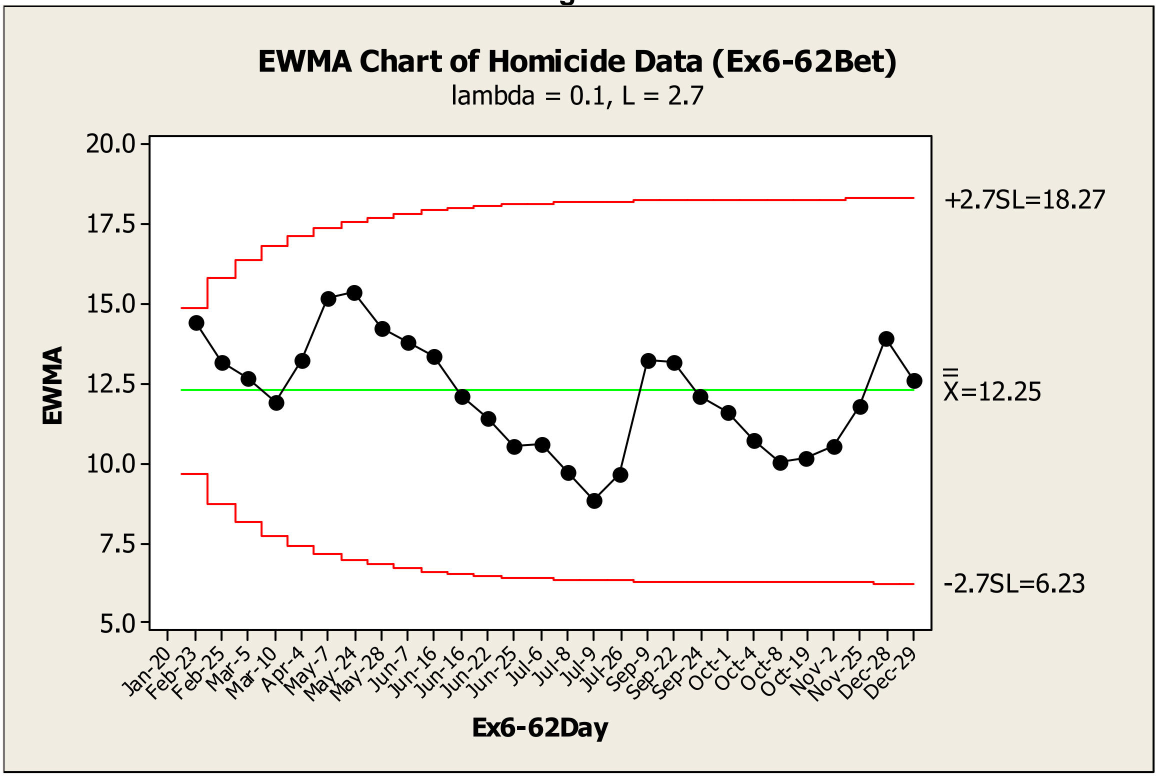
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



Process is out of control. The process average of $\hat{\mu} = 183.594$ is too far from the process target of μ_0 = 175 for the process variability. The data is grouped into three increasing levels.

8-24☺.
$\lambda = 0.1$, $L = 2.7$

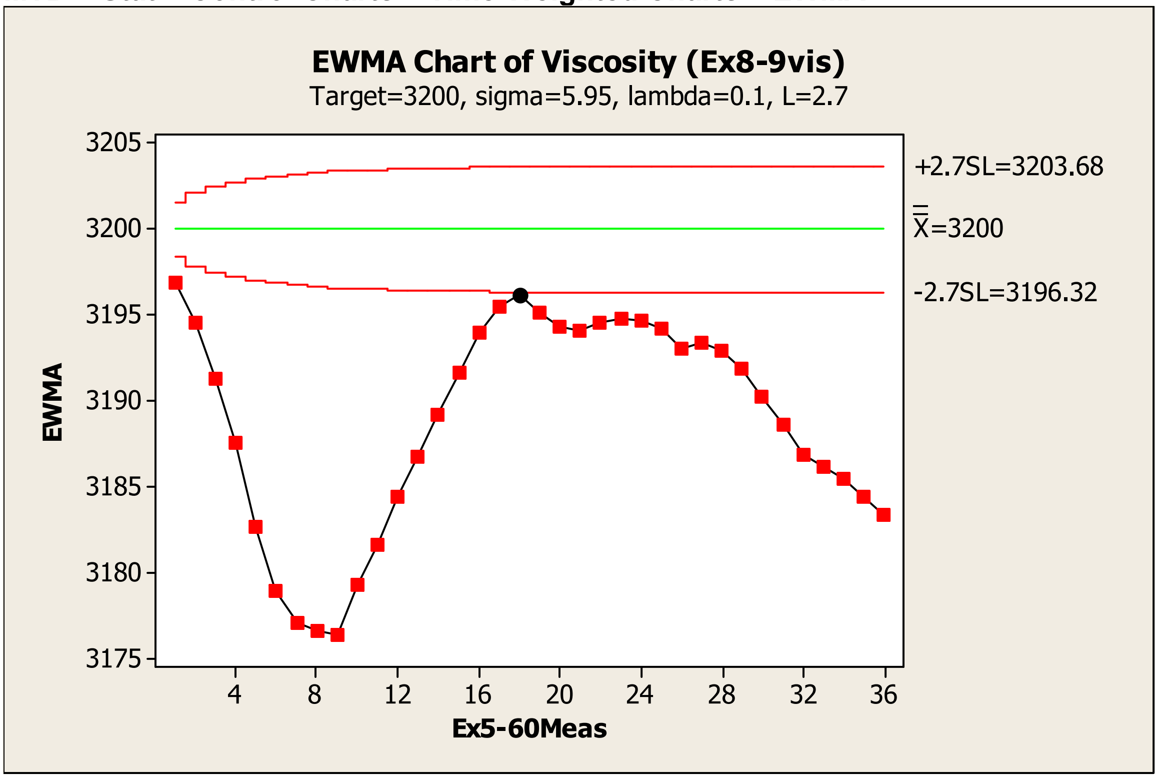
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



In Exercise 6-62, Individuals control charts of 0.2777th- and 0.25th-root transformed data showed no out-of-control signals. The EWMA chart also does not signal out of control. As mentioned in the text (Section 8.4-3), a properly designed EWMA chart is very robust to the assumption of normally distributed data.

8-25 (8-22).
$\mu_0 = 3200$, $\hat{\sigma} = 5.95$ (from Exercise 8-9), $\lambda = 0.1$, $L = 2.7$

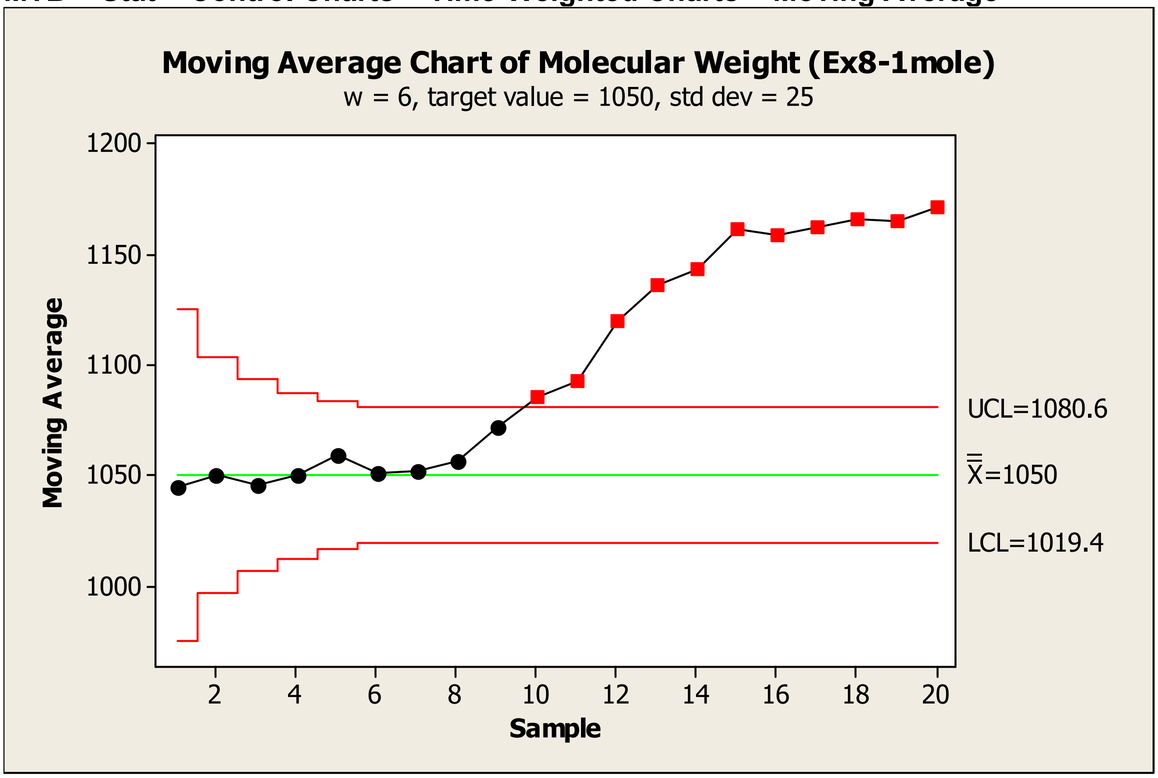
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



The process is out of control from the first sample.

8-26 (8-23).
$w = 6$, $\mu_0 = 1050$, $\sigma = 25$, CL = 1050, UCL = 1080.6, LCL = 1019.4

MTB > Stat > Control Charts > Time-Weighted Charts > Moving Average

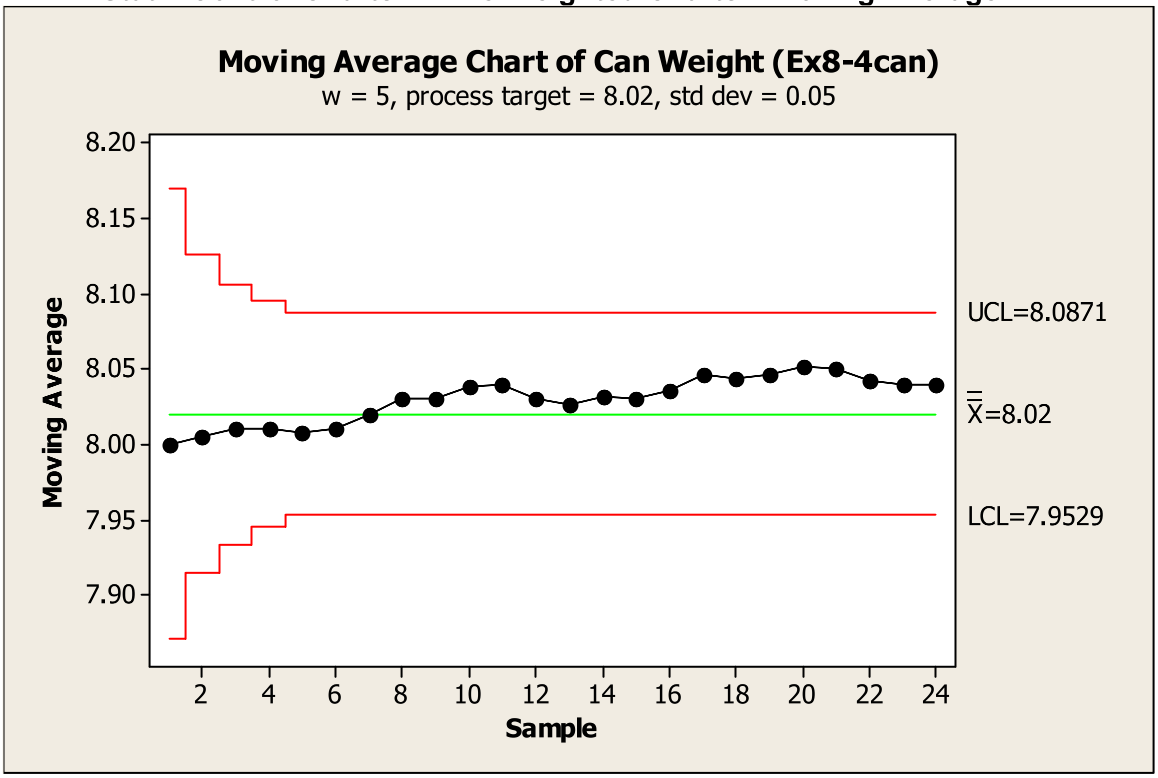



```
Test Results for Moving Average Chart of Ex8-1mole
TEST. One point beyond control limits.
Test Failed at points:  10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20
```

Process is out of control at observation 10, the same result as for Exercise 8-1.

8-27 (24).
$w = 5$, $\mu_0 = 8.02$, $\sigma = 0.05$, CL = 8.02, UCL = 8.087, LCL = 7.953

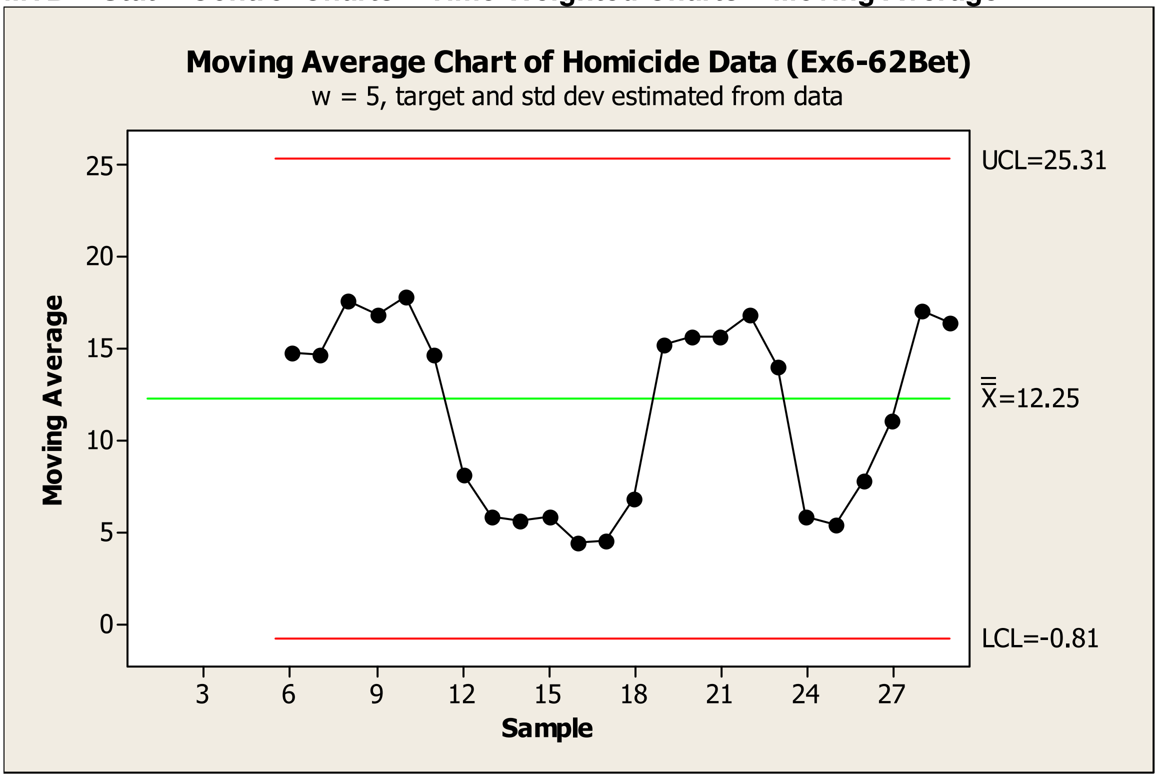
MTB > Stat > Control Charts > Time-Weighted Charts > Moving Average



The process is in control, the same result as for Exercise 8-4.

8-28☺.
$w = 5$

MTB > Stat > Control Charts > Time-Weighted Charts > Moving Average



Because these plot points are an average of five observations, the nonnormality of the individual observations should be of less concern. The approximate normality of the averages is a consequence of the Central Limit Theorem.

8-29 (8-25).

Assume that t is so large that the starting value $Z_0 = \overline{\overline{x}}$ has no effect.

$$E(Z_t) = E[\lambda \overline{x}_t + (1-\lambda)(Z_{t-1})] = E\left[\lambda \sum_{j=0}^{\infty} (1-\lambda)^j \overline{x}_{t-j}\right] = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E(\overline{x}_{t-j})$$

Since $E(\overline{x}_{t-j}) = \mu$ and $\lambda \sum_{j=0}^{\infty} (1\text{-}\lambda)^j = 1$, $E(Z_t) = \mu$

8-30 (8-26).

$$\begin{aligned}\operatorname{var}(Z_t) &= \operatorname{var}\left[\lambda \sum_{j=0}^{\infty} (1-\lambda)^j \overline{x}_{t-j}\right] \\ &= \left[\lambda^2 \sum_{j=0}^{\infty} (1-\lambda)^{2j}\right]\left[\operatorname{var}(\overline{x}_{t-j})\right] \\ &= \frac{\lambda}{2-\lambda}\left(\frac{\sigma^2}{n}\right)\end{aligned}$$

8-31 (8-27).

For the EWMA chart, the steady-state control limits are $\overline{\overline{x}} \pm 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)n}}$.

Substituting λ = 2/(w + 1), $\overline{\overline{x}} \pm 3\sigma\sqrt{\frac{\left(\frac{2}{w+1}\right)}{\left(2-\frac{2}{w+1}\right)n}} = \overline{\overline{x}} \pm 3\sigma\sqrt{\frac{1}{wn}} = \overline{\overline{x}} \pm \frac{3\sigma}{\sqrt{wn}}$,

which are the same as the limits for the MA chart.

8-32 (8-28).

The average age of the data in a w-period moving average is $\frac{1}{w}\sum_{j=0}^{w-1} j = \frac{w-1}{2}$. In the EWMA, the weight given to a sample mean j periods ago is $\lambda(1-\lambda)^j$, so the average age is $\lambda \sum_{j=0}^{\infty} (1-\lambda)^j j = \frac{1-\lambda}{\lambda}$. By equating average ages:

$$\begin{aligned}\frac{1-\lambda}{\lambda} &= \frac{w-1}{2} \\ \lambda &= \frac{2}{w+1}\end{aligned}$$

8-33 (8-29).

For $n > 1$, Control limits $= \mu_0 \pm \frac{3}{\sqrt{w}}\left(\frac{\sigma}{\sqrt{n}}\right) = \mu_0 \pm \frac{3\sigma}{\sqrt{wn}}$

8-34 (8-30).

$\bar{x}$ chart: CL = 10, UCL = 16, LCL = 4

$$\begin{aligned}\text{UCL} &= \text{CL} + k\sigma_{\bar{x}} \\ 16 &= 10 - k\sigma_{\bar{x}} \\ k\sigma_{\bar{x}} &= 6\end{aligned}$$

EWMA chart:

$$\begin{aligned}\text{UCL} &= \text{CL} + l\sigma\sqrt{\lambda/[(2-\lambda)n]} \\ &= \text{CL} + l\,\sigma/\sqrt{n}\,\sqrt{0.1/(2-0.1)} = 10 + 6(0.2294) = 11.3765\end{aligned}$$

$$\text{LCL} = 10 - 6(0.2294) = 8.6236$$

8-35 (8-31).

$\lambda = 0.4$

For EWMA, steady-state limits are $\pm L\sigma\sqrt{\lambda/(2-\lambda)}$

For Shewhart, steady-state limits are $\pm k\sigma$

$$\begin{aligned}k\sigma &= L\sigma\sqrt{\lambda/(2-\lambda)} \\ k &= L\sqrt{0.4/(2-0.4)} \\ k &= 0.5L\end{aligned}$$

8-36 (8-32).
The two alternatives to plot a CUSUM chart with transformed data are:

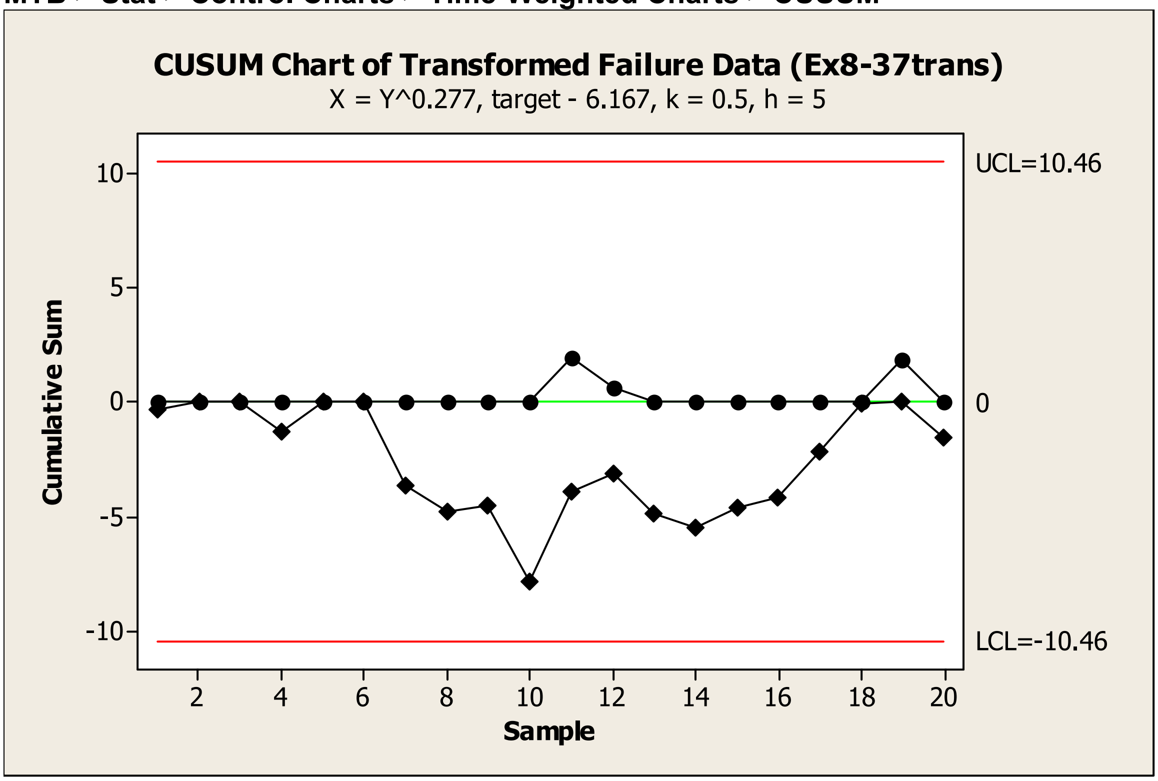
1. Transform the data, target (if given), and standard deviation (if given), then use these results in the CUSUM Chart dialog box, or
2. Transform the target (if given) and standard deviation (if given), then use the Box-Cox tab under CUSUM Options to transform the data.

The solution below uses alternative #2.

From Example 6-6, transform time-between-failures (Y) data to approximately normal distribution with $X = Y^{0.2777}$.

$T_Y = 700$, $T_X = 700^{\,0.2777} = 6.167$, $k = 0.5$, $h = 5$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



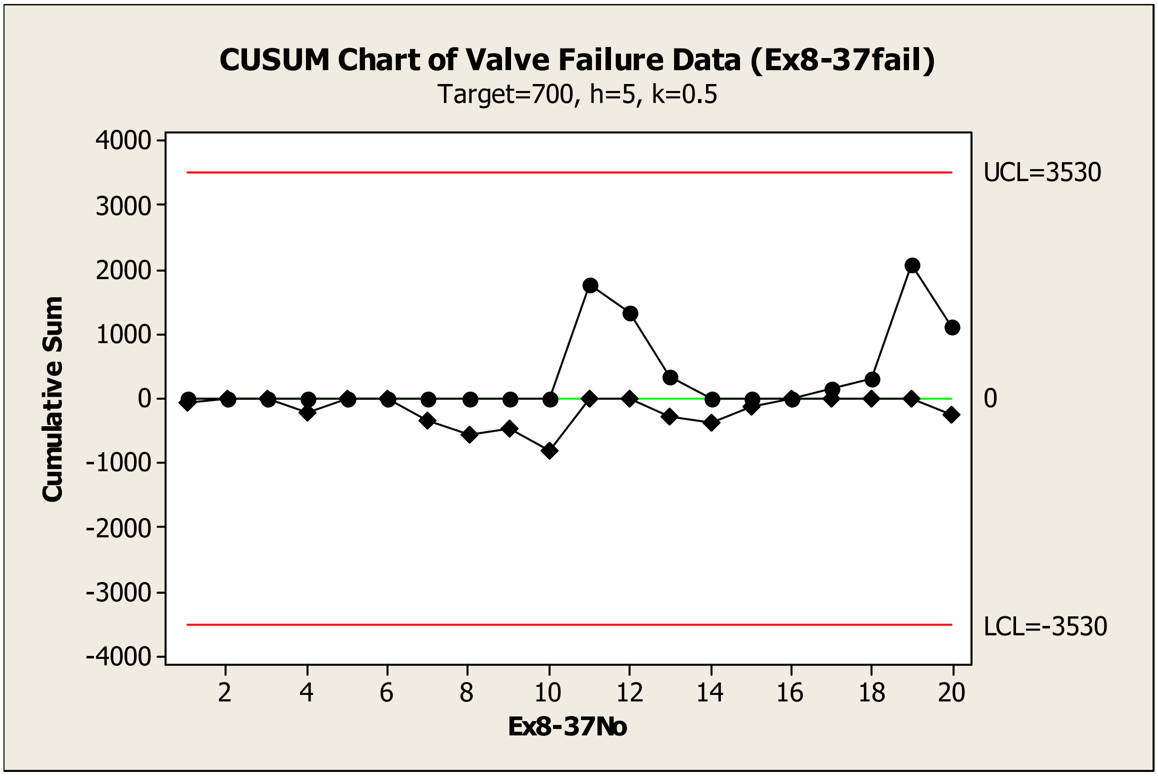
A one-sided <u>lower</u> CUSUM is needed to detect an increase in failure rate, or equivalently a decrease in the time-between-failures. Evaluate the lower CUSUM on the MINITAB chart to assess stability.

The process is in control.

8-37 (8-33).

$\mu_0 = 700$, $h = 5$, $k = 0.5$, estimate σ using the average moving range

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM,
also CUSUM options > Estimate > Average Moving Range



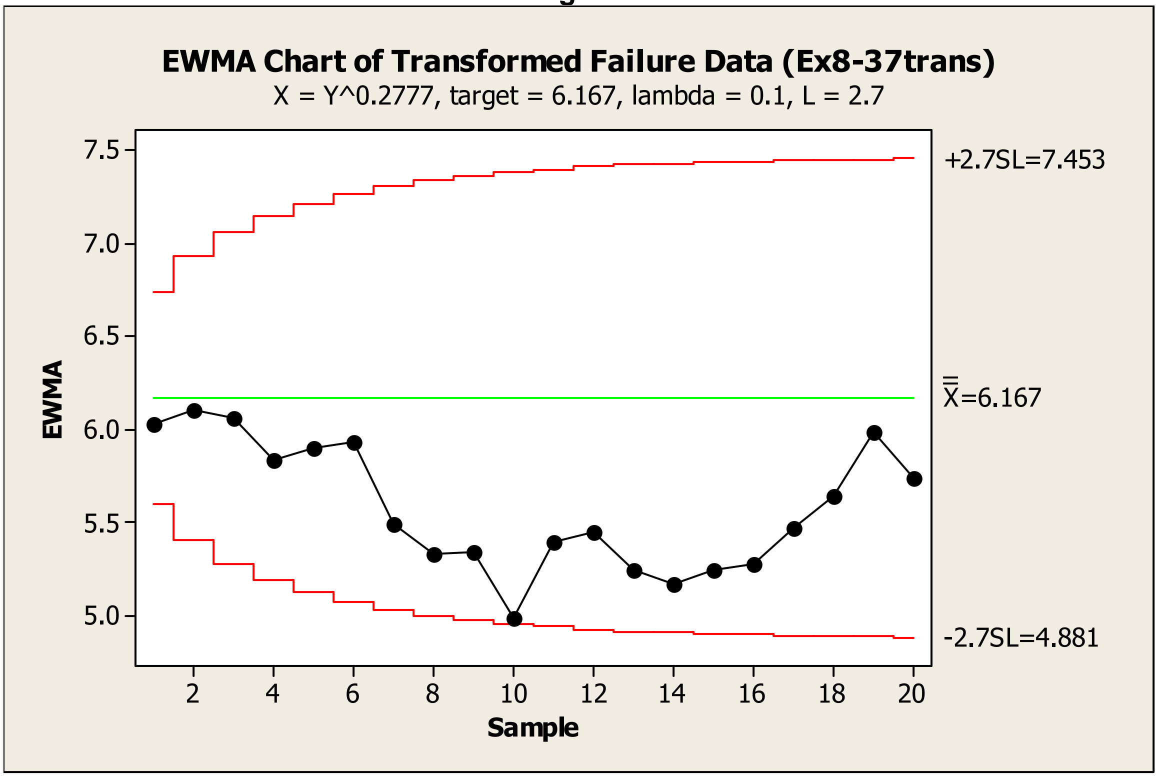
A one-sided lower CUSUM is needed to detect an increase in failure rate. Evaluate the lower CUSUM on the MINITAB chart to assess stability.

The process is in control.

Though the data are not normal, the CUSUM works fairly well for monitoring the process; this chart is very similar to the one constructed with the transformed data.

8-38 (8-34).
$\mu_0 = T_X = 700^{\,0.2777} = 6.167$, $\lambda = 0.1$, $L = 2.7$

MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



Valve failure times are in control.

8-39 (8-35).
The standard (two-sided) EWMA can be modified to form a one-sided statistic in much the same way a CUSUM can be made into a one-sided statistic. The standard (two-sided) EWMA is

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}$$

Assume that the center line is at μ_0. Then a one-sided upper EWMA is

$$z_i^+ = \max\left[\mu_0, \lambda x_i + (1-\lambda) z_{i-1}\right],$$

and the one-sided lower EWMA is

$$z_i^- = \min\left[\mu_0, \lambda x_i + (1-\lambda) z_{i-1}\right].$$