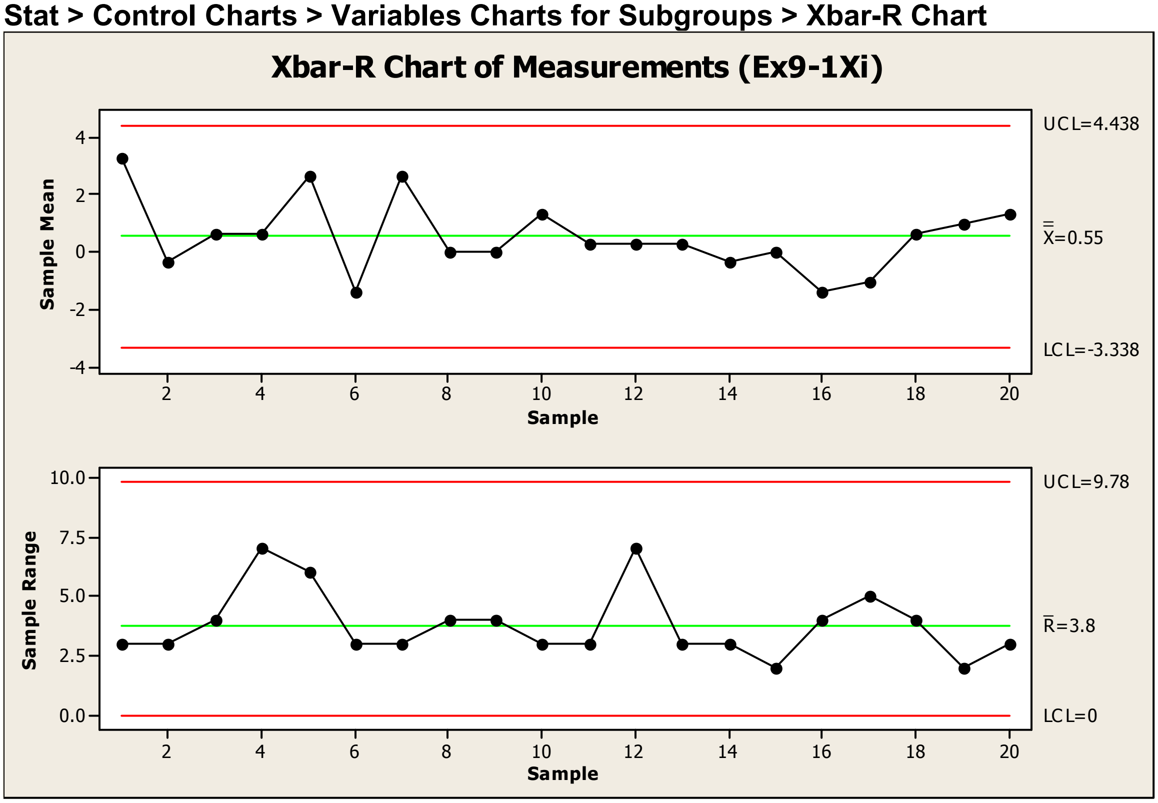
Chapter 9 Exercise Solutions

Note: Many of the exercises in this chapter were solved using Microsoft Excel 2002, not MINITAB. The solutions, with formulas, charts, etc., are in **Chap09.xls.**

9-1.
$\hat{\sigma}_A = 2.530, n_A = 15, \hat{\mu}_A = 101.40$
$\hat{\sigma}_B = 2.297, n_B = 9, \hat{\mu}_B = 60.444$
$\hat{\sigma}_C = 1.815, n_C = 18, \hat{\mu}_C = 75.333$
$\hat{\sigma}_D = 1.875, n_D = 18, \hat{\mu}_D = 50.111$
Standard deviations are approximately the same, so the DNOM chart can be used.

$\overline{R} = 3.8, \hat{\sigma} = 2.245, n = 3$
$\overline{x}$ chart: CL = 0.55, UCL = 4.44, LCL = –3.34
R chart: CL = 3.8, UCL = $D_4\overline{R}$ = 2.574 (3.8) = 9.78, LCL = 0

Stat > Control Charts > Variables Charts for Subgroups > Xbar-R Chart



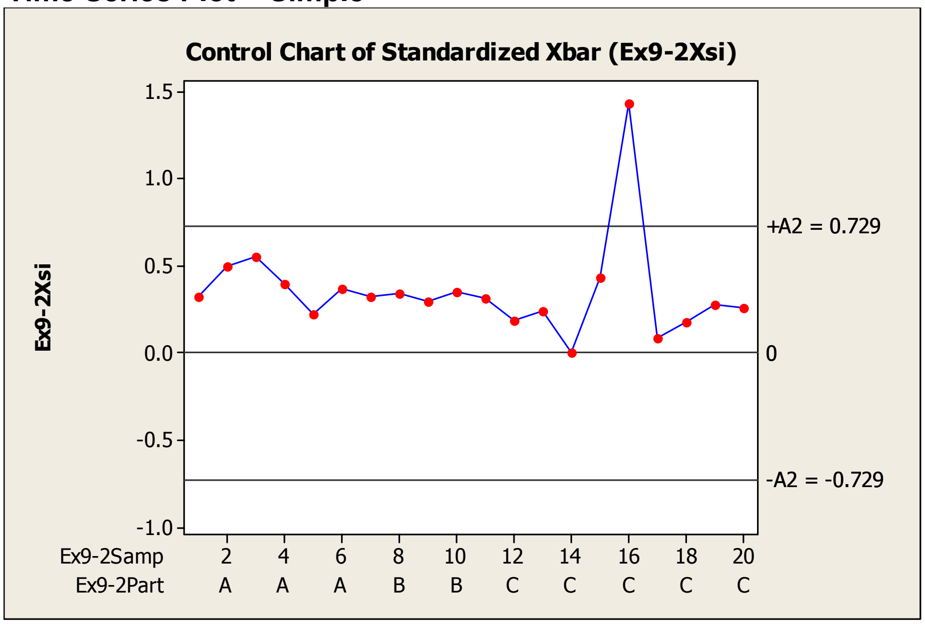
Process is in control, with no samples beyond the control limits or unusual plot patterns.

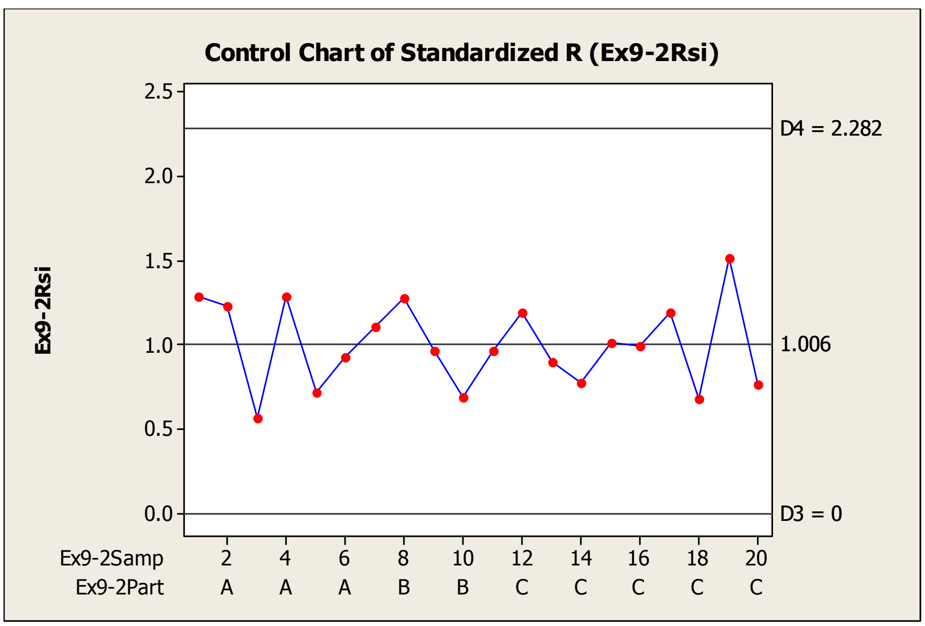
Chapter 9 Exercise Solutions

9-2.
Since the standard deviations are not the same, use a standardized $\bar{x}$ and R charts.
Calculations for standardized values are in:
Excel : workbook Chap09.xls : worksheet : Ex9-2.

$$n = 4, D_3 = 0, D_4 = 2.282, A_2 = 0.729; \quad \overline{R}_A = 19.3, \overline{R}_B = 44.8, \overline{R}_C = 278.2$$

Graph > Time Series Plot > Simple





Process is out of control at Sample 16 on the $\bar{x}$ chart.

9-3.
In a short production run situation, a standardized CUSUM could be used to detect smaller deviations from the target value. The chart would be designed so that δ, in standard deviation units, is the same for each part type. The standardized variable $(y_{i,j} - \mu_{0,j})/\sigma_j$ (where *j* represents the part type) would be used to calculate each plot statistic.

9-4.
Note: In the textbook, the 4th part on Day 246 should be "1385" not "1395".

Set up a standardized *c* chart for defect counts. The plot statistic is $Z_i = \left(c_i - \bar{c}\right)/\sqrt{\bar{c}}$, with CL = 0, UCL = +3, LCL = –3.

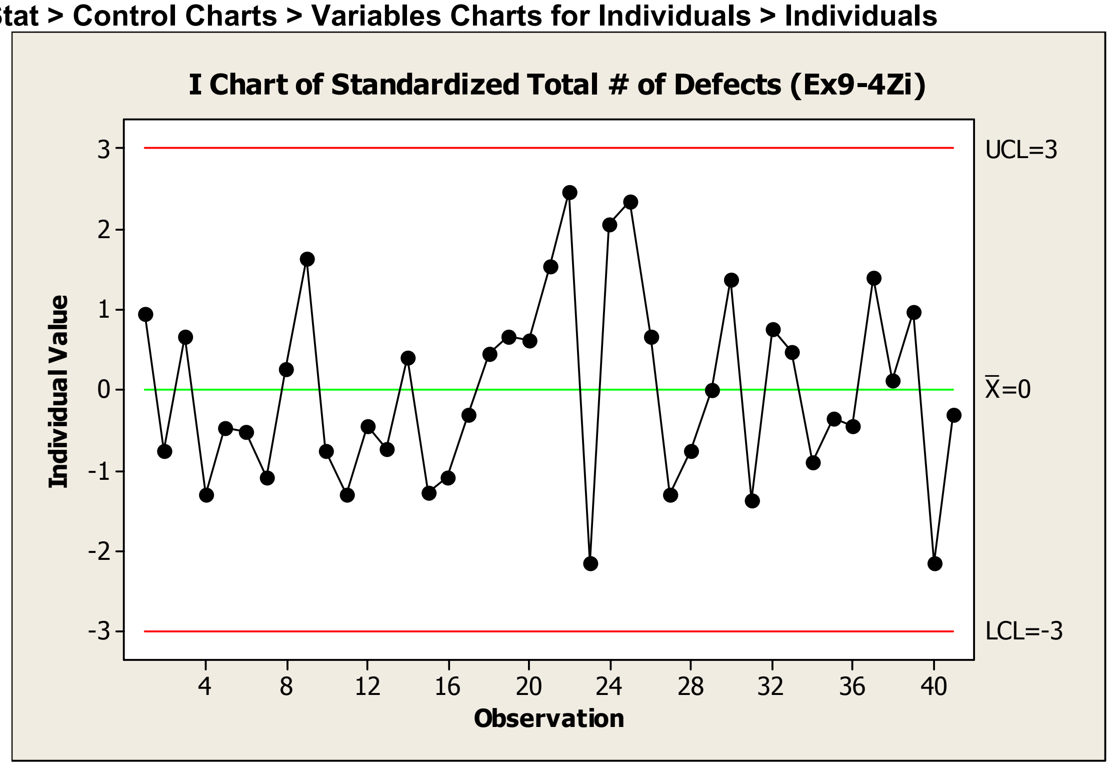
Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Rx9-4Def

Rx9-4Def	1055	13.25
	1130	64.00
	1261	12.67
	1385	26.63
	4610	4.67
	8611	50.13

$\bar{c}_{1055} = 13.25, \bar{c}_{1130} = 64.00, \bar{c}_{1261} = 12.67, \bar{c}_{1385} = 26.63, \bar{c}_{4610} = 4.67, \bar{c}_{8611} = 50.13$

Stat > Control Charts > Variables Charts for Individuals > Individuals

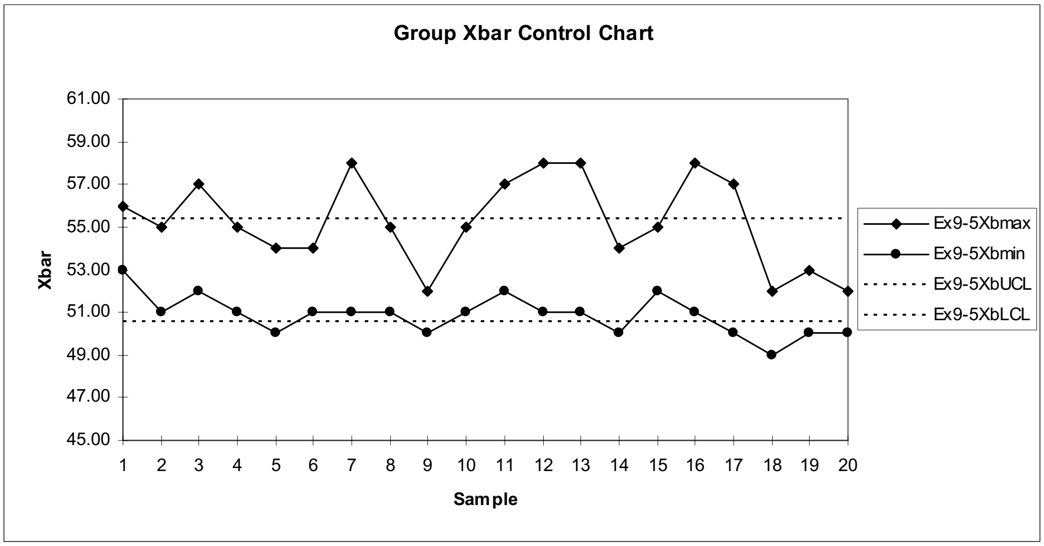


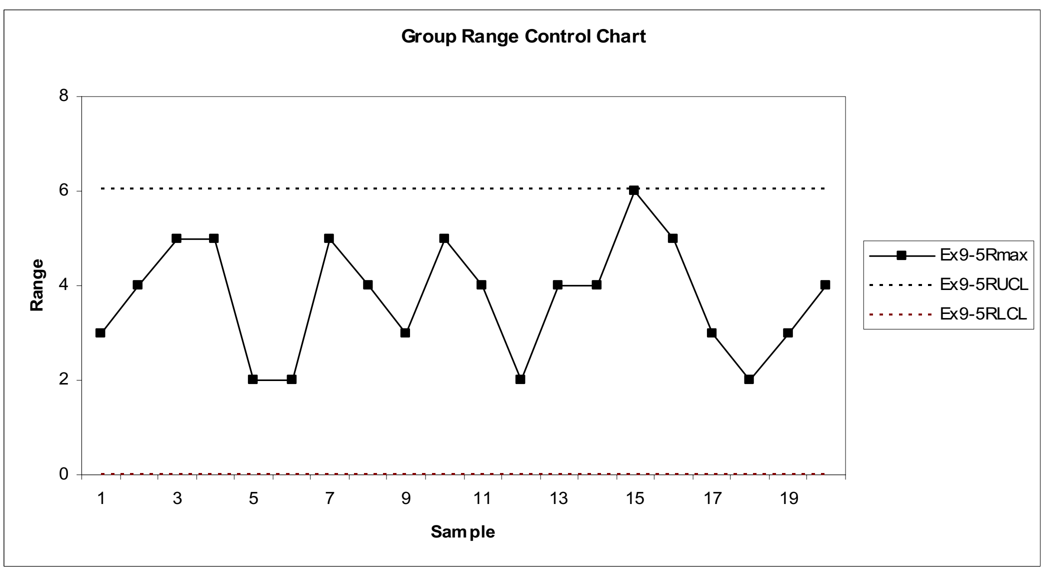
Process is in control.

9-5.

Excel : Workbook Chap09.xls : Worksheet Ex9-5

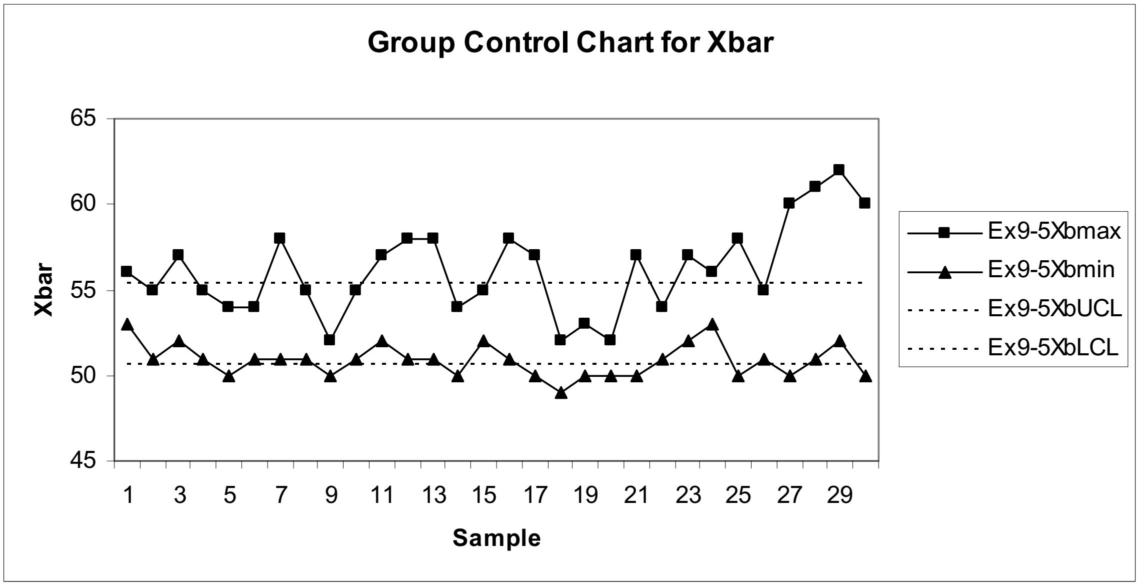
Grand Avg =	52.988	
Avg R =	2.338	
s =	4	heads
n =	3	units
A2 =	1.023	
D3 =	0	
D4 =	2.574	
Xbar UCL =	55.379	
Xbar LCL =	50.596	
R UCL =	6.017	
R LCL =	0.000	

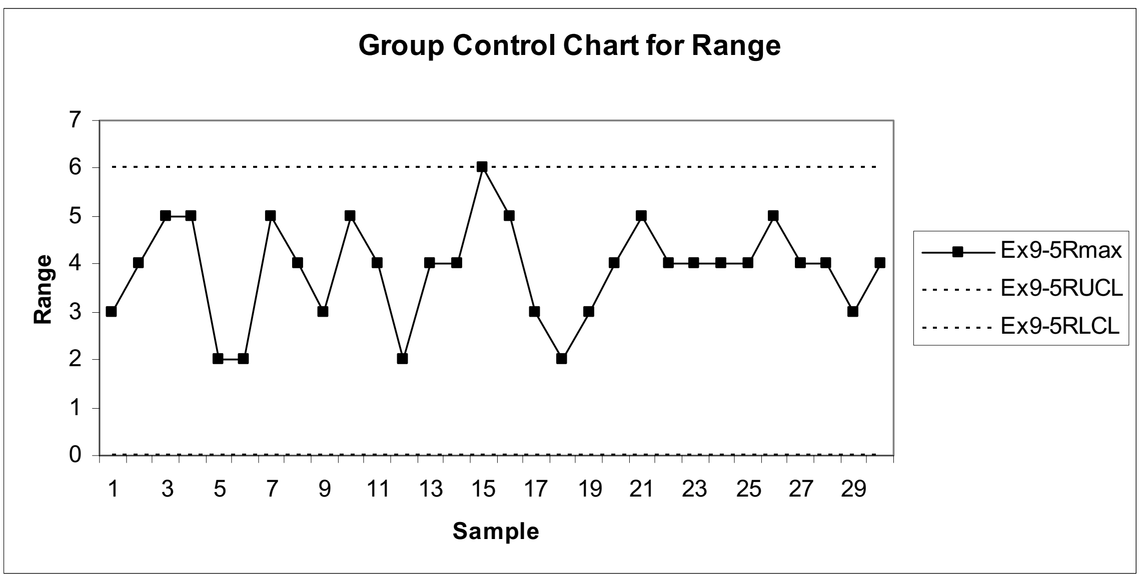




There is no situation where one single head gives the maximum or minimum value of $\overline{x}$ six times in a row. There are many values of $\overline{x}$ max and $\overline{x}$ min that are outside the control limits, so the process is out-of-control. The assignable cause affects all heads, not just a specific one.

9-6.
Excel : Workbook Chap09.xls : Worksheet Ex9-6





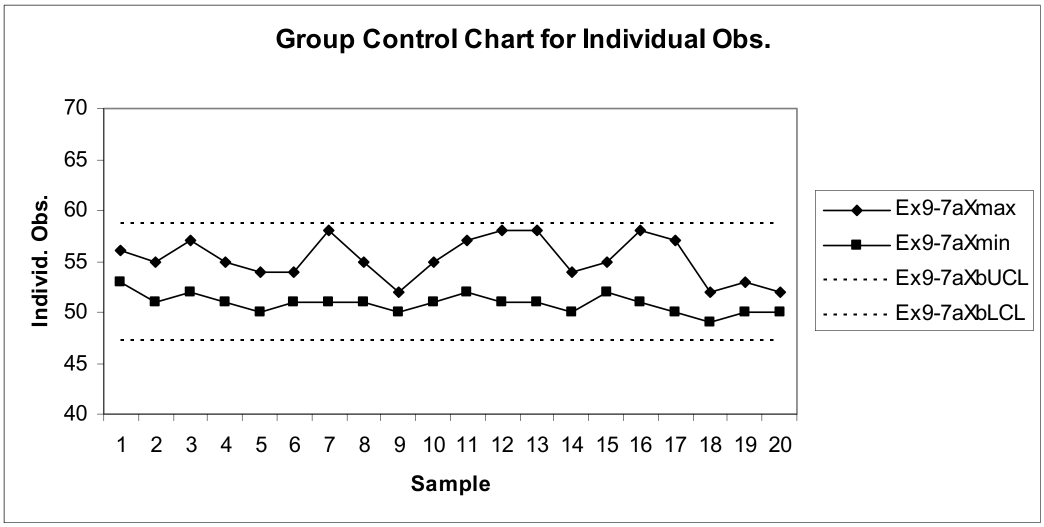
The last four samples from Head 4 are the maximum of all heads; a process change may have caused output of this head to be different from the others.

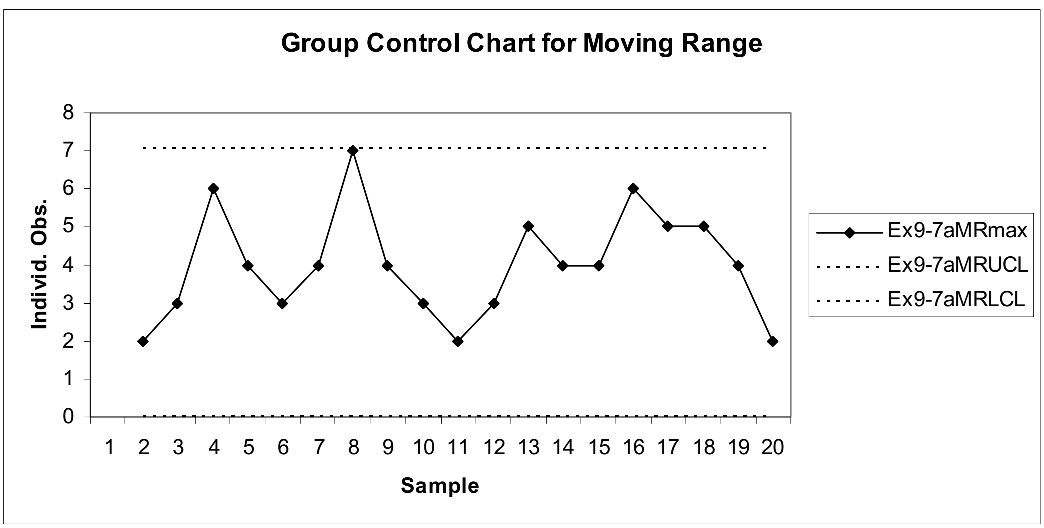
Chapter 9 Exercise Solutions

9-7.
(a)

Excel : Workbook Chap09.xls : Worksheet Ex9-7A

Grand Avg =	52.988	
Avg MR =	2.158	
s =	4	heads
n =	2	units
d2 =	1.128	
D3 =	0	
D4 =	3.267	
Xbar UCL =	58.727	
Xbar LCL =	47.248	
R UCL =	7.050	
R LCL =	0.000	



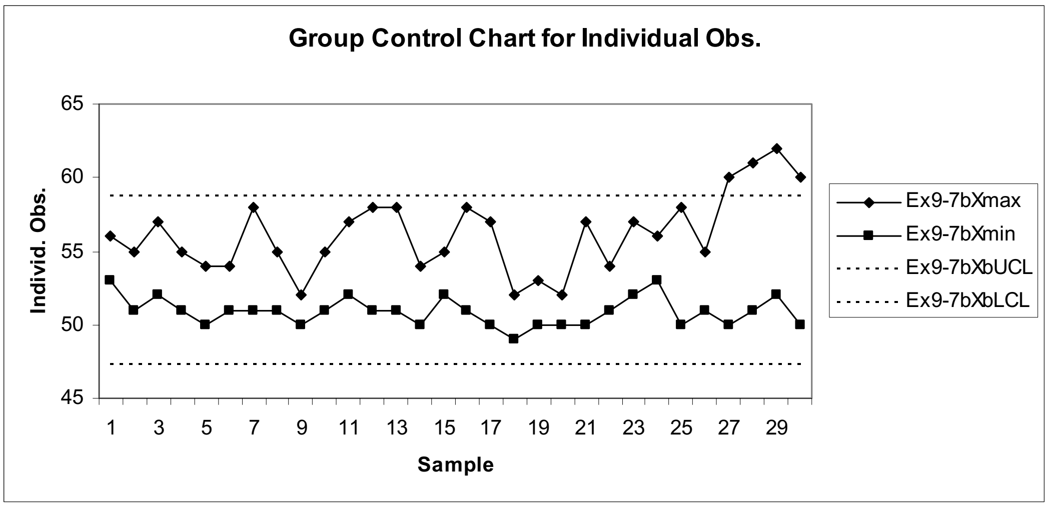


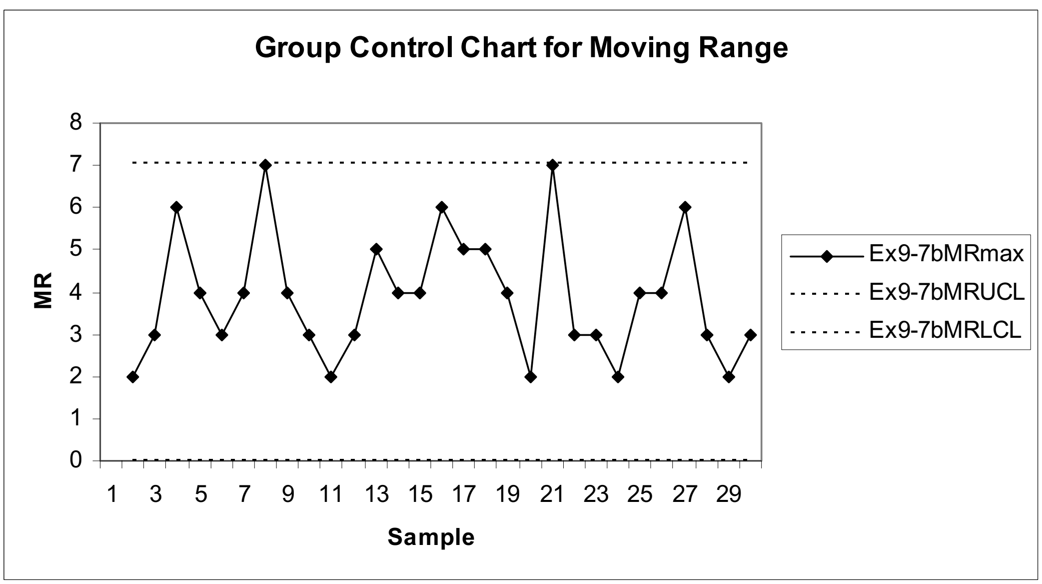
See the discussion in Exercise 9-5.

9-7 continued
(b)

Excel : Workbook Chap09.xls : Worksheet Ex9-7b

Grand Avg =	52.988	
Avg MR =	2.158	
s =	4	heads
n =	2	units
d2 =	1.128	
D3 =	0	
D4 =	3.267	
Xbar UCL =	58.727	
Xbar LCL =	47.248	
R UCL =	7.050	
R LCL =	0.000	

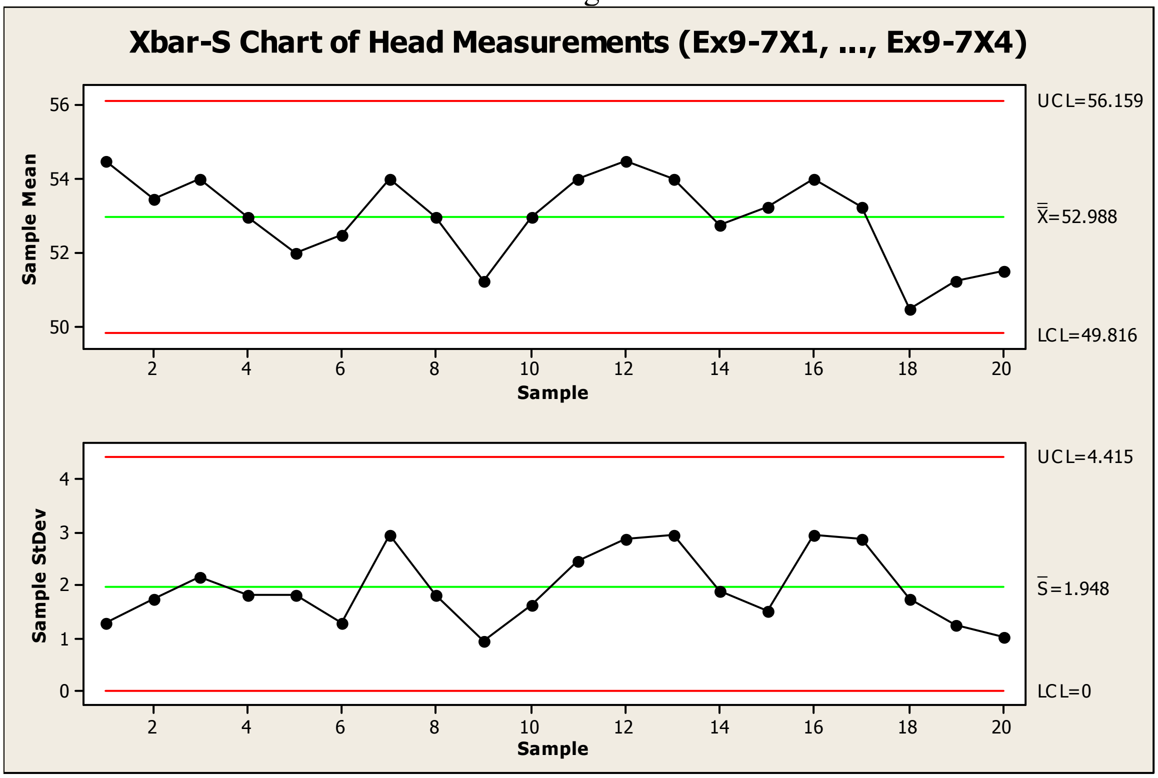




The last four samples from Head 4 remain the maximum of all heads; indicating a potential process change.

9-7 continued
(c)

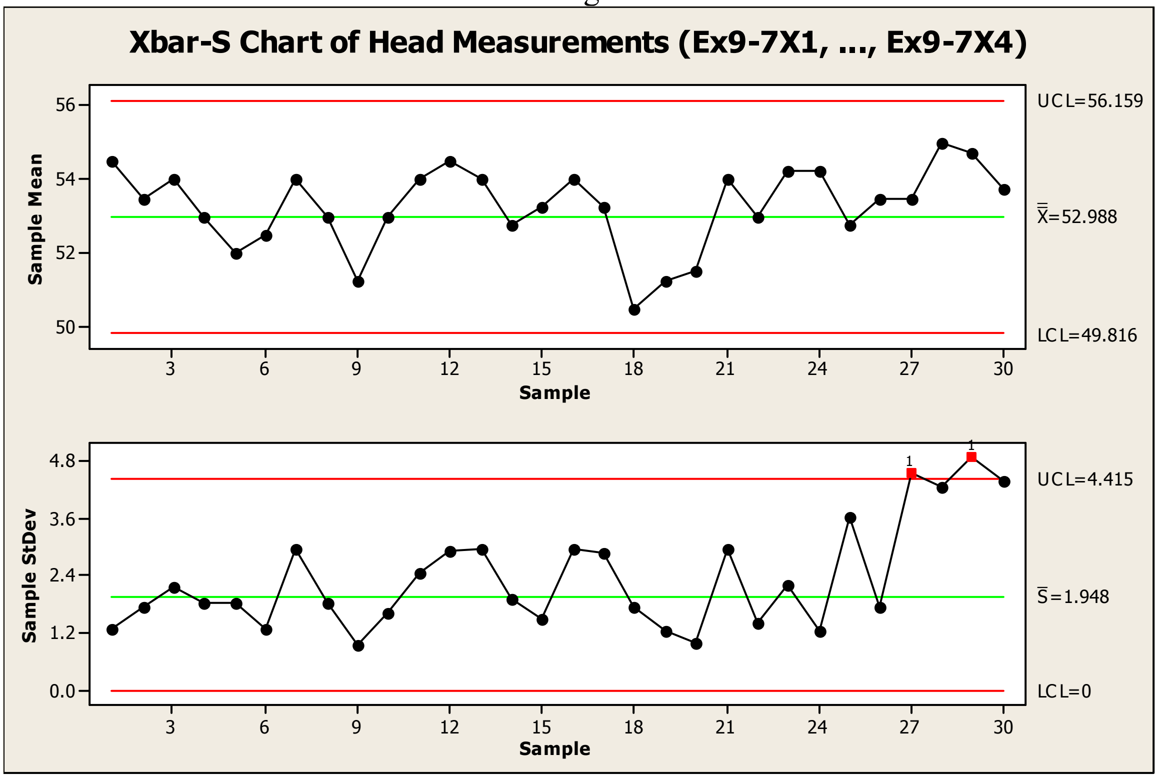
Stat > Control Charts > Variables Charts for Subgroups > Xbar-S Chart
Note: Use "Sbar" as the method for estimating standard deviation.



Failure to recognize the multiple stream nature of the process had led to control charts that fail to identify the out-of-control conditions in this process.

9-7 continued
(d)

Stat > Control Charts > Variables Charts for Subgroups > Xbar-S Chart
Note: Use “Sbar” as the method for estimating standard deviation.



Test Results for S Chart of Ex9-7X1, ..., Ex9-7X4
`TEST 1. One point more than 3.00 standard deviations from center line.`
`Test Failed at points:  27, 29`

Only the *S* chart gives any indication of out-of-control process.

9-8.

Stat > Basic Statistics > Display Descriptive Statistics

Descriptive Statistics: Ex9-8Xbar, Ex9-8R

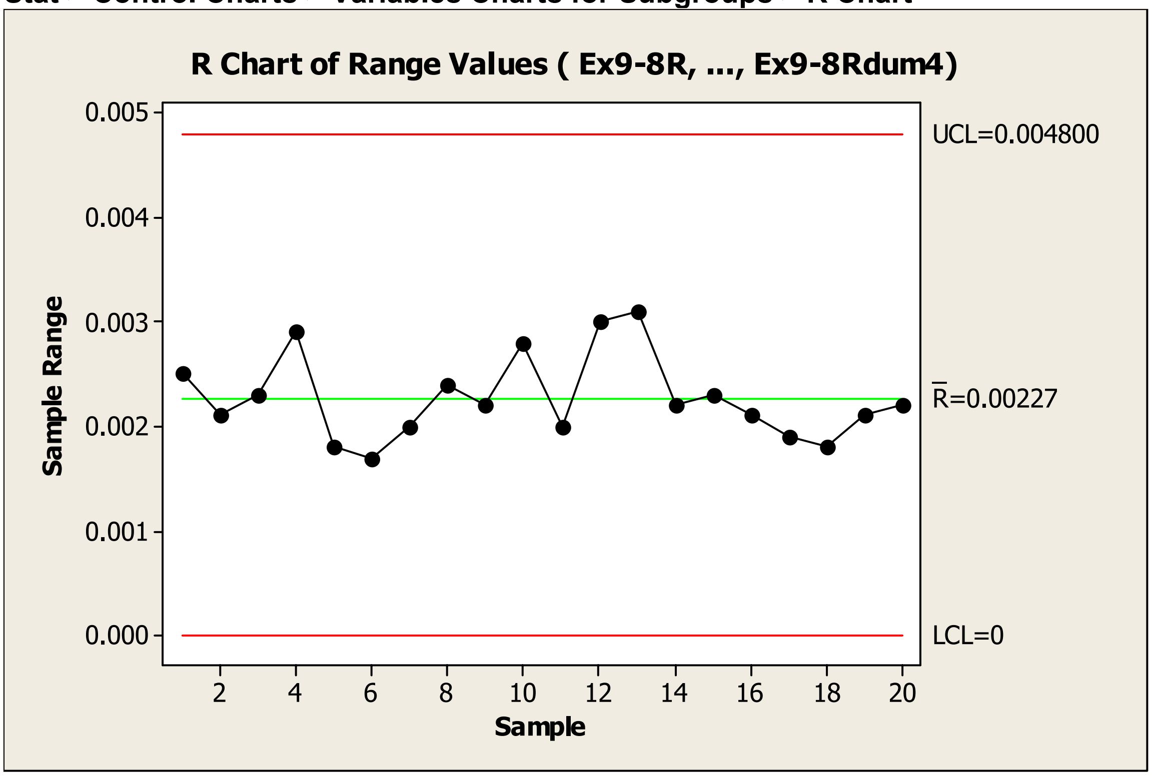
```
Variable       Mean
Ex9-8Xbar   0.55025
Ex9-8R     0.002270
```

$n = 5$

$\bar{\bar{x}} = 0.55025, \bar{R} = 0.00227, \hat{\sigma} = \bar{R} / d_2 = 0.00227 / 2.326 = 0.000976$

$\widehat{\text{PCR}} = (\text{USL-LSL})/6\hat{\sigma} = (0.552 - 0.548)/[6(0.000976)] = 6.83$

Stat > Control Charts > Variables Charts for Subgroups > R Chart



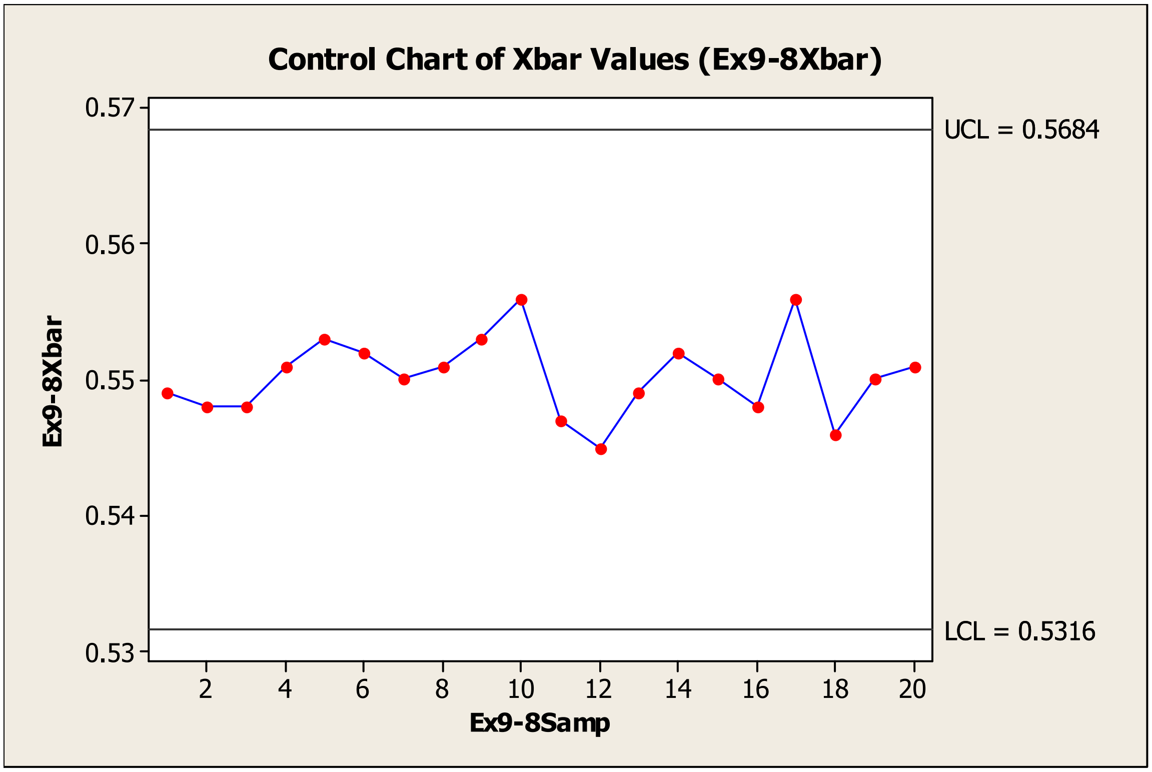
The process variability, as shown on the *R* chart is in control.

9-8 continued
(a)
3-sigma limits:
$\delta = 0.01, Z_\delta = Z_{0.01} = 2.33$

$$\text{UCL} = \text{USL} - \left(Z_\delta - 3\big/\sqrt{n}\right)\hat{\sigma} = (0.550 + 0.020) - \left(2.33 - 3\big/\sqrt{20}\right)(0.000976) = 0.5684$$

$$\text{LCL} = \text{LSL} + \left(Z_\delta - 3\big/\sqrt{n}\right)\hat{\sigma} = (0.550 - 0.020) + \left(2.33 - 3\big/\sqrt{20}\right)(0.000976) = 0.5316$$

Graph > Time Series Plot > Simple
Note: Reference lines have been used set to the control limit values.



The process mean falls within the limits that define 1% fraction nonconforming.

Notice that the control chart does not have a centerline. Since this type of control scheme allows the process mean to vary over the interval—with the assumption that the overall process performance is not appreciably affected—a centerline is not needed.

9-8 continued

(b)

$\gamma = 0.01, Z_\gamma = Z_{0.01} = 2.33$

$1-\beta = 0.90, Z_\beta = z_{0.10} = 1.28$

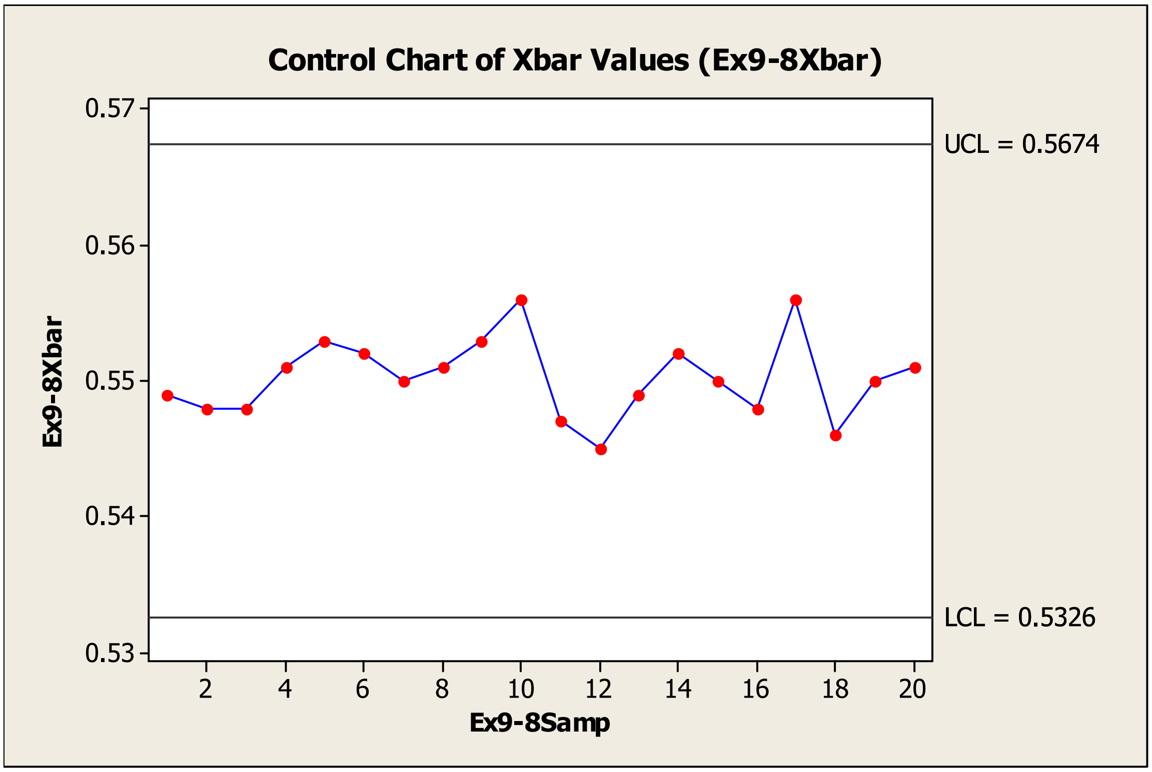
$$\text{UCL} = \text{USL} - \left(Z_\gamma + Z_\beta / \sqrt{n}\right)\hat{\sigma} = (0.550+0.020) - \left(2.33 + 1.28/\sqrt{20}\right)(0.000976) = 0.5674$$

$$\text{LCL} = \text{LSL} + \left(Z_\gamma + Z_\beta / \sqrt{n}\right)\hat{\sigma} = (0.550-0.020) + \left(2.33 + 1.28/\sqrt{20}\right)(0.000976) = 0.5326$$

Chart control limits for part (b) are slightly narrower than for part (a).

Graph > Time Series Plot > Simple

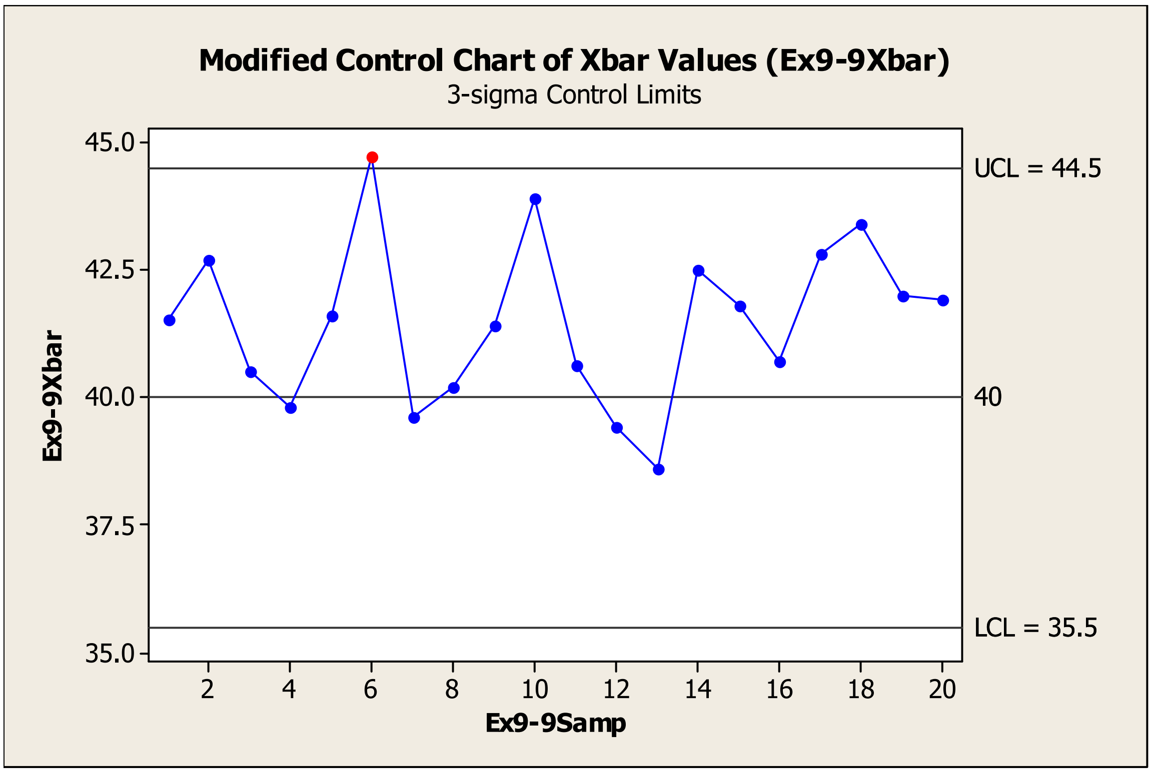
Note: Reference lines have been used set to the control limit values.



The process mean falls within the limits defined by 0.90 probability of detecting a 1% fraction nonconforming.

9-9.
(a)
3-sigma limits:
$n = 5, \delta = 0.001, Z_\delta = Z_{0.001} = 3.090$
$\text{USL} = 40 + 8 = 48, \text{LSL} = 40 - 8 = 32$
$\text{UCL} = \text{USL} - \left(Z_\delta - 3/\sqrt{n}\right)\sigma = 48 - \left(3.090 - 3/\sqrt{5}\right)(2.0) = 44.503$
$\text{LCL} = \text{LSL} + \left(Z_\delta - 3/\sqrt{n}\right)\sigma = 32 + \left(3.090 - 3/\sqrt{5}\right)(2.0) = 35.497$

Graph > Time Series Plot > Simple
Note: Reference lines have been used set to the control limit values.



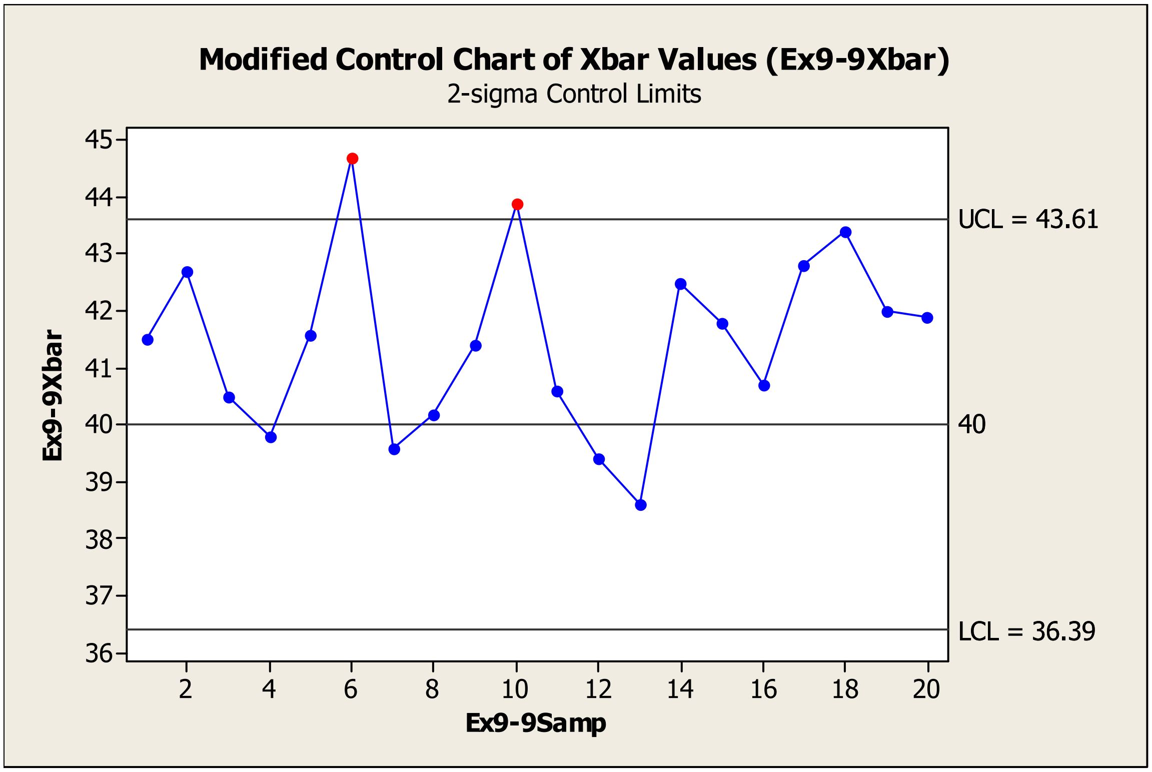
Process is out of control at sample #6.

9-9 continued
(b)
2-sigma limits:

$$\text{UCL} = \text{USL} - \left(Z_\delta - 2\big/\sqrt{n}\right)\sigma = 48 - \left(3.090 - 2\big/\sqrt{5}\right)(2.0) = 43.609$$

$$\text{LCL} = \text{LSL} + \left(Z_\delta - 2\big/\sqrt{n}\right)\sigma = 32 + \left(3.090 - 2\big/\sqrt{5}\right)(2.0) = 36.391$$

Graph > Time Series Plot > Simple
Note: Reference lines have been used set to the control limit values.



With 3-sigma limits, sample #6 exceeds the UCL, while with 2-sigma limits both samples #6 and #10 exceed the UCL.

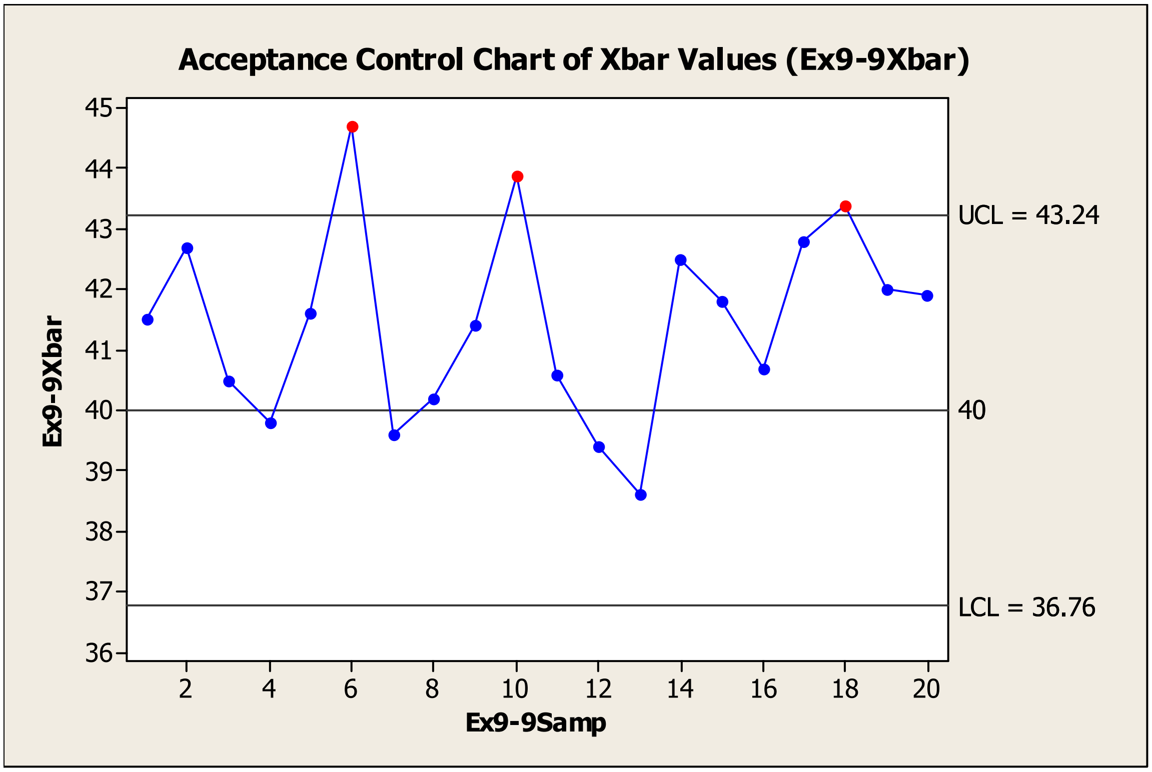
9-9 continued
(c)
$\gamma = 0.05,\ Z_{\gamma} = Z_{0.05} = 1.645$

$1-\beta = 0.95,\ Z_{\beta} = Z_{0.05} = 1.645$

$$\text{UCL} = \text{USL} - \left(Z_{\gamma} + z_{\beta}\big/\sqrt{n}\right)\sigma = 48 - \left(1.645 + 1.645\big/\sqrt{5}\right)(2.0) = 43.239$$

$$\text{LCL} = \text{LSL} + \left(Z_{\gamma} + z_{\beta}\big/\sqrt{n}\right)\sigma = 32 + \left(1.645 + 1.645\big/\sqrt{5}\right)(2.0) = 36.761$$

Graph > Time Series Plot > Simple
Note: Reference lines have been used set to the control limit values.



Sample #18 also signals an out-of-control condition.

9-10.
Design an acceptance control chart.

Accept in-control fraction nonconforming = 0.1% $\rightarrow \delta = 0.001$, $Z_\delta = Z_{0.001} = 3.090$
with probability $1-\alpha = 0.95 \rightarrow \alpha = 0.05$, $Z_\alpha = Z_{0.05} = 1.645$
Reject at fraction nonconforming = 2% $\rightarrow \gamma = 0.02$, $Z_\gamma = Z_{0.02} = 2.054$
with probability $1-\beta = 0.90 \rightarrow \beta = 0.10$, $Z_\beta = Z_{0.10} = 1.282$

$$n = \left(\frac{Z_\alpha + Z_\beta}{Z_\delta - Z_\gamma} \right)^2 = \left(\frac{1.645 + 1.282}{3.090 - 2.054} \right)^2 = 7.98 \approx 8$$

$$\text{UCL} = \text{USL} - \left(Z_\gamma + Z_\beta \big/ \sqrt{n} \right)\sigma = \text{USL} - \left(2.054 + 1.282 \big/ \sqrt{8} \right)\sigma = \text{USL} - 2.507\sigma$$
$$\text{LCL} = \text{LSL} + \left(Z_\gamma + Z_\beta \big/ \sqrt{n} \right)\sigma = \text{LSL} + \left(2.054 + 1.282 \big/ \sqrt{8} \right)\sigma = \text{LSL} + 2.507\sigma$$

9-11.
$\mu = 0,\ \sigma = 1.0,\ n = 5,\ \delta = 0.00135,\ Z_\delta = Z_{0.00135} = 3.00$

For 3-sigma limits, $Z_\alpha = 3$

$$\text{UCL} = \text{USL} - \left(z_\delta - z_\alpha \big/ \sqrt{n}\right)\sigma = \text{USL} - \left(3.000 - 3\big/\sqrt{5}\right)(1.0) = \text{USL} - 1.658$$

$$\Pr\{\text{Accept}\} = \Pr\{\bar{x} < \text{UCL}\} = \Phi\left(\frac{\text{UCL} - \mu_0}{\sigma\big/\sqrt{n}}\right) = \Phi\left(\frac{\text{USL} - 1.658 - \mu_0}{1.0\big/\sqrt{5}}\right) = \Phi\left((\Delta - 1.658)\sqrt{5}\right)$$

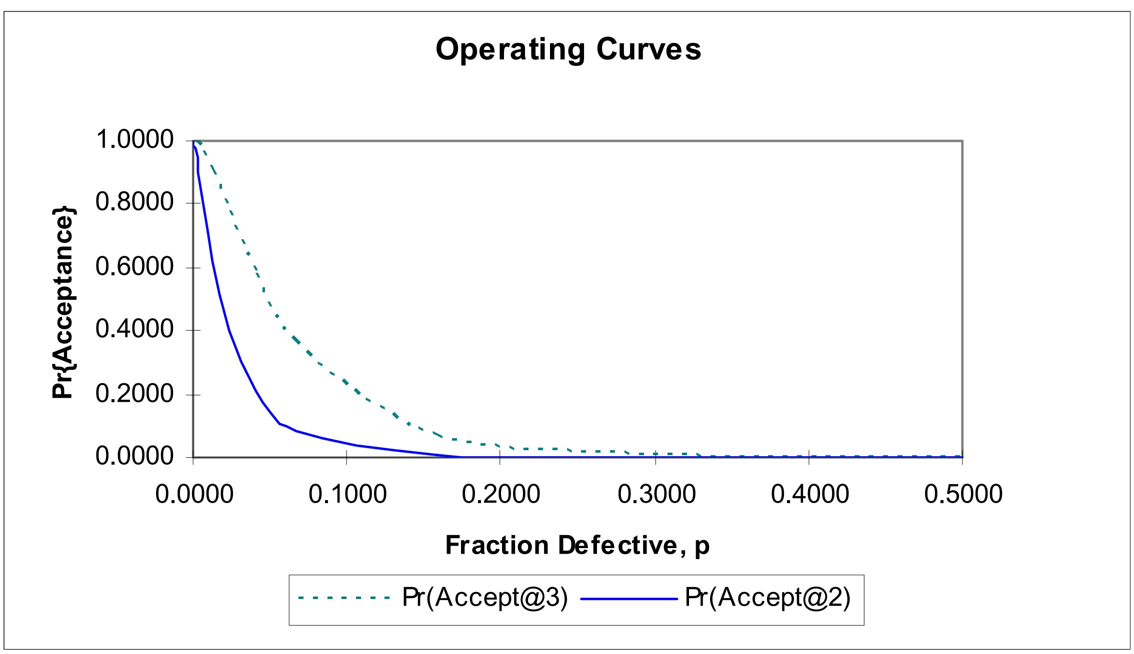
where $\quad \Delta = \text{USL} - \mu_0$

For 2-sigma limits, $Z_\alpha = 2 \Rightarrow \Pr\{\text{Accept}\} = \Phi\left((\Delta - 2.106)\sqrt{5}\right)$

$$p = \Pr\{x > \text{USL}\} = 1 - \Pr\{x \le \text{USL}\} = 1 - \Phi\left(\frac{\text{USL} - \mu_0}{\sigma}\right) = 1 - \Phi(\Delta)$$

Excel : Workbook Chap09.xls : Worksheet Ex9-11

DELTA=USL-mu0	CumNorm(DELTA)	p	Pr(Accept@3)	Pr(Accept@2)
3.50	0.9998	0.0002	1.0000	0.9991
3.25	0.9994	0.0006	0.9998	0.9947
3.00	0.9987	0.0013	0.9987	0.9772
2.50	0.9938	0.0062	0.9701	0.8108
2.25	0.9878	0.0122	0.9072	0.6263
2.00	0.9772	0.0228	0.7778	0.4063
1.75	0.9599	0.0401	0.5815	0.2130
1.50	0.9332	0.0668	0.3619	0.0877
1.00	0.8413	0.1587	0.0706	0.0067
0.50	0.6915	0.3085	0.0048	0.0002
0.25	0.5987	0.4013	0.0008	0.0000
0.00	0.5000	0.5000	0.0001	0.0000



9-12.
Design a modified control chart.

$n = 8$, USL = 8.01, LSL = 7.99, $S = 0.001$
$\delta = 0.00135$, $Z_\delta = Z_{0.00135} = 3.000$
For 3-sigma control limits, $Z_\alpha = 3$

$$\text{UCL} = \text{USL} - \left(Z_\delta - Z_\alpha/\sqrt{n}\right)\sigma = 8.01 - \left(3.000 - 3/\sqrt{8}\right)(0.001) = 8.008$$
$$\text{LCL} = \text{LSL} + \left(Z_\delta - Z_\alpha/\sqrt{n}\right)\sigma = 7.99 + \left(3.000 - 3/\sqrt{8}\right)(0.001) = 7.992$$

9-13.
Design a modified control chart.

$n = 4$, USL = 70, LSL = 30, $S = 4$
$\delta = 0.01$, $Z_\delta = 2.326$
$1 - \alpha = 0.995$, $\alpha = 0.005$, $Z_\alpha = 2.576$

$$\text{UCL} = \text{USL} - \left(Z_\delta - Z_\alpha/\sqrt{n}\right)\sigma = (50 + 20) - \left(2.326 - 2.576/\sqrt{4}\right)(4) = 65.848$$
$$\text{LCL} = \text{LSL} + \left(Z_\delta - Z_\alpha/\sqrt{n}\right)\sigma = (50 - 20) + \left(2.326 - 2.576/\sqrt{4}\right)(4) = 34.152$$

9-14.
Design a modified control chart.

$n = 4$, USL = 820, LSL = 780, $S = 4$
$\delta = 0.01$, $Z_\delta = 2.326$
$1 - \alpha = 0.90$, $\alpha = 0.10$, $Z_\alpha = 1.282$

$$\text{UCL} = \text{USL} - \left(Z_\delta - Z_\alpha/\sqrt{n}\right)\sigma = (800 + 20) - \left(2.326 - 1.282/\sqrt{4}\right)(4) = 813.26$$
$$\text{LCL} = \text{LSL} + \left(Z_\delta - Z_\alpha/\sqrt{n}\right)\sigma = (800 - 20) + \left(2.326 - 1.282/\sqrt{4}\right)(4) = 786.74$$

9-15.

$n = 4,\ \overline{R} = 8.236,\ \overline{\overline{x}} = 620.00$

(a)

$\hat{\sigma}_x = \overline{R}/d_2 = 8.236/2.059 = 4.000$

(b)

$$\begin{aligned}
\hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\
&= \Pr\{x < 595\} + \left[1 - \Pr\{x \le 625\}\right] \\
&= \Phi\left(\frac{595-620}{4.000}\right) + \left[1 - \Phi\left(\frac{625-620}{4.000}\right)\right] \\
&= 0.0000 + \left[1 - 0.8944\right] \\
&= 0.1056
\end{aligned}$$

(c)

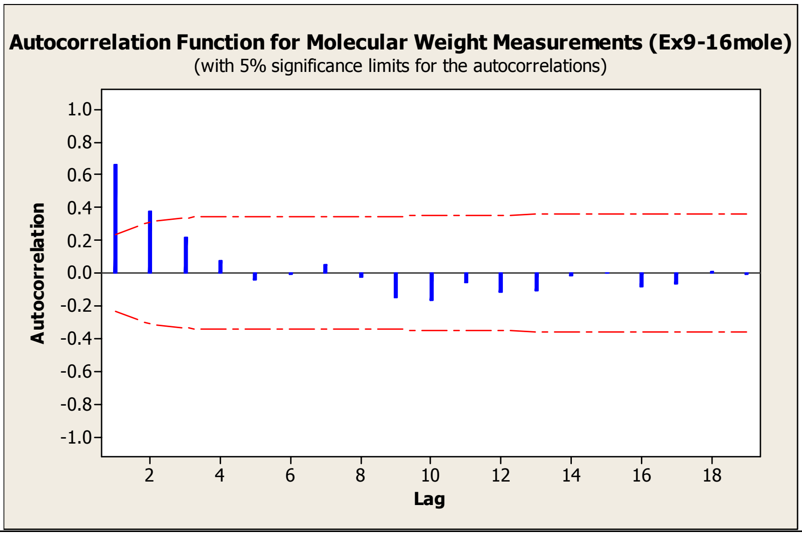
$\delta = 0.005,\ Z_\delta = Z_{0.005} = 2.576$

$\alpha = 0.01,\ Z_\alpha = Z_{0.01} = 2.326$

$\text{UCL} = \text{USL} - \left(Z_\delta - Z_\alpha/\sqrt{n}\right)\sigma = 625 - \left(2.576 - 2.326/\sqrt{4}\right)4 = 619.35$

$\text{LCL} = \text{LSL} + \left(Z_\delta - Z_\alpha/\sqrt{n}\right)\sigma = 595 + \left(2.576 - 2.326/\sqrt{4}\right)4 = 600.65$

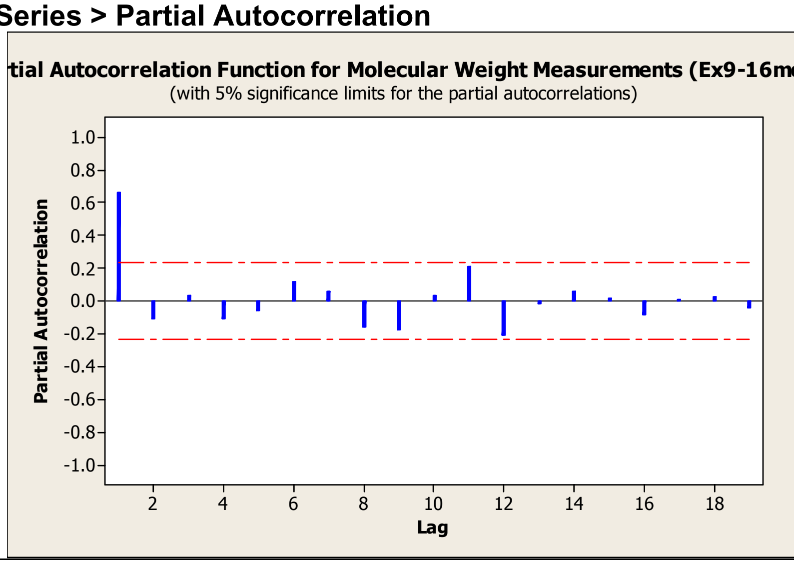
9-16. Note: In the textbook, the 5th column, the 5th row should be "2000" not "2006".
(a)
Stat > Time Series > Autocorrelation



Autocorrelation Function: Ex9-16mole

Lag	ACF	T	LBQ
1	0.658253	5.70	33.81
2	0.373245	2.37	44.84
3	0.220536	1.30	48.74
4	0.072562	0.42	49.16
5	-0.039599	-0.23	49.29 …

Stat > Time Series > Partial Autocorrelation



Partial Autocorrelation Function: Ex9-16mole

Lag	PACF	T
1	0.658253	5.70
2	-0.105969	-0.92
3	0.033132	0.29
4	-0.110802	-0.96
5	-0.055640	-0.48 …

The decaying sine wave of the ACFs combined with a spike at lag 1 for the PACFs suggests an autoregressive process of order 1, AR(1).

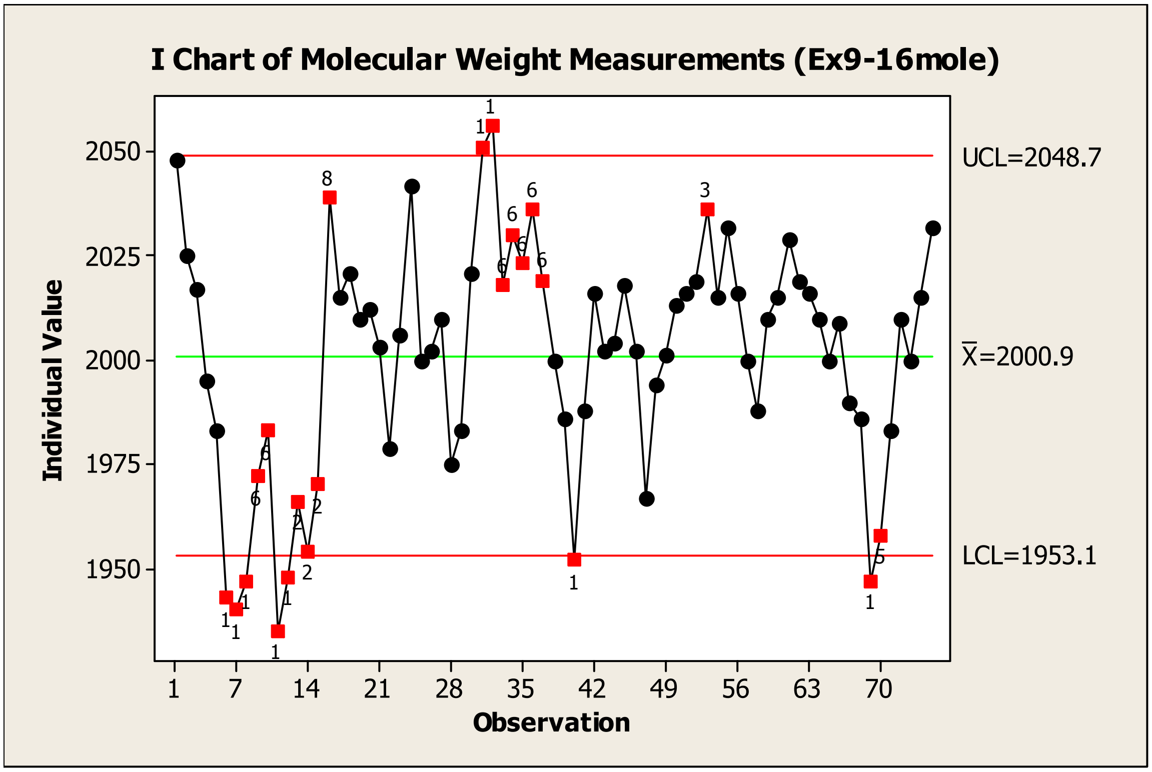
9-16 continued

(b)

x chart: CL = 2001, UCL = 2049, LCL = 1953

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 17.97/1.128 = 15.93$

Stat > Control Charts > Variables Charts for Individuals > Individuals

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Test Results for I Chart of Ex9-16mole
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  6, 7, 8, 11, 12, 31, 32, 40, 69
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  12, 13, 14, 15
TEST 3. 6 points in a row all increasing or all decreasing.
Test Failed at points:  7, 53
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
        one side of CL).
Test Failed at points:  7, 8, 12, 13, 14, 32, 70
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
        one side of CL).
Test Failed at points:  8, 9, 10, 11, 12, 13, 14, 15, 33, 34, 35, 36, 37
TEST 8. 8 points in a row more than 1 standard deviation from center line
        (above and below CL).
Test Failed at points:  12, 13, 14, 15, 16, 35, 36, 37
```

The process is out of control on the x chart, violating many runs tests, with big swings and very few observations actually near the mean.

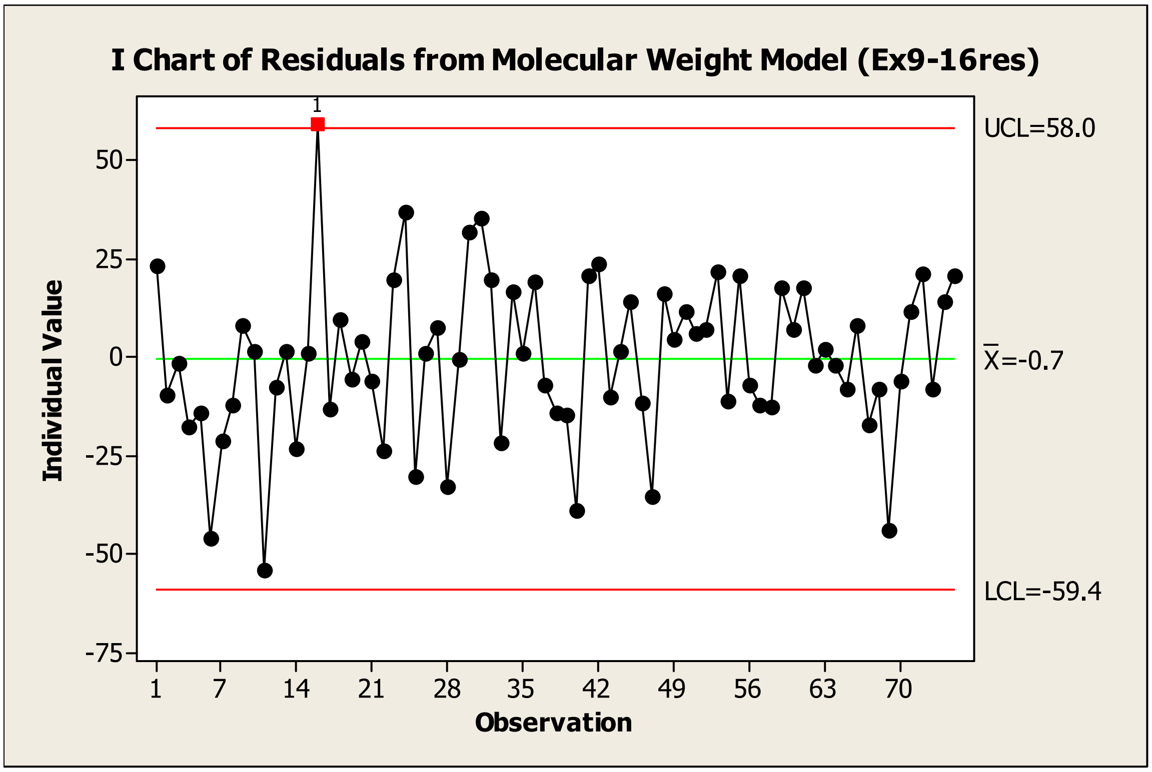
9-16 continued
(c)
Stat > Time Series > ARIMA

ARIMA Model: Ex9-16mole

```
Estimates at each iteration
Iteration       SSE      Parameters
        0  50173.7  0.100  1800.942
        1  41717.0  0.250  1500.843
        2  35687.3  0.400  1200.756
        3  32083.6  0.550   900.693
        4  30929.9  0.675   650.197
        5  30898.4  0.693   613.998
        6  30897.1  0.697   606.956
        7  30897.1  0.698   605.494
        8  30897.1  0.698   605.196
Relative change in each estimate less than 0.0010

Final Estimates of Parameters
Type          Coef  SE Coef       T      P
AR   1      0.6979   0.0852    8.19  0.000
Constant   605.196    2.364  256.02  0.000
Mean       2003.21     7.82…
```

Stat > Control Charts > Variables Charts for Individuals > Individuals



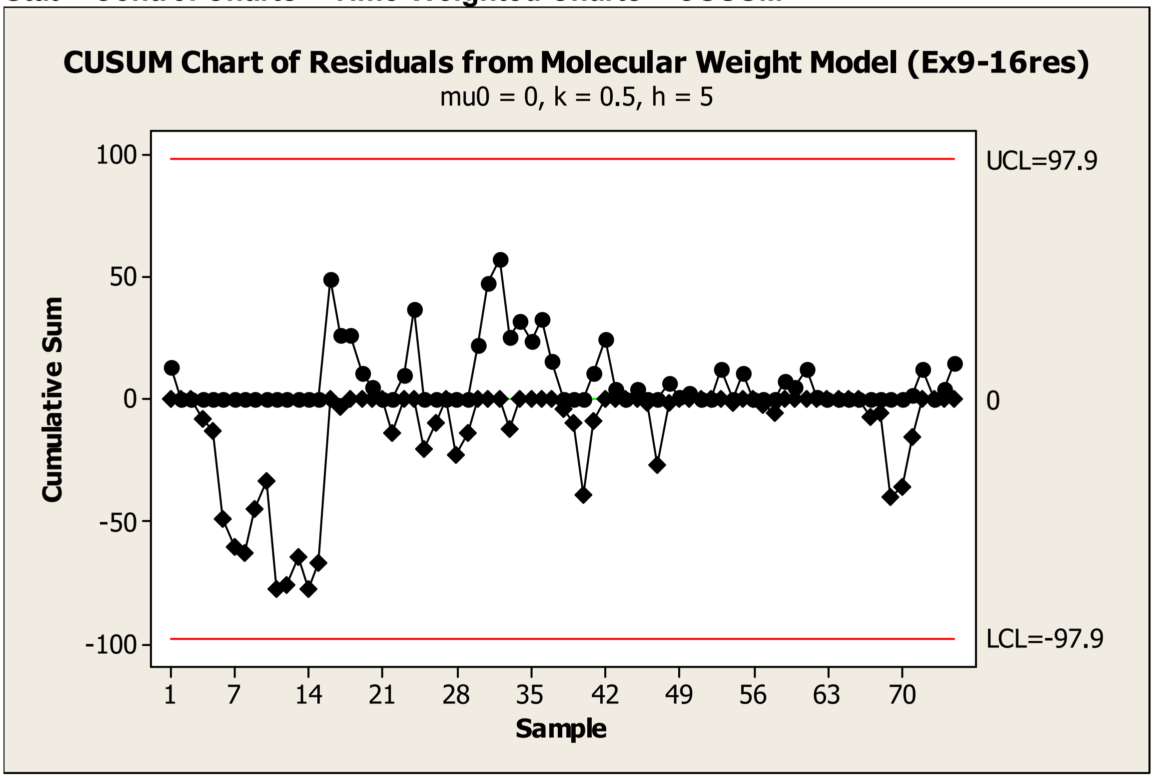
Test Results for I Chart of Ex9-16res

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  16
```

Observation 16 signals out of control above the upper limit. There are no other violations of special cause tests.

9-17.
Let $\mu_0 = 0$, $\delta = 1$ sigma, $k = 0.5$, $h = 5$.

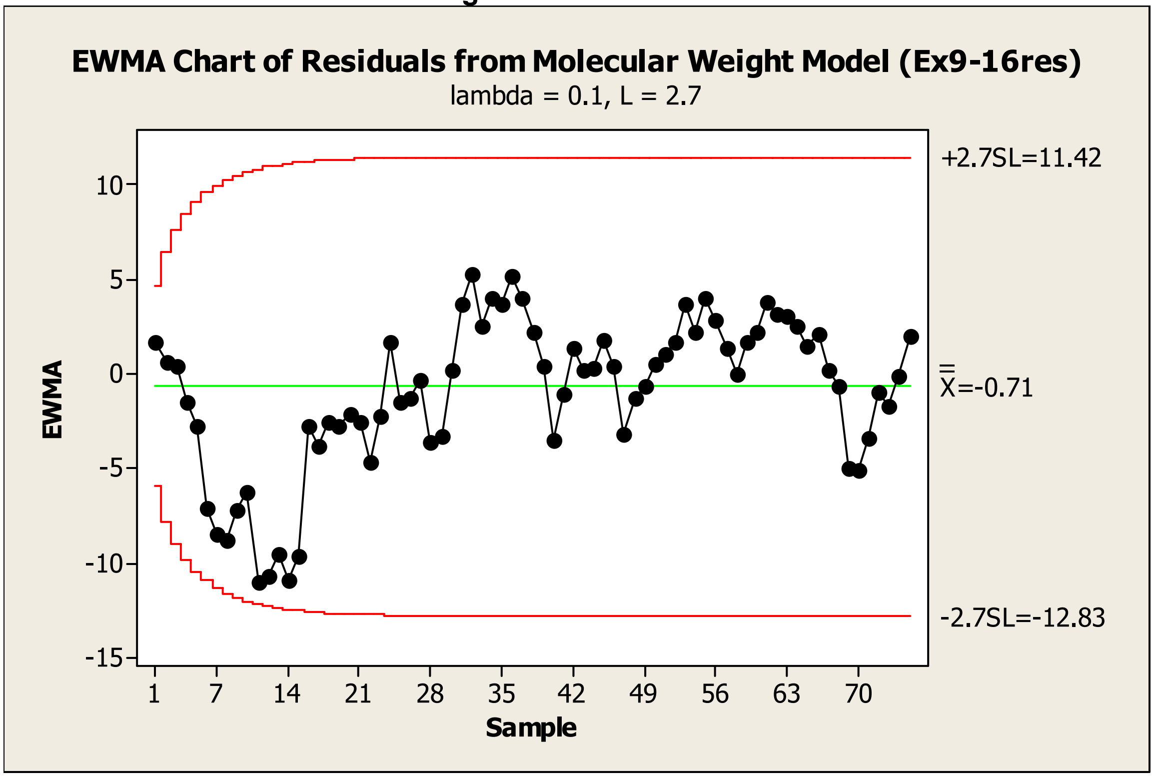
Stat > Control Charts > Time-Weighted Charts > CUSUM



No observations exceed the control limit. The residuals are in control.

9-18.
Let $\lambda = 0.1$ and $L = 2.7$ (approximately the same as a CUSUM with $k = 0.5$ and $h = 5$).

Stat > Control Charts > Time-Weighted Charts > EWMA



Process is in control.

9-19.

To find the optimal λ, fit an ARIMA (0,1,1) (= EWMA = IMA(1,1)).

Stat > Time Series > ARIMA

ARIMA Model: Ex9-16mole

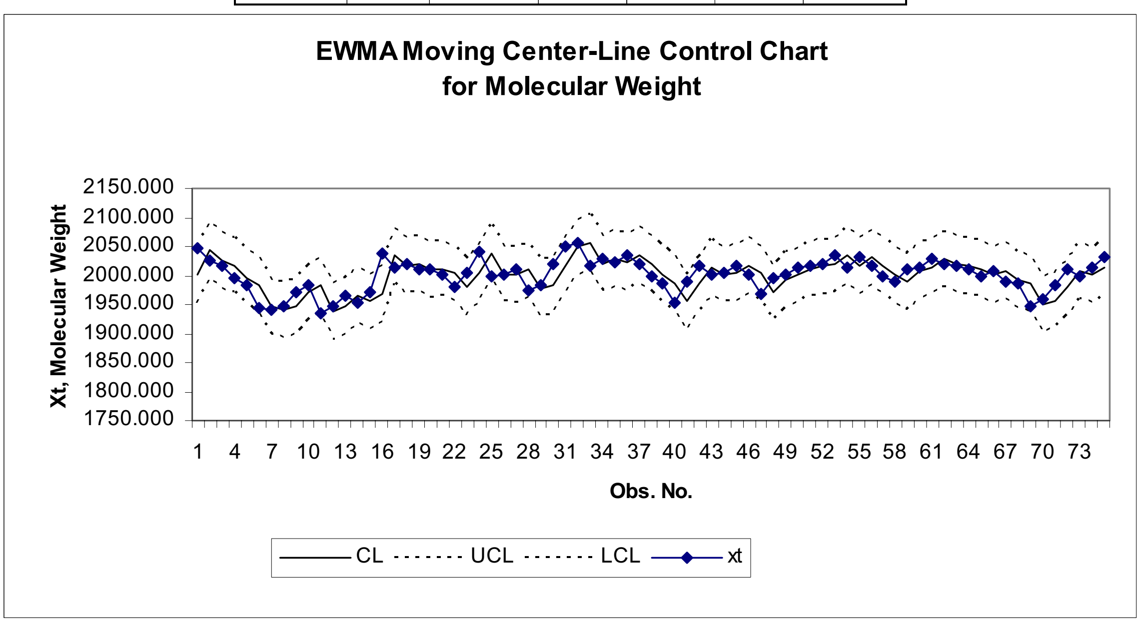
```
…
Final Estimates of Parameters
Type          Coef  SE Coef      T      P
MA   1      0.0762   0.1181   0.65  0.521
Constant    -0.211    2.393  -0.09  0.930
…
```

$\lambda = 1 - \text{MA1} = 1 - 0.0762 = 0.9238$

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 17.97/1.128 = 15.93$

Excel : Workbook Chap09.xls : Worksheet Ex9-19

t	xt	zt	CL	UCL	LCL	OOC?
0		2000.947				
1	2048	2044.415	2000.947	2048.749	1953.145	No
2	2025	2026.479	2044.415	2092.217	1996.613	No
3	2017	2017.722	2026.479	2074.281	1978.677	No
4	1995	1996.731	2017.722	2065.524	1969.920	No
5	1983	1984.046	1996.731	2044.533	1948.929	No
6	1943	1946.128	1984.046	2031.848	1936.244	No
7	1940	1940.467	1946.128	1993.930	1898.326	No
8	1947	1946.502	1940.467	1988.269	1892.665	No
9	1972	1970.057	1946.502	1994.304	1898.700	No
10	1983	1982.014	1970.057	2017.859	1922.255	No
11	1935	1938.582	1982.014	2029.816	1934.212	No
12	1948	1947.282	1938.582	1986.384	1890.780	No
13	1966	1964.574	1947.282	1995.084	1899.480	No
14	1954	1954.806	1964.574	2012.376	1916.772	No
15	1970	1968.842	1954.806	2002.608	1907.004	No
16	2039	2033.654	1968.842	2016.644	1921.040	above UCL
…						

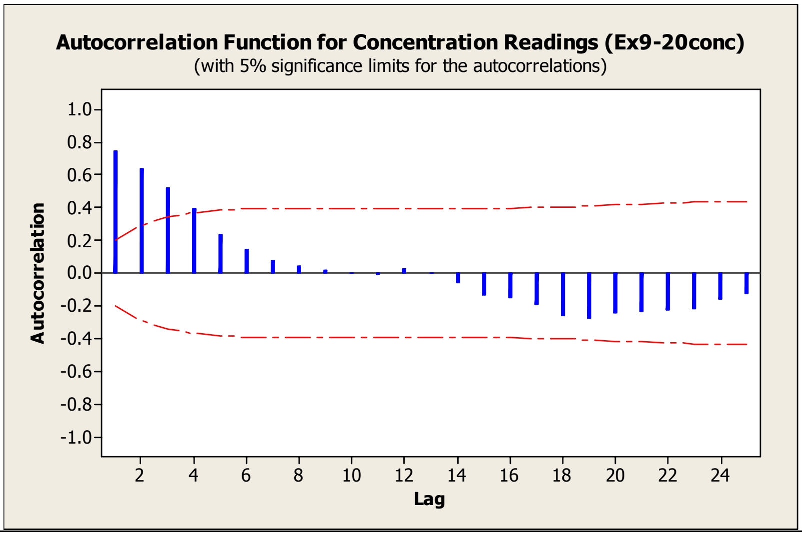


Observation 6 exceeds the upper control limit compared to one out-of-control signal at observation 16 on the Individuals control chart.

9-20

(a)

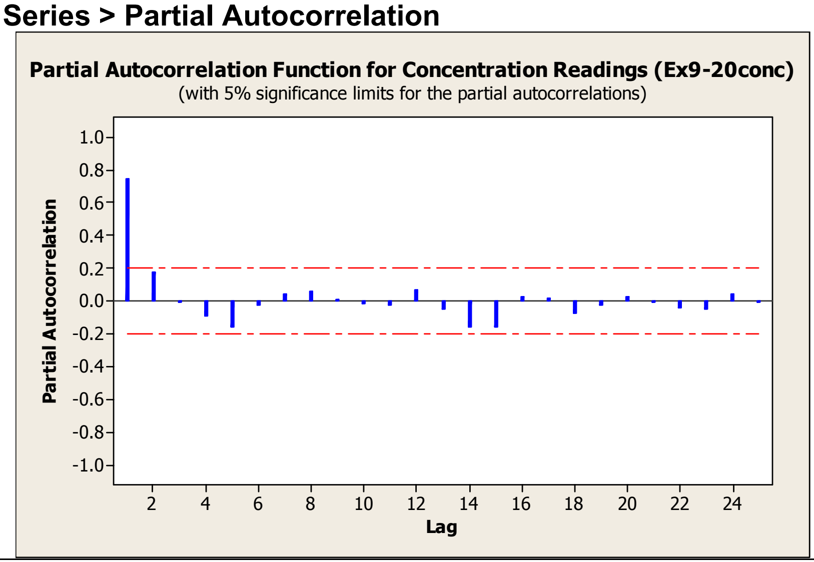
Stat > Time Series > Autocorrelation



Autocorrelation Function: Ex9-20conc

Lag	ACF	T	LBQ
1	0.746174	7.46	57.36
2	0.635375	4.37	99.38
3	0.520417	3.05	127.86
4	0.390108	2.10	144.03
5	0.238198	1.23	150.12 …

Stat > Time Series > Partial Autocorrelation



Partial Autocorrelation Function: Ex9-20conc

Lag	PACF	T
1	0.746174	7.46
2	0.177336	1.77
3	-0.004498	-0.04
4	-0.095134	-0.95
5	-0.158358	-1.58 …

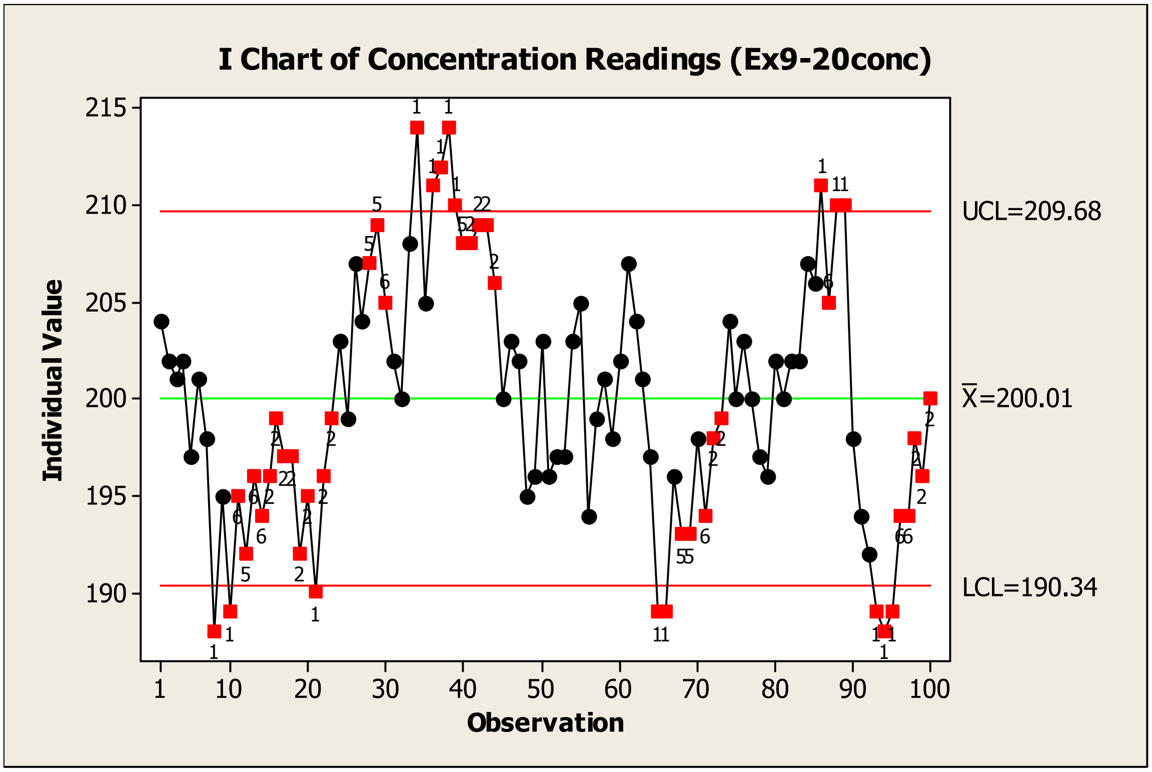
The decaying sine wave of the ACFs combined with a spike at lag 1 for the PACFs suggests an autoregressive process of order 1, AR(1).

9-20 continued

(b)

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 3.64/1.128 = 3.227$

Stat > Control Charts > Variables Charts for Individuals > Individuals




```
Test Results for I Chart of Ex9-20conc
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  8, 10, 21, 34, 36, 37, 38, 39, 65, 66, 86, 88, 89, 93,
                        94, 95
TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  15, 16, 17, 18, 19, 20, 21, 22, 23, 41, 42, 43, 44, 72,
                        73, 98, 99, 100
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  10, 12, 21, 28, 29, 34, 36, 37, 38, 39, 40, 41, 42, 43,
                        66, 68, 69, 86, 88, 89, 93, 94, 95
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  11, 12, 13, 14, 15, 22, 29, 30, 36, 37, 38, 39, 40, 41,
                        42, 43, 44, 68, 69, 71, 87, 88, 89, 94, 95, 96, 97, 99
TEST 8. 8 points in a row more than 1 standard deviation from center line
     (above and below CL).
Test Failed at points:  15, 40, 41, 42, 43, 44
```

The process is out of control on the x chart, violating many runs tests, with big swings and very few observations actually near the mean.

9-20 continued

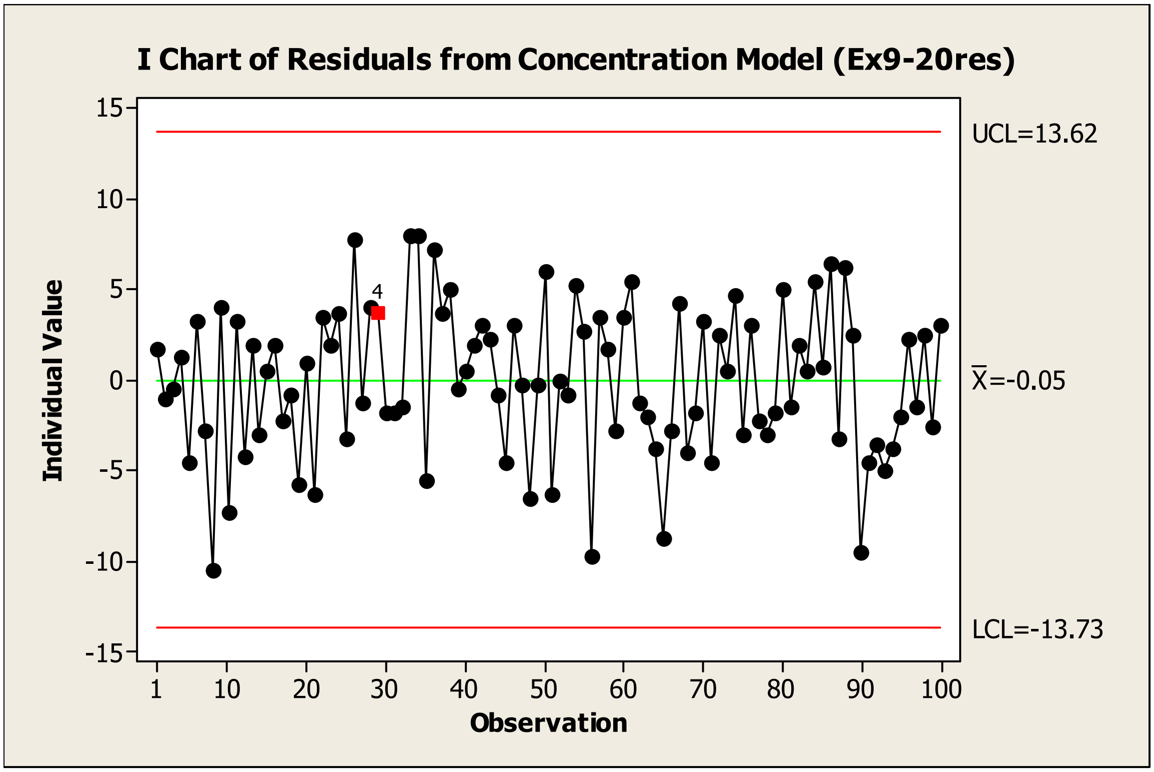
(c)

Stat > Time Series > ARIMA

ARIMA Model: Ex9-20conc

```
…
Final Estimates of Parameters
Type          Coef   SE Coef       T       P
AR   1      0.7493    0.0669   11.20   0.000
Constant   50.1734    0.4155  120.76   0.000
Mean       200.122     1.657
…
```

Stat > Control Charts > Variables Charts for Individuals > Individuals

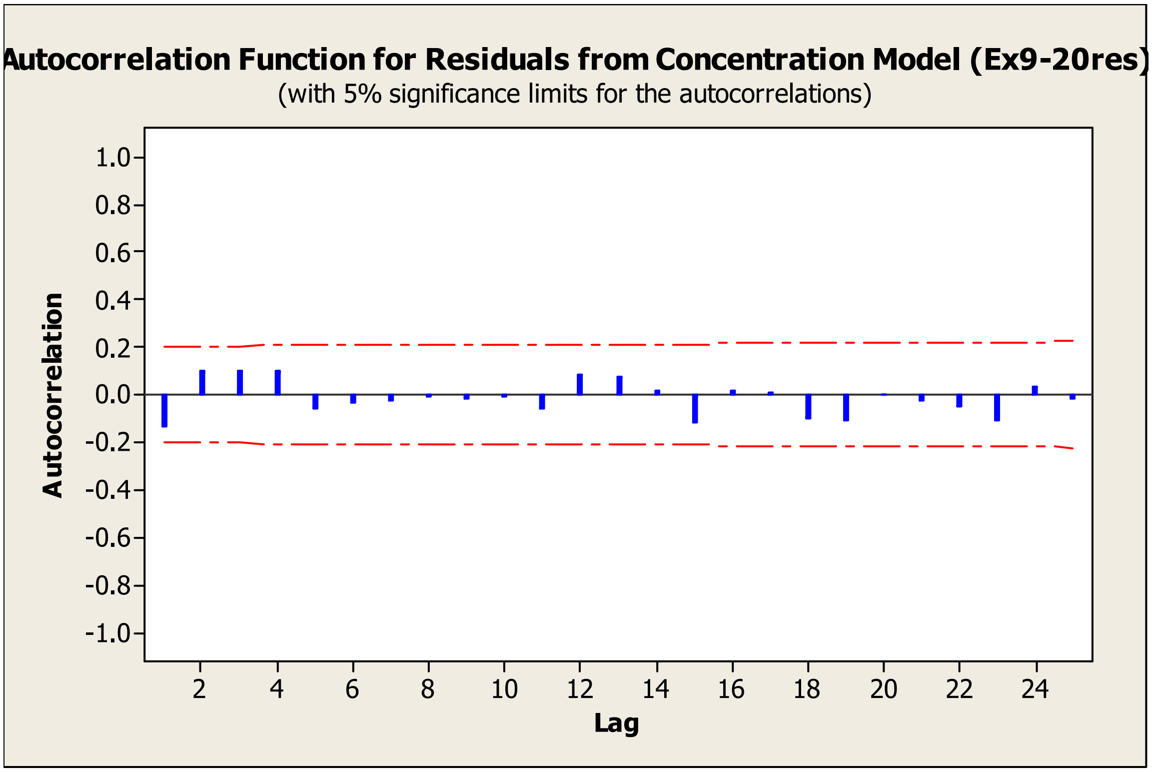


Test Results for I Chart of Ex9-20res

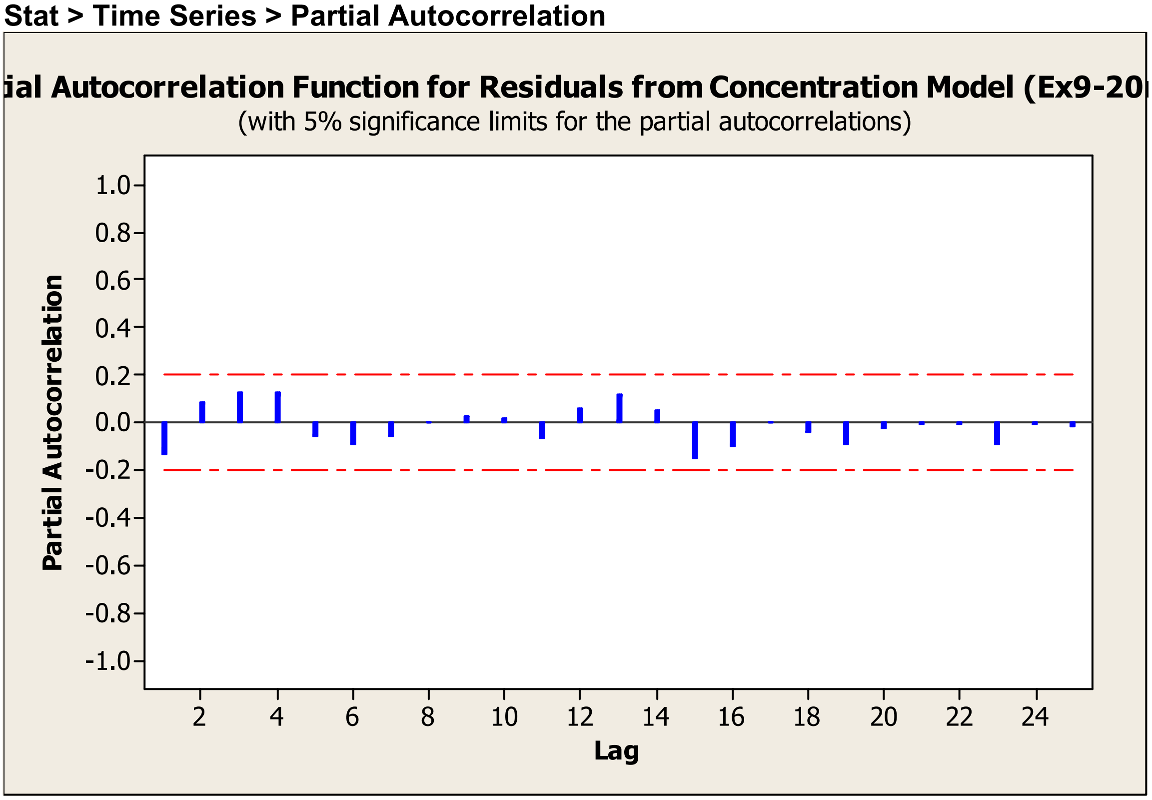
```
TEST 4. 14 points in a row alternating up and down.
Test Failed at points:   29
```

Observation 29 signals out of control for test 4, however this is not unlikely for a dataset of 100 observations. Consider the process to be in control.

9-20 continued
(d)
Stat > Time Series > Autocorrelation

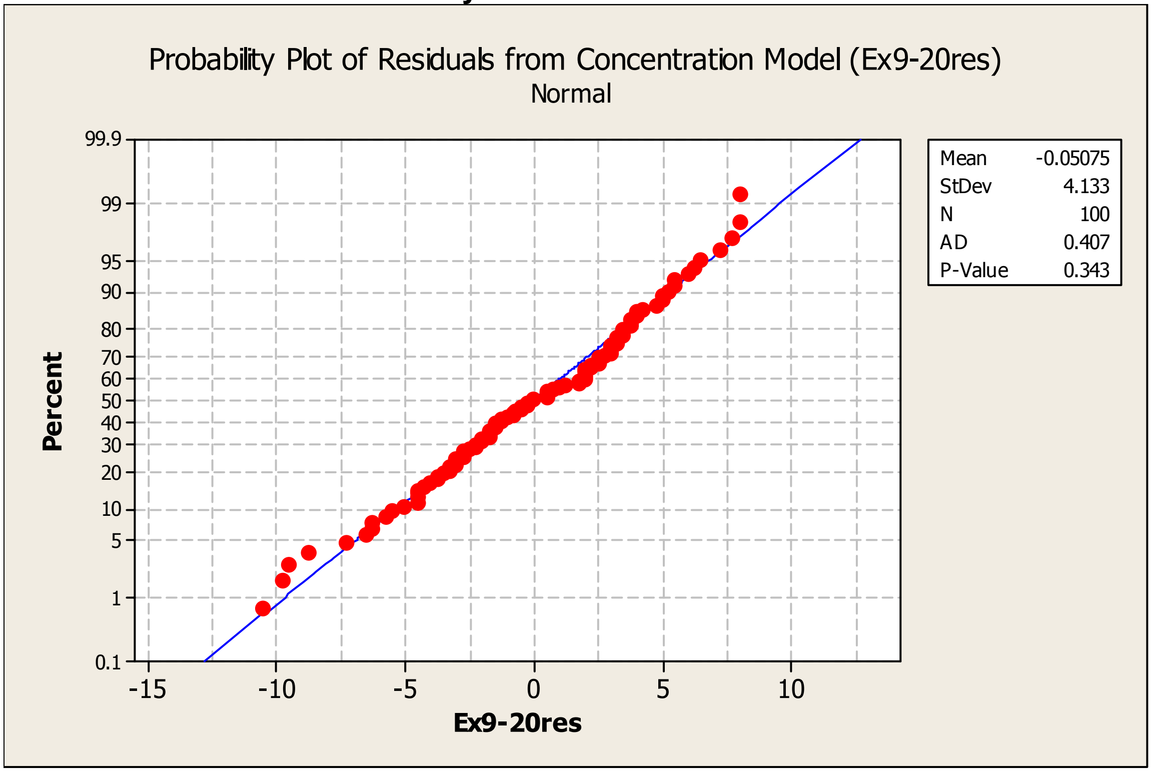


Stat > Time Series > Partial Autocorrelation



9-20 (d) continued

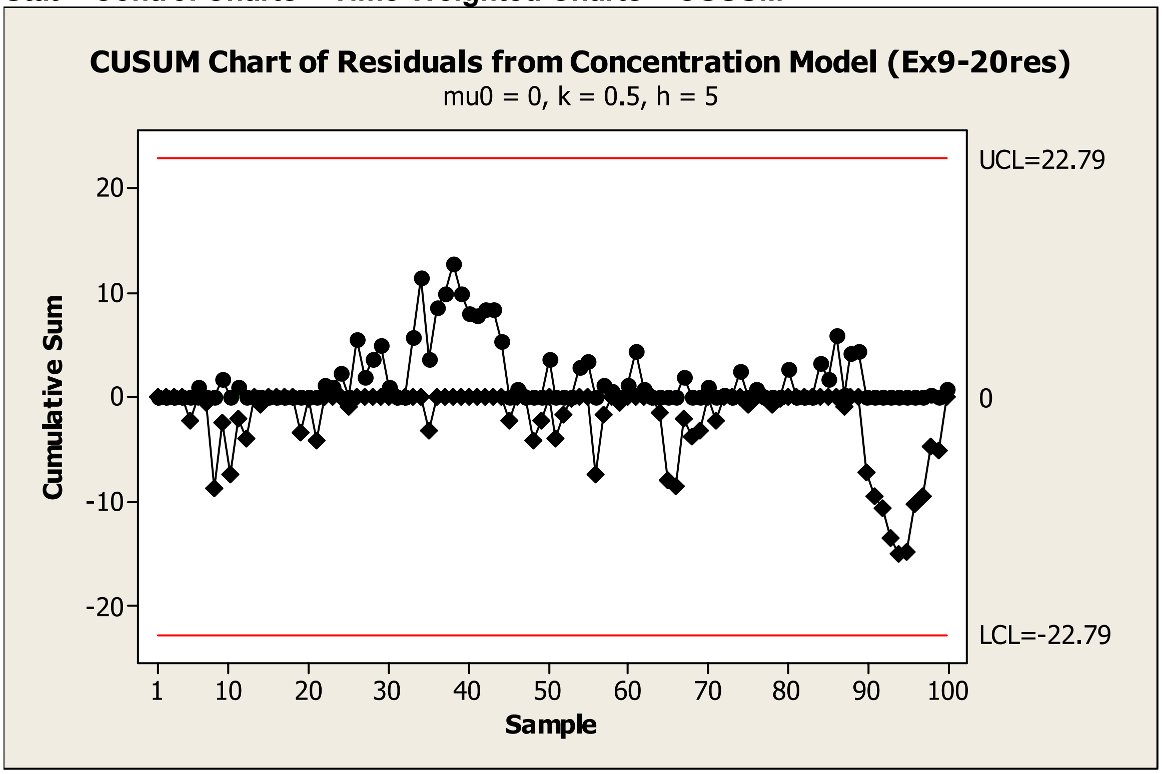
Stat > Basic Statistics > Normality Test



Visual examination of the ACF, PACF and normal probability plot indicates that the residuals are normal and uncorrelated.

9-21.
Let $\mu_0 = 0$, $\delta = 1$ sigma, $k = 0.5$, $h = 5$.

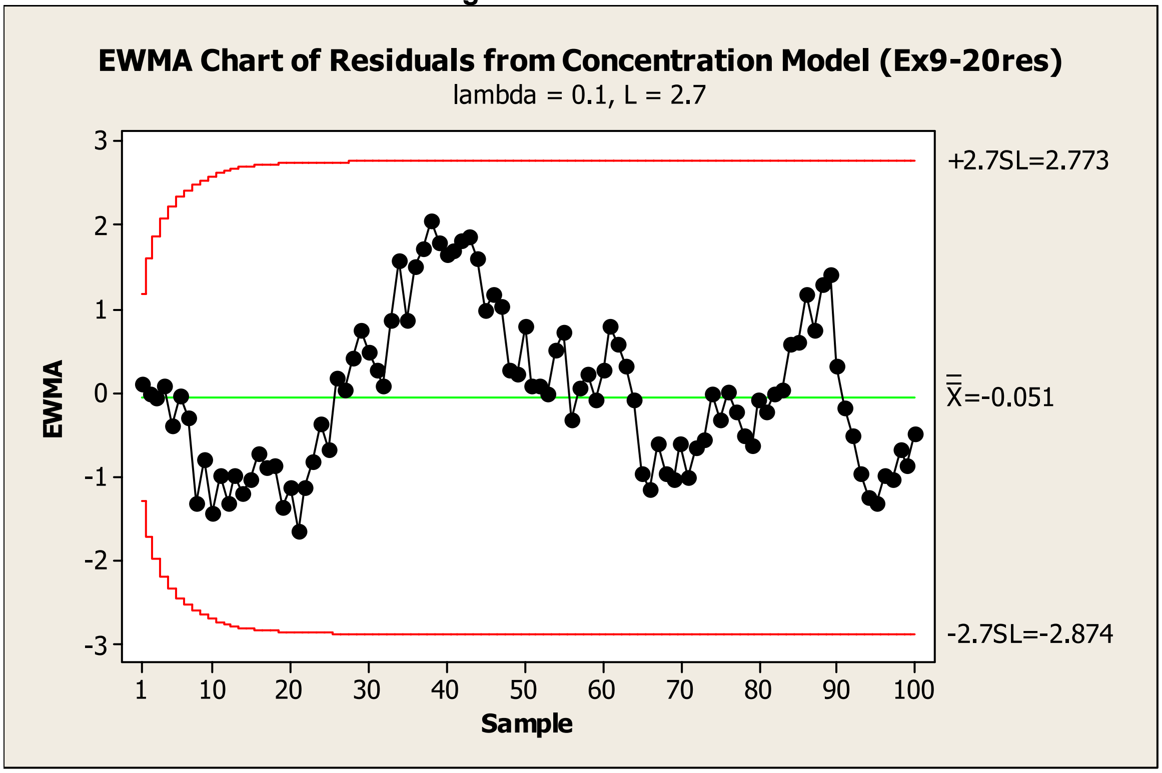
Stat > Control Charts > Time-Weighted Charts > CUSUM



No observations exceed the control limit. The residuals are in control, and the AR(1) model for concentration should be a good fit.

9-22.
Let $\lambda = 0.1$ and $L = 2.7$ (approximately the same as a CUSUM with $k = 0.5$ and $h = 5$).

Stat > Control Charts > Time-Weighted Charts > EWMA



No observations exceed the control limit. The residuals are in control.

9-23.

To find the optimal λ, fit an ARIMA (0,1,1) (= EWMA = IMA(1,1)).

Stat > Time Series > ARIMA

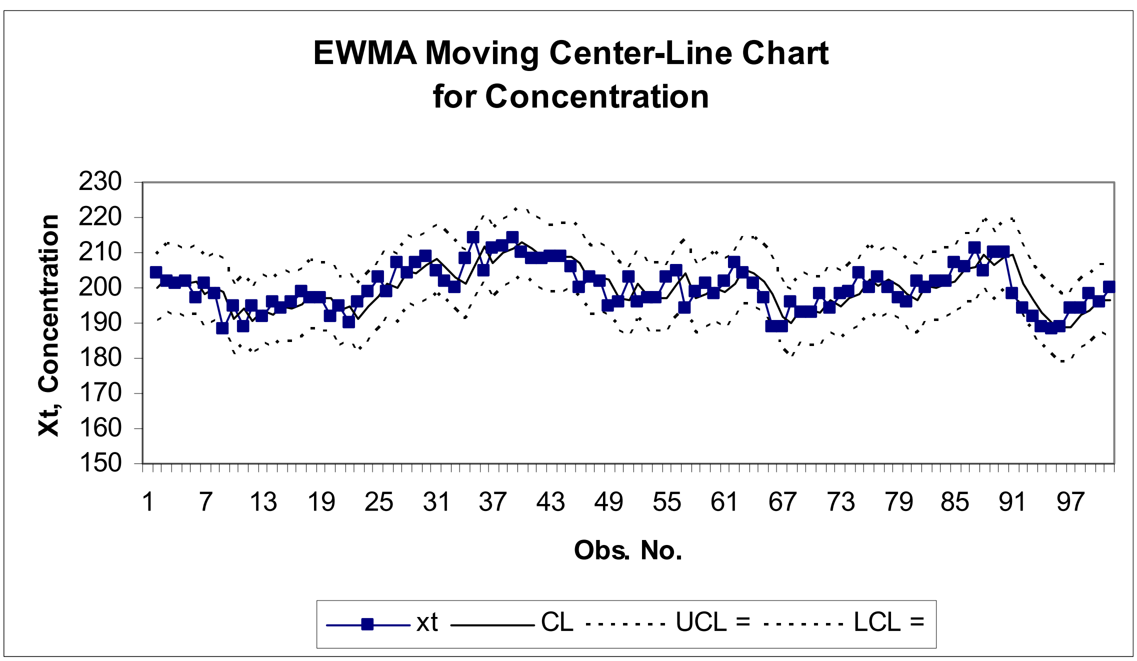
```
ARIMA Model: Ex9-20conc
…
Final Estimates of Parameters
Type           Coef   SE Coef      T      P
MA   1       0.2945    0.0975   3.02  0.003
Constant    -0.0452    0.3034  -0.15  0.882
…
```

$$\lambda = 1 - \text{MA1} = 1 - 0.2945 = 0.7055$$

$$\hat{\sigma} = \overline{\text{MR}}/d_2 = 3.64/1.128 = 3.227$$

Excel : Workbook Chap09.xls : Worksheet Ex9-23

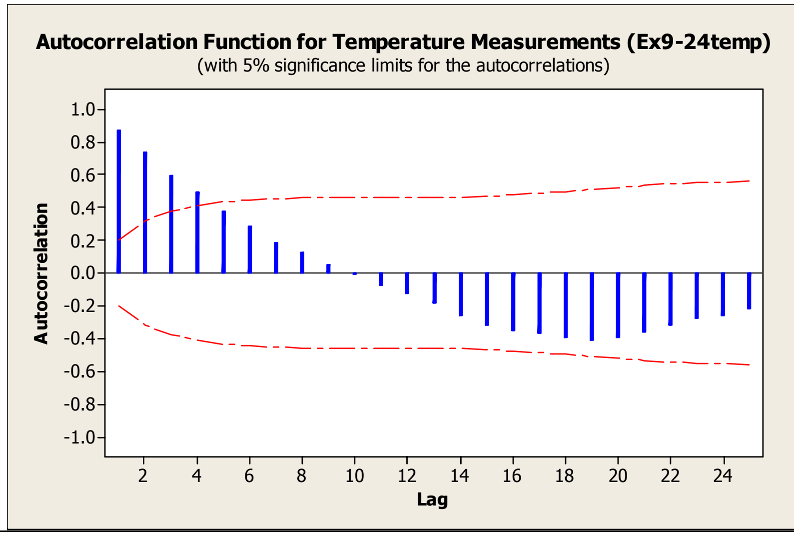
lamda =	0.706	sigma^ =	3.23			
t	xt	zt	CL	UCL =	LCL =	OOC?
0		200.010				
1	204	202.825	200.010	209.691	190.329	0
2	202	202.243	202.825	212.506	193.144	0
3	201	201.366	202.243	211.924	192.562	0
4	202	201.813	201.366	211.047	191.685	0
5	197	198.418	201.813	211.494	192.132	0
6	201	200.239	198.418	208.099	188.737	0
7	198	198.660	200.239	209.920	190.558	0
8	188	191.139	198.660	208.341	188.979	below LCL
9	195	193.863	191.139	200.820	181.458	0
10	189	190.432	193.863	203.544	184.182	0
…						



The control chart of concentration data signals out of control at three observations (8, 56, 90).

9-24.

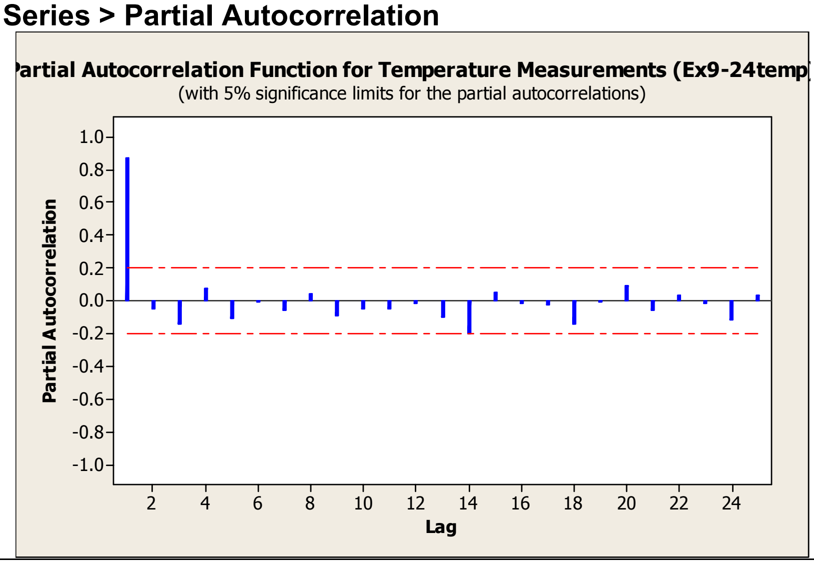
(a) **Stat > Time Series > Autocorrelation**



Autocorrelation Function: Ex9-24temp

Lag	ACF	T	LBQ
1	0.865899	8.66	77.25
2	0.737994	4.67	133.94
3	0.592580	3.13	170.86
4	0.489422	2.36	196.31
5	0.373763	1.71	211.31…

Stat > Time Series > Partial Autocorrelation



Partial Autocorrelation Function: Ex9-24temp

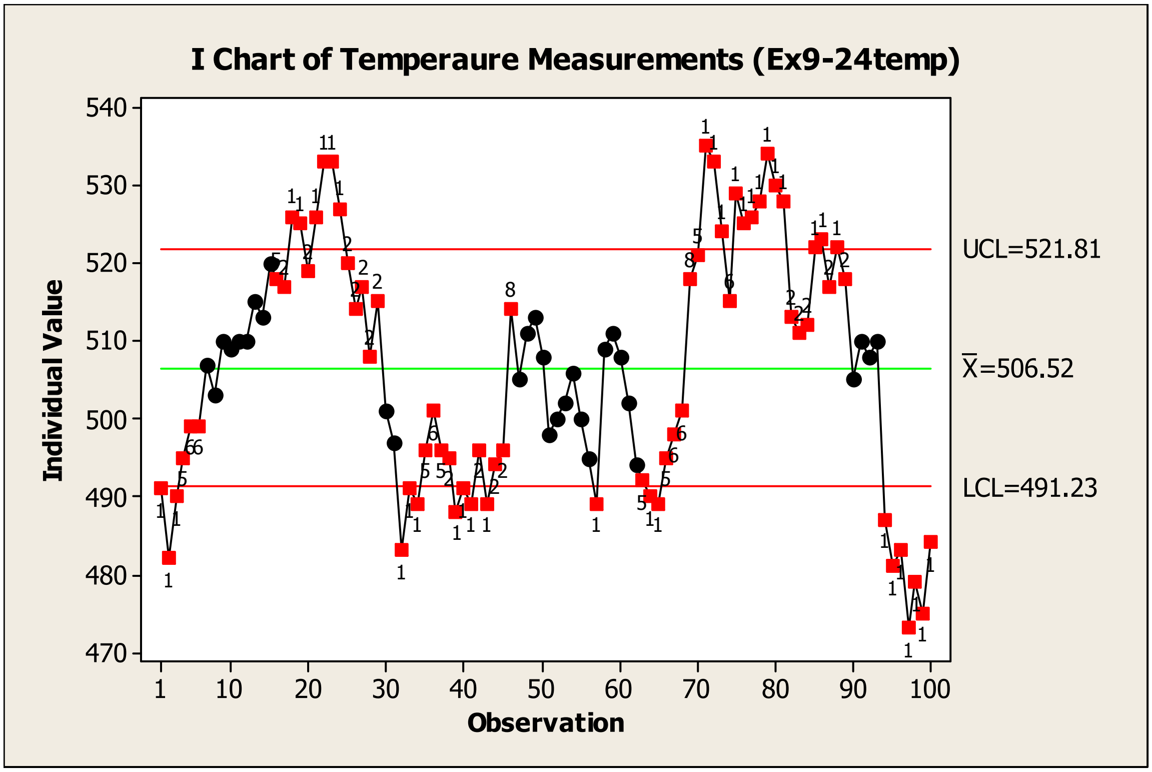
Lag	PACF	T
1	0.865899	8.66
2	-0.047106	-0.47
3	-0.143236	-1.43
4	0.078040	0.78
5	-0.112785	-1.13…

Slow decay of ACFs with sinusoidal wave indicates autoregressive process. PACF graph suggest order 1.

9-24 continued

(b)

Stat > Control Charts > Variables Charts for Individuals > Individuals




```
Test Results for I Chart of Ex9-24temp
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  1, 2, 3, 18, 19, 21, 22, 23, 24, 32, 33, 34, …

TEST 2. 9 points in a row on same side of center line.
Test Failed at points:  17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, …

TEST 3. 6 points in a row all increasing or all decreasing.
Test Failed at points:  65, 71

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  2, 3, 4, 16, 17, 18, 19, 20, 21, 22, 23, 24, …

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  4, 5, 6, 16, 17, 18, 19, 20, 21, 22, 23, 24, …

TEST 8. 8 points in a row more than 1 standard deviation from center line
     (above and below CL).
Test Failed at points:  20, 21, 22, 23, 24, 25, 26, 27, 36, 37, 38, 39, …
```

Process is out of control, violating many of the tests for special causes. The temperature measurements appear to wander over time.

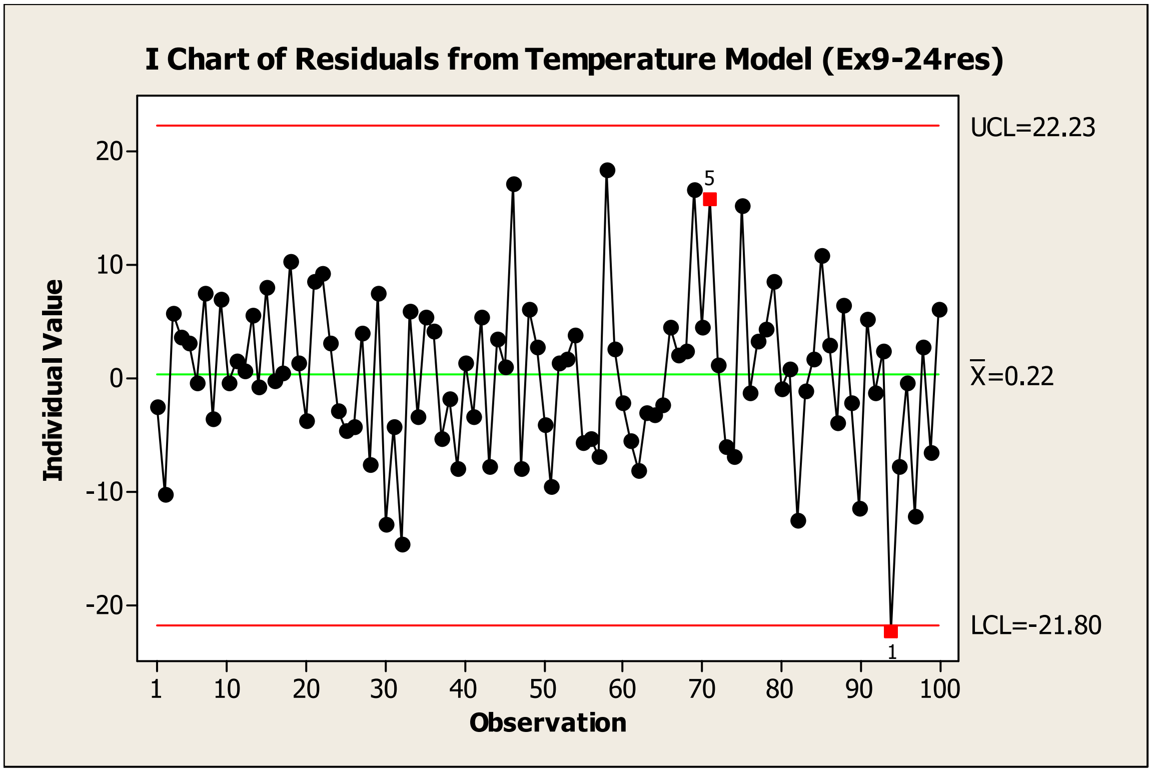
9-24 continued

(c) **Stat > Time Series > ARIMA**

ARIMA Model: Ex9-24temp

```
…
Final Estimates of Parameters
Type          Coef   SE Coef      T      P
AR    1     0.8960    0.0480  18.67  0.000
Constant   52.3794    0.7263  72.12  0.000
Mean       503.727     6.985
…
```

Stat > Control Charts > Variables Charts for Individuals > Individuals

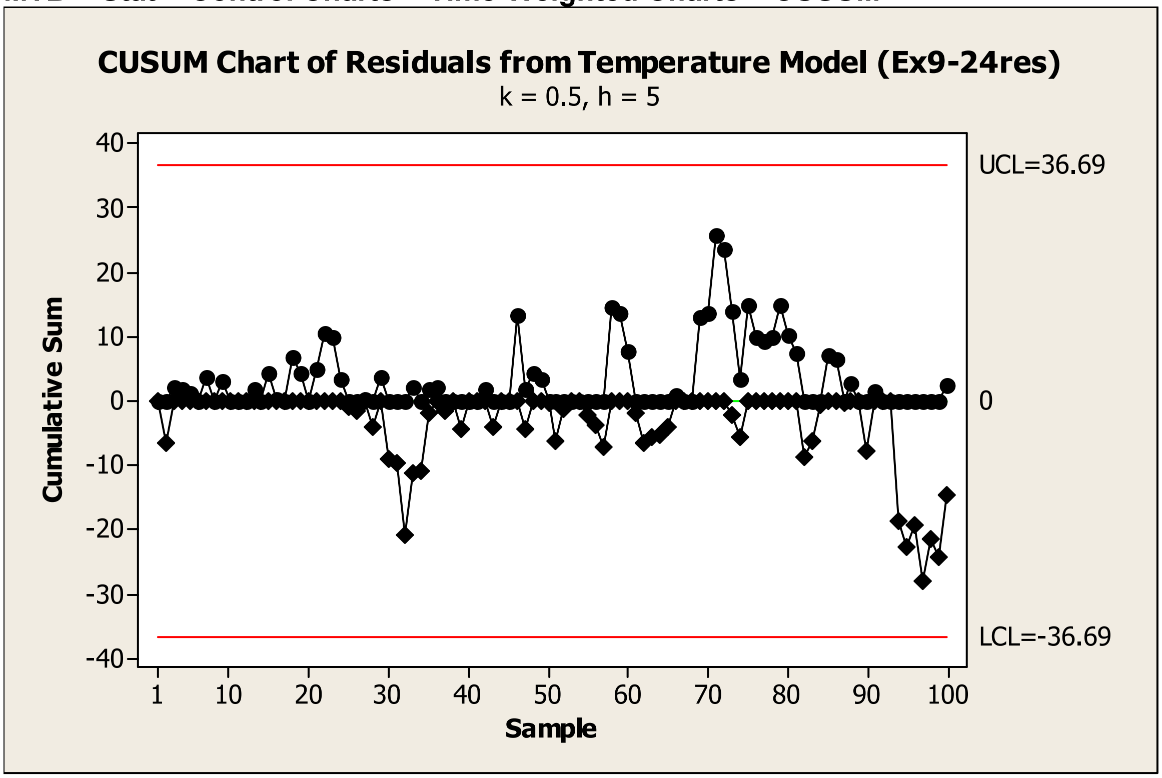


Test Results for I Chart of Ex9-24res

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  94
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  71
```

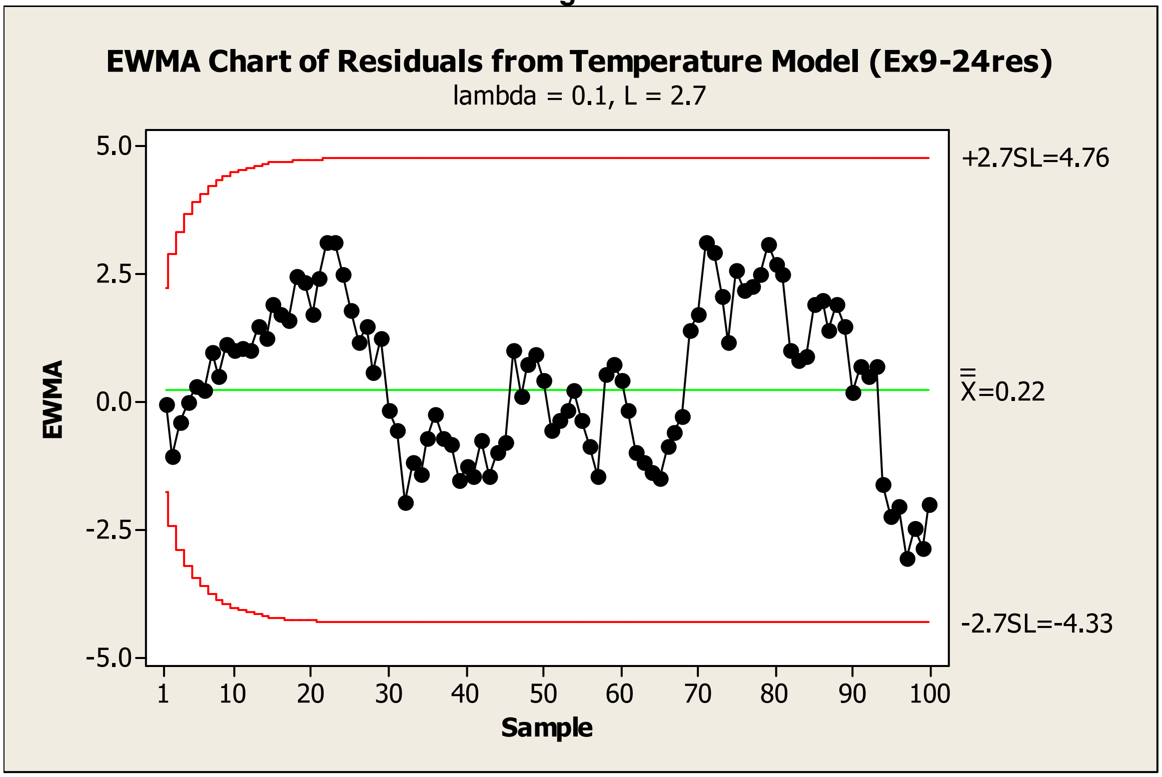
Observation 94 signals out of control above the upper limit, and observation 71 fails Test 5. The residuals do not exhibit cycles in the original temperature readings, and points are distributed between the control limits. The chemical process is in control.

9-25.
MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



No observations exceed the control limits. The residuals are in control, indicating the process is in control. This is the same conclusion as applying an Individuals control chart to the model residuals.

9-26.
MTB > Stat > Control Charts > Time-Weighted Charts > EWMA



No observations exceed the control limits. The residuals are in control, indicating the process is in control. This is the same conclusion as applying the Individuals and CUSUM control charts to the model residuals.

9-27.
To find the optimal λ, fit an ARIMA (0,1,1) (= EWMA = IMA(1,1)).

Stat > Time Series > ARIMA

ARIMA Model: Ex9-24temp

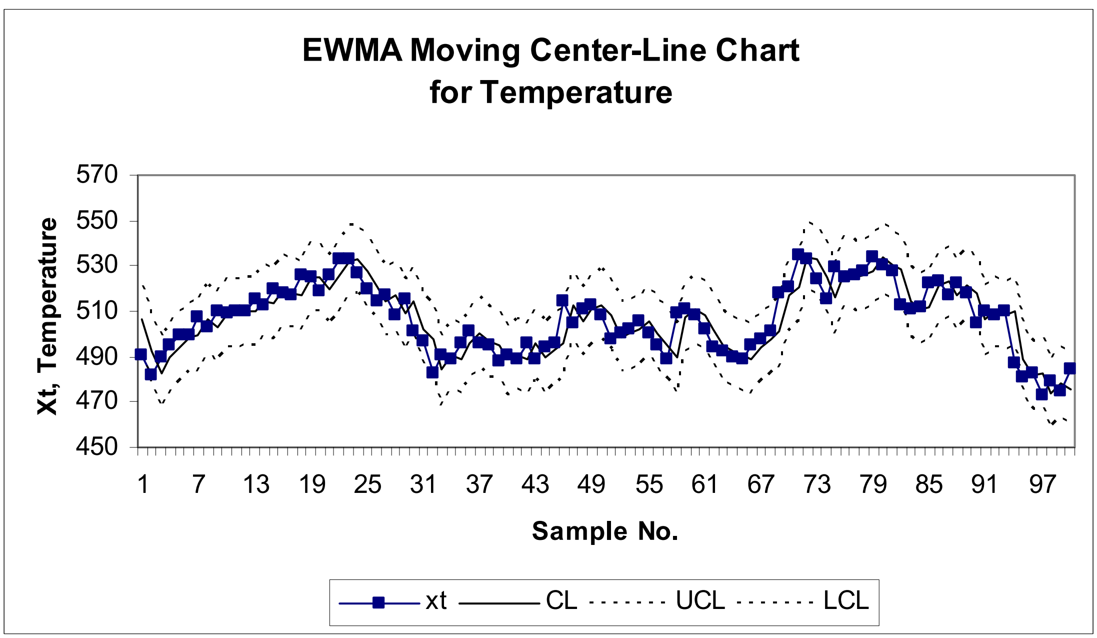
```
…
Final Estimates of Parameters
Type          Coef   SE Coef      T      P
MA   1      0.0794    0.1019   0.78  0.438
Constant   -0.0711    0.6784  -0.10  0.917
…
```

$\lambda = 1 - \text{MA1} = 1 - 0.0794 = 0.9206$

$\hat{\sigma} = \overline{\text{MR}}/d_2 = 5.75/1.128 = 5.0975$ (from a Moving Range chart with CL = 5.75)

Excel : Workbook Chap09.xls : Worksheet Ex9-27

		lambda =	0.921	sigma^ =	5.098	
t	xt	zt	CL	UCL	LCL	OOC?
0		506.520				
1	491	492.232	506.520	521.813	491.227	below LCL
2	482	482.812	492.232	507.525	476.940	0
3	490	489.429	482.812	498.105	467.520	0
4	495	494.558	489.429	504.722	474.137	0
5	499	498.647	494.558	509.850	479.265	0
6	499	498.972	498.647	513.940	483.355	0
7	507	506.363	498.972	514.265	483.679	0
8	503	503.267	506.363	521.655	491.070	0
9	510	509.465	503.267	518.560	487.974	0
10	509	509.037	509.465	524.758	494.173	0



A few observations exceed the upper limit (46, 58, 69) and the lower limit (1, 94), similar to the two out-of-control signals on the Individuals control chart (71, 94).

9-28.

(a)

When the data are positively autocorrelated, adjacent observations will tend to be similar, therefore making the moving ranges smaller. This would tend to produce an estimate of the process standard deviation that is too small.

(b)

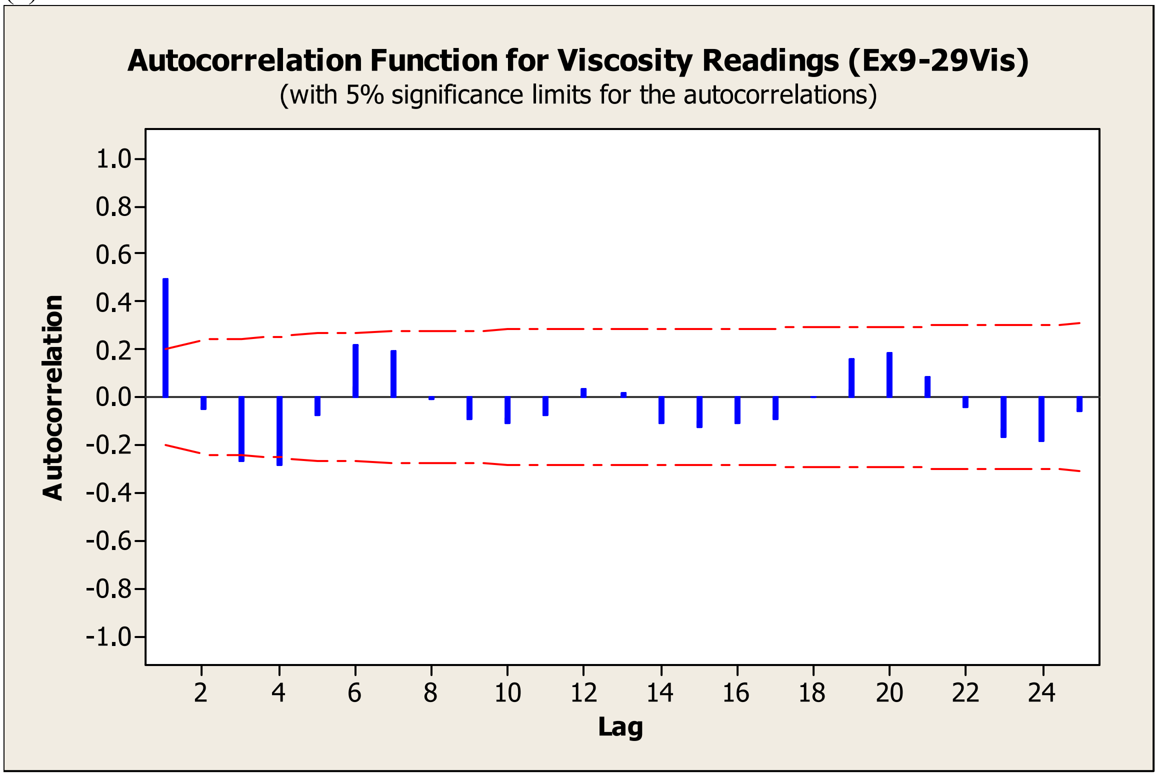
S^2 is still an unbiased estimator of σ^2 when the data are positively autocorrelated. There is nothing in the derivation of the expected value of $S^2 = \sigma^2$ that depends on an assumption of independence.

(c)

If assignable causes are present, it is not good practice to estimate σ^2 from S^2. Since it is difficult to determine whether a process generating autocorrelated data – or really any process – is in control, it is generally a bad practice to use S^2 to estimate σ^2.

9-29.

(a) **Stat > Time Series > Autocorrelation**



Autocorrelation Function: Ex9-29Vis

Lag	ACF	T	LBQ
1	0.494137	4.94	25.16
2	-0.049610	-0.41	25.41
3	-0.264612	-2.17	32.78
4	-0.283150	-2.22	41.29
5	-0.071963	-0.54	41.85

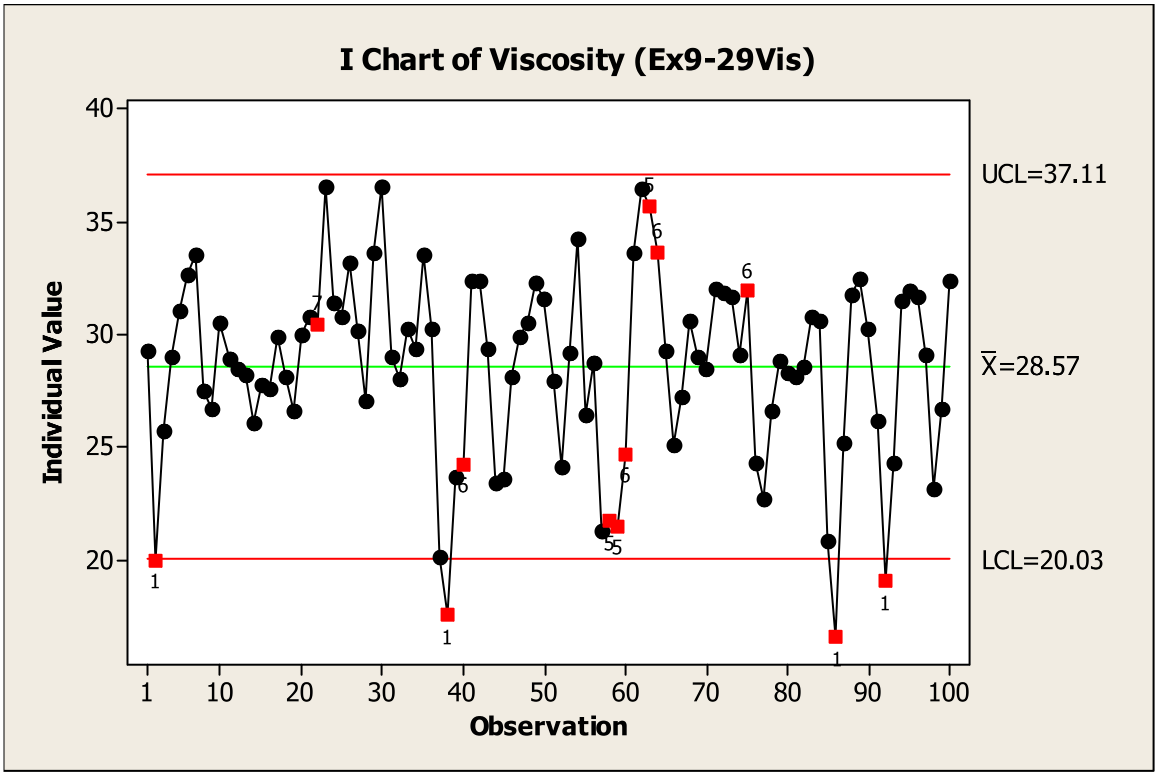
…

$r_1 = 0.49$, indicating a strong positive correlation at lag 1. There is a serious problem with autocorrelation in viscosity readings.

9-29 continued

(b)

Stat > Control Charts > Variables Charts for Individuals > Individuals



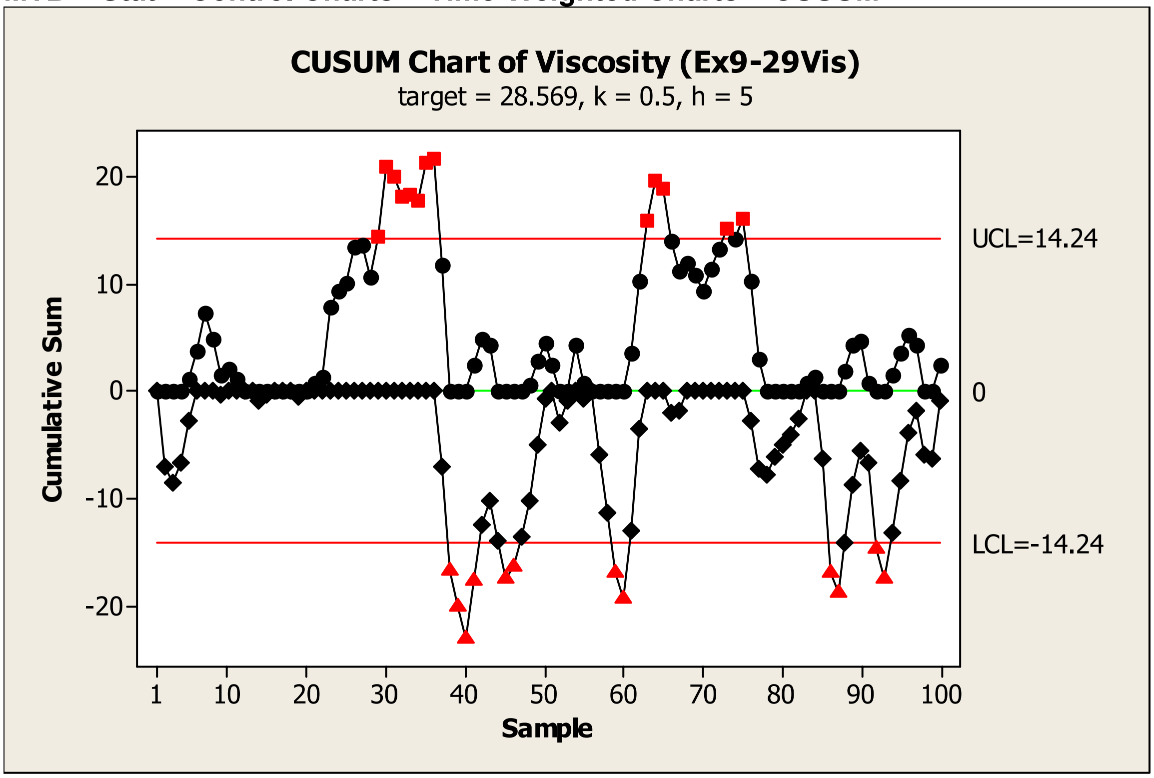
Test Results for I Chart of Ex9-29Vis

```
TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points:  2, 38, 86, 92
TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on
     one side of CL).
Test Failed at points:  38, 58, 59, 63, 86
TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on
     one side of CL).
Test Failed at points:  40, 60, 64, 75
TEST 7. 15 points within 1 standard deviation of center line (above and below
     CL).
Test Failed at points:  22
TEST 8. 8 points in a row more than 1 standard deviation from center line
     (above and below CL).
Test Failed at points:  64
```

Process is out of control, violating many of the tests for special causes. The viscosity measurements appear to wander over time.

9-29 continued
(c)
Let target = $\mu_0 = 28.569$

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM

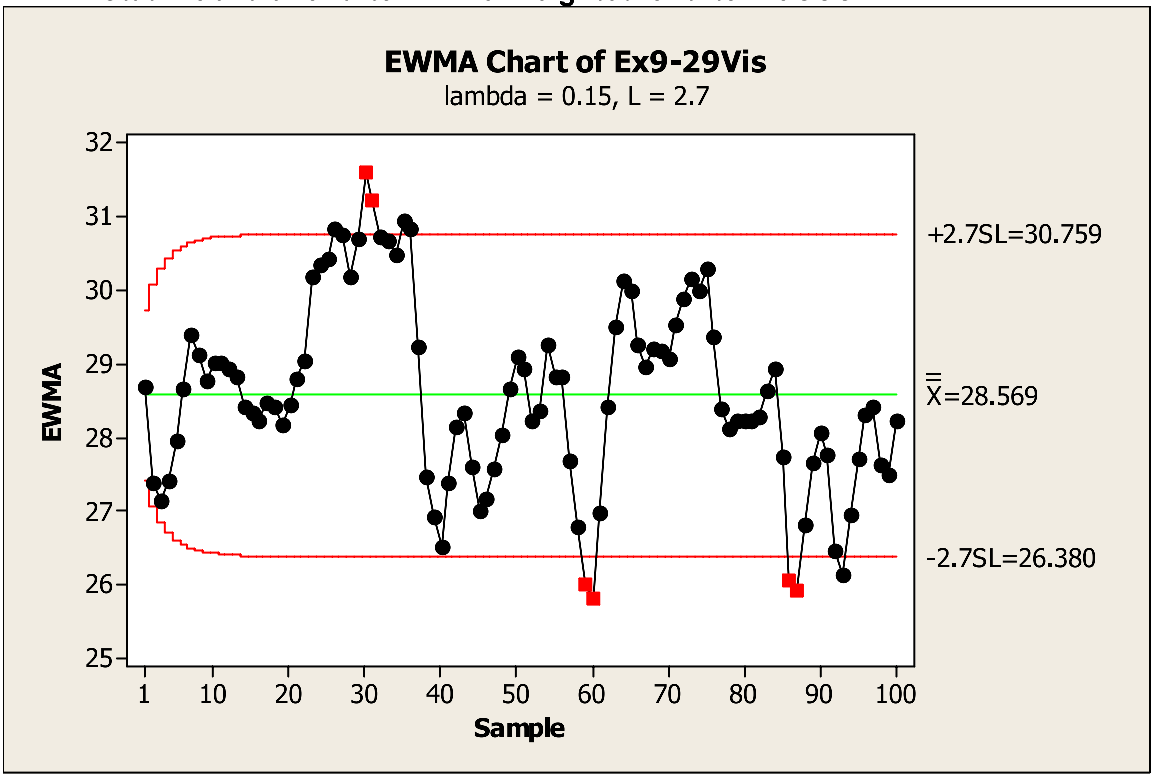


Several observations are out of control on both the lower and upper sides.

9-29 continued

(d)

MTB > Stat > Control Charts > Time-Weighted Charts > CUSUM



The process is not in control. There are wide swings in the plot points and several are beyond the control limits.

9-29 continued
(e)
To find the optimal λ, fit an ARIMA (0,1,1) (= EWMA = IMA(1,1)).

Stat > Time Series > ARIMA

ARIMA Model: Ex9-29Vis

```
...
Final Estimates of Parameters
Type          Coef  SE Coef      T      P
MA    1    -0.1579   0.1007  -1.57  0.120
Constant    0.0231   0.4839   0.05  0.962
```

$\lambda = 1 - \text{MA1} = 1 - (-0.1579) = 1.1579$

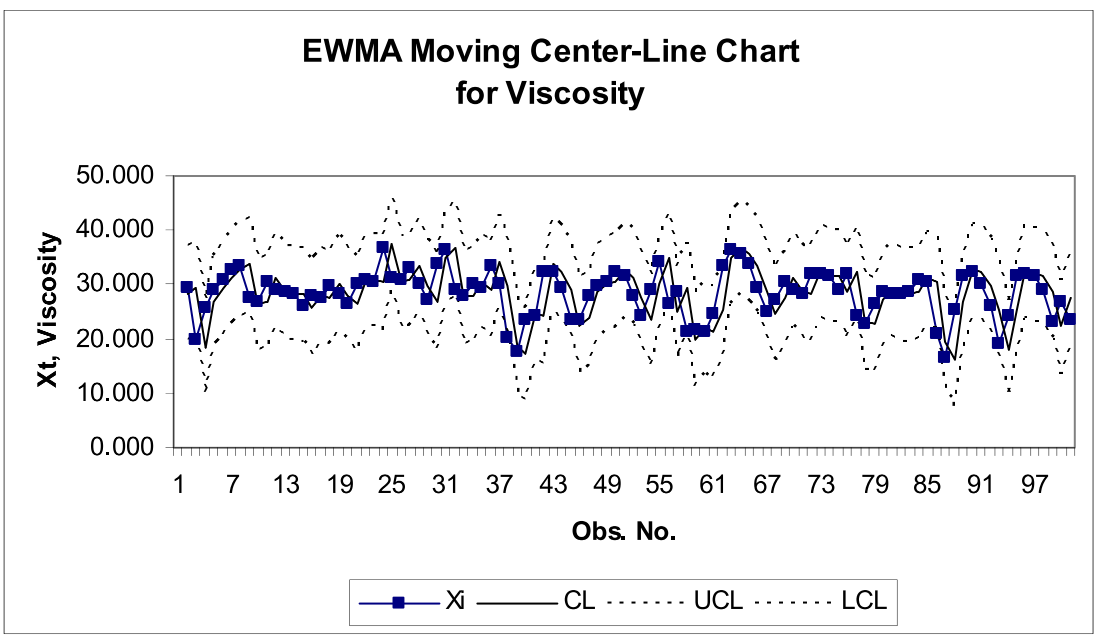
$\hat{\sigma} = \overline{\text{MR}}/d_2 = 3.21/1.128 = 2.8457$ (from a Moving Range chart with CL = 5.75)

Excel : Workbook Chap09.xls : Worksheet Ex9-29

lambda = 1.158 sigma^ = 2.85

I	Xi	Zi	CL	UCL	LCL	OOC?
0		28.479				
1	29.330	29.464	28.479	37.022	19.937	0
2	19.980	18.482	29.464	38.007	20.922	below LCL
3	25.760	26.909	18.482	27.025	9.940	0
4	29.000	29.330	26.909	35.452	18.367	0
5	31.030	31.298	29.330	37.873	20.788	0
6	32.680	32.898	31.298	39.841	22.756	0
7	33.560	33.665	32.898	41.441	24.356	0
8	27.500	26.527	33.665	42.207	25.122	0
9	26.750	26.785	26.527	35.069	17.984	0
10	30.550	31.144	26.785	35.328	18.243	0

...



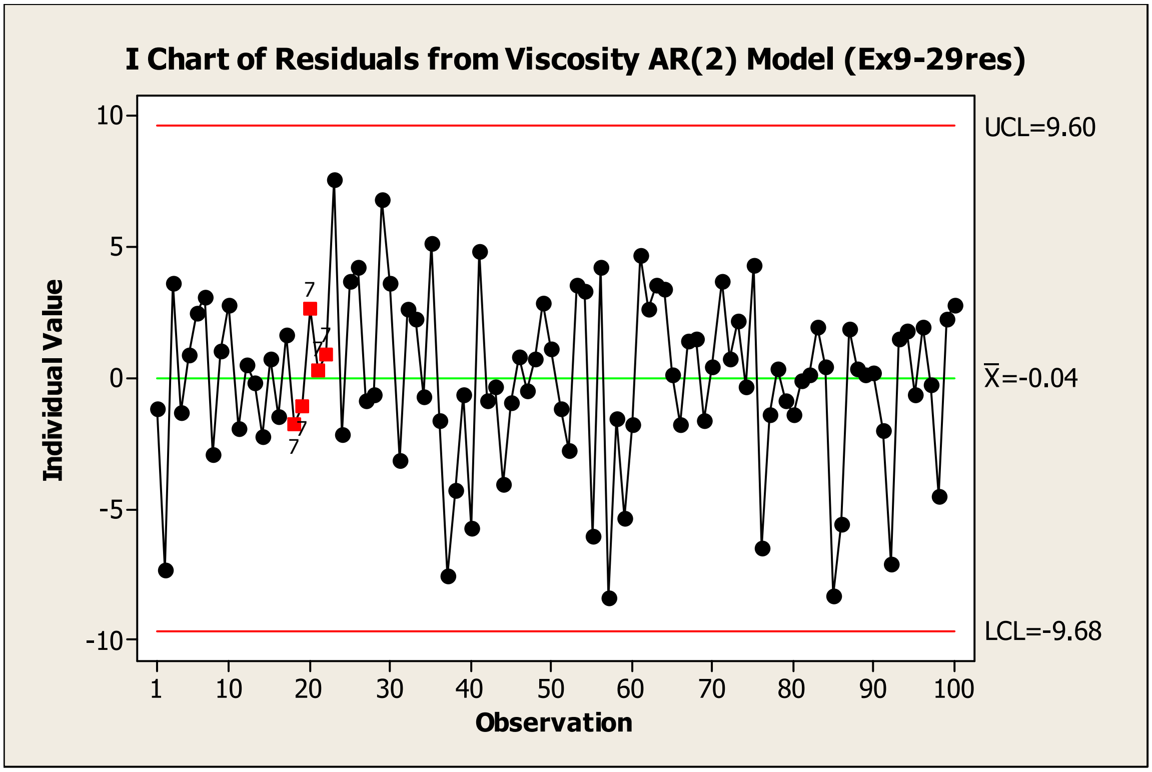
A few observations exceed the upper limit (87) and the lower limit (2, 37, 55, 85).

9-29 continued
(f)

Stat > Time Series > ARIMA
ARIMA Model: Ex9-29Vis

```
…
Final Estimates of Parameters
Type          Coef   SE Coef      T       P
AR    1     0.7193    0.0923   7.79   0.000
AR    2    -0.4349    0.0922  -4.72   0.000
Constant   20.5017    0.3278  62.54   0.000
Mean       28.6514    0.4581
…
```

Stat > Control Charts > Variables Charts for Individuals > Individuals



Test Results for I Chart of Ex9-29res

```
TEST 7. 15 points within 1 standard deviation of center line (above and below
     CL).
Test Failed at points:  18, 19, 20, 21, 22
```

The model residuals signal a potential issue with viscosity around observation 20. Otherwise the process appears to be in control, with a good distribution of points between the control limits and no patterns.

9-30.
$\lambda = 0.01$/hr or $1/\lambda = 100$hr; $\delta = 2.0$
a_1 = \$0.50/sample; a_2 = \$0.10/unit; a'_3 = \$5.00; a_3 = \$2.50; a_4 = \$100/hr
g = 0.05hr/sample; D = 2hr

(a)
Excel : workbook Chap09.xls : worksheet Ex9-30a
$n = 5$, $k = 3$, $h = 1$, $\alpha = 0.0027$

$$\beta = \Phi\left(\frac{(\mu_0 + k\,\sigma/\sqrt{n}) - (\mu_0 + 2\sigma)}{\sigma/\sqrt{n}}\right) - \Phi\left(\frac{(\mu_0 - k\,\sigma/\sqrt{n}) - (\mu_0 + 2\sigma)}{\sigma/\sqrt{n}}\right)$$
$$= \Phi\left(3 - 2\sqrt{5}\right) - \Phi\left(-3 - 2\sqrt{5}\right)$$
$$= \Phi(-1.472) - \Phi(-7.472)$$
$$= 0.0705 - 0.0000$$
$$= 0.0705$$
$$\tau \cong \frac{h}{2} - \frac{\lambda h^2}{12} = 0.4992$$
$$\frac{\alpha e^{-\lambda h}}{\left(1 - e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h} = 0.27$$
$E(L)$ = \$3.79/hr

(b)
$n = 3$, $k_{\text{opt}} = 2.210$, $h_{\text{opt}} = 1.231$, $\alpha = 0.027$, $1 - \beta = 0.895$
$E(L)$ = \$3.6098/hr

9-31.

$\lambda = 0.01$/hr or $1/\lambda = 100$hr; $\delta = 2.0$

a_1 = \$0.50/sample; a_2 = \$0.10/unit; a'_3 = \$50; a_3 = \$25; a_4 = \$100/hr

g = 0.05hr/sample; D = 2hr

(a)

Excel : workbook Chap09.xls : worksheet Ex9-31

$n = 5$, $k = 3$, $h = 1$, $\alpha = 0.0027$

$$\beta = \Phi\left(\frac{(\mu_0 + k\sigma/\sqrt{n}) - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n}}\right) - \Phi\left(\frac{(\mu_0 - k\sigma/\sqrt{n}) - (\mu_0 + \delta\sigma)}{\sigma/\sqrt{n}}\right)$$

$$= \Phi\left(k - \delta\sqrt{n}\right) - \Phi\left(-k - \delta\sqrt{n}\right)$$

$$= \Phi\left(3 - 2\sqrt{5}\right) - \Phi\left(-3 - 2\sqrt{5}\right)$$

$$= \Phi(-1.472) - \Phi(-7.472)$$

$$= 0.0705 - 0.0000$$

$$= 0.0705$$

$$\tau \cong \frac{h}{2} - \frac{\lambda h^2}{12} = \frac{1}{2} - \frac{0.01(1^2)}{12} = 0.4992$$

$$\frac{\alpha e^{-\lambda h}}{\left(1 - e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h} = \frac{0.0027}{0.01(1)} = 0.27$$

$E(L)$ = \$4.12/hr

(b)

$n = 5$, $k = 3$, $h = 0.5$, $\alpha = 0.0027$, $\beta = 0.0705$

$$\tau \cong \frac{h}{2} - \frac{\lambda h^2}{12} = \frac{0.5}{2} - \frac{0.01(0.5^2)}{12} = 0.2498$$

$$\frac{\alpha e^{-\lambda h}}{\left(1 - e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h} = \frac{0.0027}{0.01(0.5)} = 0.54$$

$E(L)$ = \$4.98/hr

(c)

$n = 5$, $k_{\text{opt}} = 3.080$, $h_{\text{opt}} = 1.368$, $\alpha = 0.00207$, $1 - \beta = 0.918$

$E(L)$ = \$4.01392/hr

9-32.
Excel : workbook Chap09.xls : worksheet Ex9-32
D_0 = 2hr, D_1 = 2hr
V_0 = \$500, Δ = \$25
$n = 5$, $k = 3$, $h = 1$, $\alpha = 0.0027$, $\beta = 0.0705$
$E(L)$ = \$13.16/hr

9-33.
Excel : workbook Chap09.xls : worksheet Ex9-33
$\lambda = 0.01$/hr or $1/\lambda = 100$hr
$\delta = 2.0$
a_1 = \$2/sample
a_2 = \$0.50/unit
a'_3 = \$75
a_3 = \$50
a_4 = \$200/hr
g = 0.05 hr/sample
D = 1 hr

(a)
$n = 5$, $k = 3$, $h = 0.5$, $\alpha = 0.0027$

$$\begin{aligned}\beta &= \Phi\left(k-\delta\sqrt{n}\right)-\Phi\left(-k-\delta\sqrt{n}\right)\\ &= \Phi\left(3-1\sqrt{5}\right)-\Phi\left(-3-1\sqrt{5}\right)\\ &= \Phi(-1.472)-\Phi(-7.472)\\ &= 0.775-0.0000\\ &= 0.775\end{aligned}$$

$$\tau \cong \frac{h}{2}-\frac{\lambda h^2}{12}=\frac{0.5}{2}-\frac{0.01(0.5^2)}{12}=0.2498$$

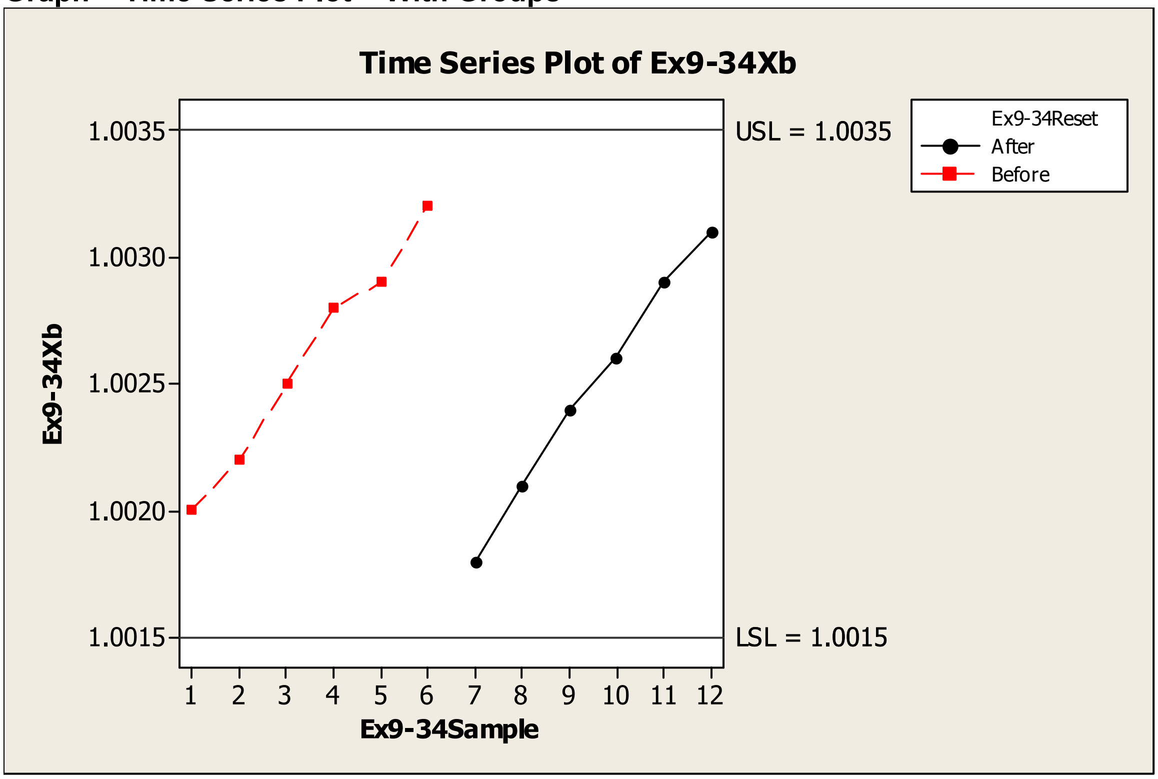
$$\frac{\alpha e^{-\lambda h}}{\left(1-e^{-\lambda h}\right)} \cong \frac{\alpha}{\lambda h}=\frac{0.0027}{0.01(0.5)}=0.54$$

$E(L)$ = \$16.17/hr

(b)
$n = 10$, $k_{\text{opt}} = 2.240$, $h_{\text{opt}} = 2.489018$, $\alpha = 0.025091$, $1 - \beta = 0.8218083$
$E(L)$ = \$10.39762/hr

9-34.
It is good practice visually examine data in order to understand the type of tool wear occurring. The plot below shows that the tool has been reset to approximately the same level as initially and the rate of tool wear is approximately the same after reset.

Graph > Time Series Plot > With Groups



$n = 5;\ \bar{R} = 0.00064;\ \hat{\sigma} = \bar{R}/d_2 = 0.00064/2.326 = 0.00028$

CL = $\bar{R}$ = 0.00064, UCL = $D_4\bar{R}$ = 2.114(0.00064) = 0.00135, LCL = 0

$\bar{x}$ chart initial settings:
CL = LSL + 3σ = 1.0015 + 3(0.00028) = 1.00234
$\text{UCL} = \text{CL} + 3\sigma_{\bar{x}} = 1.00234 + 3\left(0.00028/\sqrt{5}\right) = 1.00272$

$\text{LCL} = \text{CL} - 3\sigma_{\bar{x}} = 1.00234 - 3\left(0.00028/\sqrt{5}\right) = 1.00196$

$\bar{x}$ chart at tool reset:
CL = USL – 3σ = 1.0035 – 3(0.00028) = 1.00266 (maximum permissible average)
$\text{UCL} = \text{CL} + 3\sigma_{\bar{x}} = 1.00266 + 3\left(0.00028/\sqrt{5}\right) = 1.00304$

$\text{LCL} = \text{CL} - 3\sigma_{\bar{x}} = 1.00266 - 3\left(0.00028/\sqrt{5}\right) = 1.00228$