# Chapter 2 Exercise Solutions

Several exercises in this chapter differ from those in the 4th edition. An "*" following the exercise number indicates that the description has changed (e.g., new values). A second exercise number in parentheses indicates that the exercise number has changed. For example, "2-16* (2-9)" means that exercise 2-16 was 2-9 in the 4th edition, and that the description also differs from the 4th edition (in this case, asking for a time series plot instead of a digidot plot). New exercises are denoted with an "☺".

2-1*.

(a)

$$\bar{x} = \sum_{i=1}^{n} x_i \Big/ n = \left(16.05 + 16.03 + \cdots + 16.07\right) \Big/ 12 = 16.029 \text{ oz}$$

(b)

$$s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 \Big/ n}{n-1}} = \sqrt{\frac{(16.05^2 + \cdots + 16.07^2) - (16.05 + \cdots + 16.07)^2 / 12}{12-1}} = 0.0202 \text{ oz}$$

**MTB > Stat > Basic Statistics > Display Descriptive Statistics**

**Descriptive Statistics: Ex2-1**

```
Variable   N  N*    Mean  SE Mean   StDev  Minimum      Q1  Median      Q3
Ex2-1     12   0  16.029  0.00583  0.0202   16.000  16.013  16.025  16.048
Variable  Maximum
Ex2-1      16.070
```

2-2.

(a)

$$\bar{x} = \sum_{i=1}^{n} x_i \Big/ n = \left(50.001 + 49.998 + \cdots + 50.004\right) \Big/ 8 = 50.002 \text{ mm}$$

(b)

$$s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 \Big/ n}{n-1}} = \sqrt{\frac{(50.001^2 + \cdots + 50.004^2) - (50.001 + \cdots + 50.004)^2 / 8}{8-1}} = 0.003 \text{ mm}$$

**MTB > Stat > Basic Statistics > Display Descriptive Statistics**

**Descriptive Statistics: Ex2-2**

```
Variable  N  N*    Mean  SE Mean    StDev  Minimum      Q1  Median      Q3
Ex2-2     8   0  50.002  0.00122  0.00344   49.996  49.999  50.003  50.005
Variable  Maximum
Ex2-2      50.006
```

2-3.

(a)

$$\bar{x} = \sum_{i=1}^{n} x_i \Big/ n = (953 + 955 + \cdots + 959)\big/9 = 952.9 \text{ °F}$$

(b)

$$s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 \Big/ n}{n-1}} = \sqrt{\frac{(953^2 + \cdots + 959^2) - (953 + \cdots + 959)^2/9}{9-1}} = 3.7 \text{ °F}$$

**MTB > Stat > Basic Statistics > Display Descriptive Statistics**

**Descriptive Statistics: Ex2-3**

| Variable | N | N* | Mean | SE Mean | StDev | Minimum | Q1 | Median | Q3 |
|---|---|---|---|---|---|---|---|---|---|
| Ex2-3 | 9 | 0 | 952.89 | 1.24 | 3.72 | 948.00 | 949.50 | 953.00 | 956.00 |

| Variable | Maximum |
|---|---|
| Ex2-3 | 959.00 |

2-4.

(a)

In ranked order, the data are {948, 949, 950, 951, **953**, 954, 955, 957, 959}. The sample median is the middle value.

(b)

Since the median is the value dividing the ranked sample observations in half, it remains the same regardless of the size of the largest measurement.

2-5.

**MTB > Stat > Basic Statistics > Display Descriptive Statistics**

**Descriptive Statistics: Ex2-5**

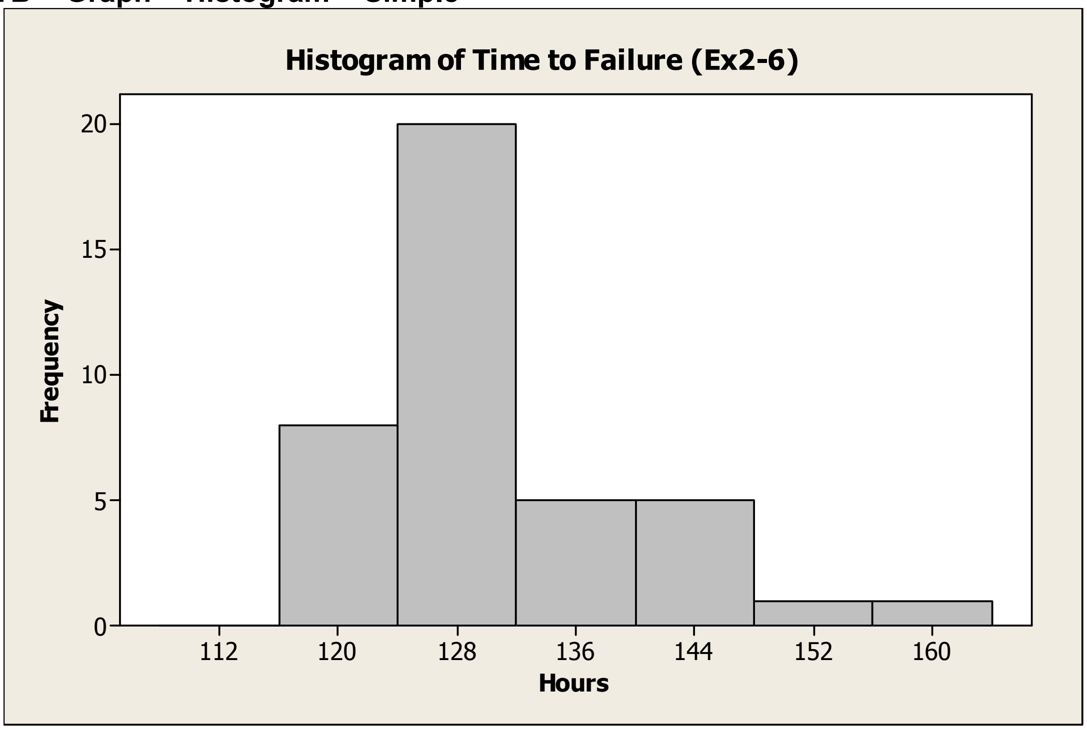
| Variable | N | N* | Mean | SE Mean | StDev | Minimum | Q1 | Median | Q3 |
|---|---|---|---|---|---|---|---|---|---|
| Ex2-5 | 8 | 0 | 121.25 | 8.00 | 22.63 | 96.00 | 102.50 | 117.00 | 144.50 |

| Variable | Maximum |
|---|---|
| Ex2-5 | 156.00 |

2-6.
(a), (d)
**MTB > Stat > Basic Statistics > Display Descriptive Statistics**

**Descriptive Statistics: Ex2-6**
```
Variable   N  N*    Mean  SE Mean  StDev  Minimum      Q1  Median      Q3
Ex2-6     40   0  129.98     1.41   8.91   118.00  124.00  128.00  135.25
Variable  Maximum
Ex2-6      160.00
```

(b)
Use $\sqrt{n} = \sqrt{40} \cong 7$ bins
**MTB > Graph > Histogram > Simple**



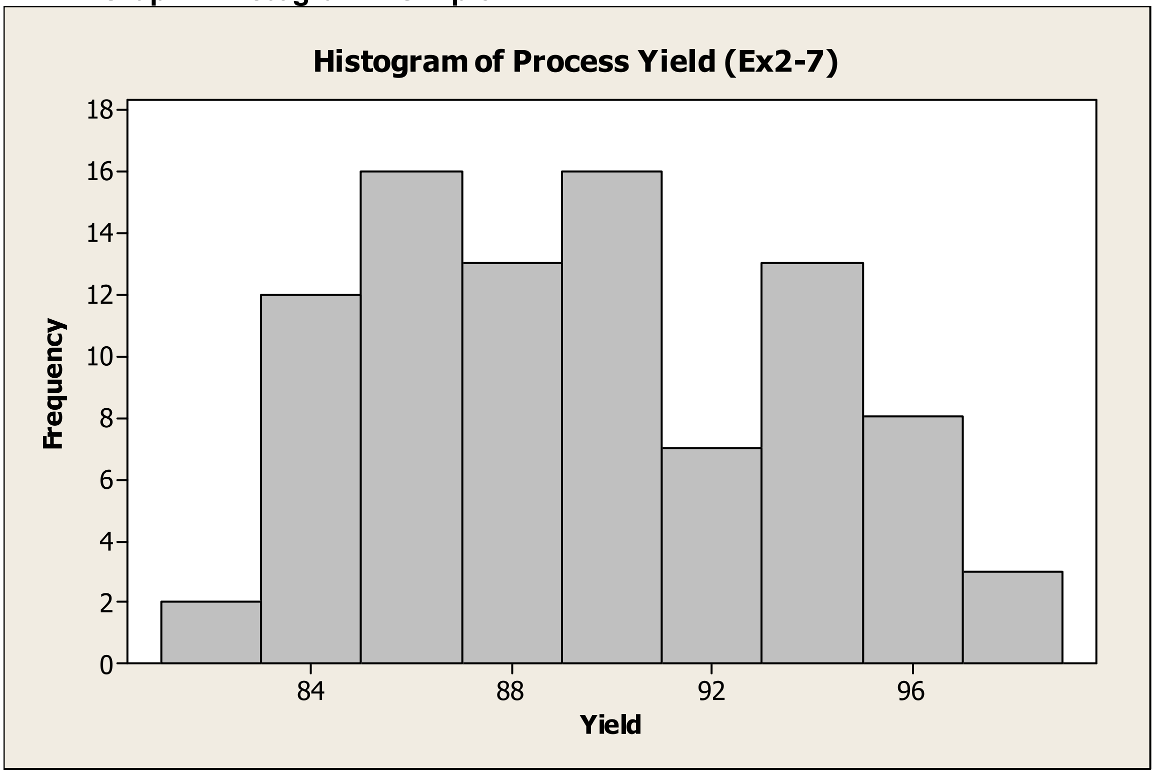

(c)
**MTB > Graph > Stem-and-Leaf**

**Stem-and-Leaf Display: Ex2-6**
```
Stem-and-leaf of Ex2-6  N  = 40
Leaf Unit = 1.0
 2   11  89
 5   12  011
 8   12  233
 17  12  444455555
 19  12  67
(5)  12  88999
 16  13  0111
 12  13  33
 10  13
 10  13  677
 7   13
 7   14  001
 4   14  22
HI 151, 160
```

2-7.

Use $\sqrt{n} = \sqrt{90} \cong 9$ bins

**MTB > Graph > Histogram > Simple**



2-8.

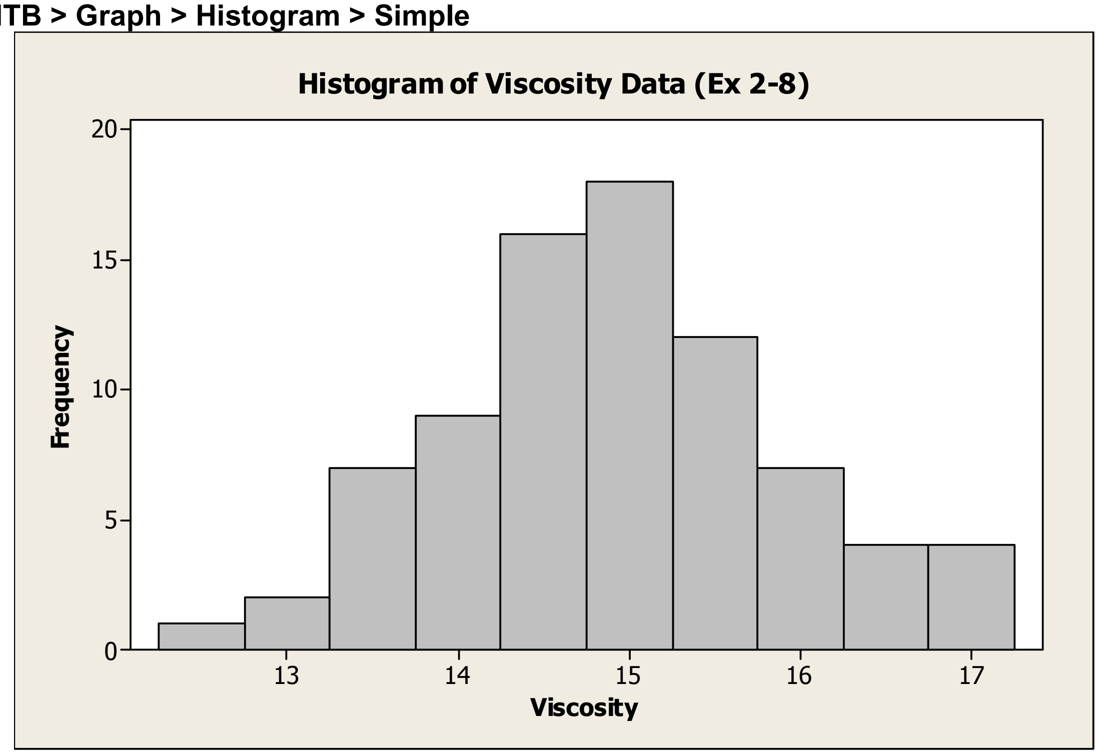
(a)

```
    Stem-and-Leaf Plot
  2    12o|68
  6    13*|3134
 12    13o|776978
 28    14*|3133101332423404
(15)   14o|585669589889695
 37    15*|3324223422112232
 21    15o|568987666
 12    16*|144011
  6    16o|85996
  1    17*|0
Stem  Freq|Leaf
```

(b)

Use $\sqrt{n} = \sqrt{80} \cong 9$ bins

**MTB > Graph > Histogram > Simple**




Note that the histogram has 10 bins. The number of bins can be changed by editing the X scale. However, if 9 bins are specified, MINITAB generates an 8-bin histogram. Constructing a 9-bin histogram requires manual specification of the bin cut points. Recall that this formula is an approximation, and therefore either 8 or 10 bins should suffice for assessing the distribution of the data.

2-8(c) continued

**MTB > %hbins 12.5 17 .5 c7**

| Row | Intervals | Frequencies | Percents |
|---|---|---|---|
| 1 | 12.25 to 12.75 | 1 | 1.25 |
| 2 | 12.75 to 13.25 | 2 | 2.50 |
| 3 | 13.25 to 13.75 | 7 | 8.75 |
| 4 | 13.75 to 14.25 | 9 | 11.25 |
| 5 | 14.25 to 14.75 | 16 | 20.00 |
| 6 | 14.75 to 15.25 | 18 | 22.50 |
| 7 | 15.25 to 15.75 | 12 | 15.00 |
| 8 | 15.75 to 16.25 | 7 | 8.75 |
| 9 | 16.25 to 16.75 | 4 | 5.00 |
| 10 | 16.75 to 17.25 | 4 | 5.00 |
| 11 | Totals | 80 | 100.00 |

(d)

**MTB > Graph > Stem-and-Leaf**

**Stem-and-Leaf Display: Ex2-8**

```
Stem-and-leaf of Ex2-8  N  = 80
Leaf Unit = 0.10
 2    12  68
 6    13  1334
 12   13  677789
 28   14  0011122333333444
(15)  14  555566688889999
 37   15  1122222222333344
 21   15  566667889
 12   16  011144
 6    16  56899
 1    17  0
```

median observation rank is (0.5)(80) + 0.5 = 40.5

$x_{0.50}$ = (14.9 + 14.9)/2 = 14.9

Q1 observation rank is (0.25)(80) + 0.5 = 20.5

Q1 = (14.3 + 14.3)/2 = 14.3

Q3 observation rank is (0.75)(80) + 0.5 = 60.5

Q3 = (15.6 + 15.5)/2 = 15.55

(d)

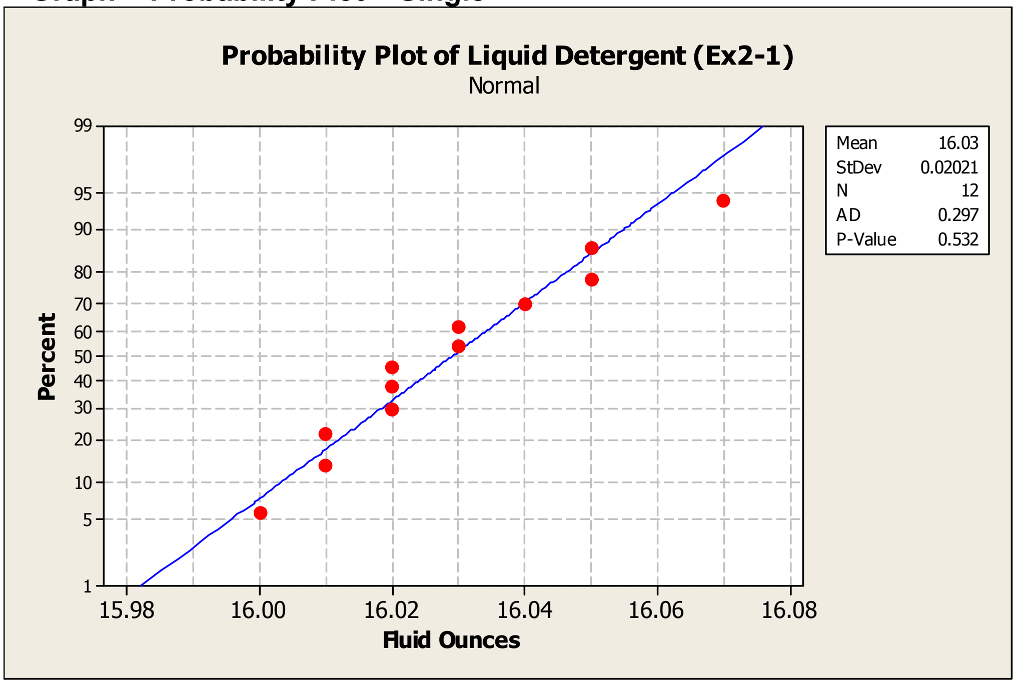
10$^{th}$ percentile observation rank = (0.10)(80) + 0.5 = 8.5

$x_{0.10}$ = (13.7 + 13.7)/2 = 13.7

90$^{th}$ percentile observation rank is (0.90)(80) + 0.5 = 72.5

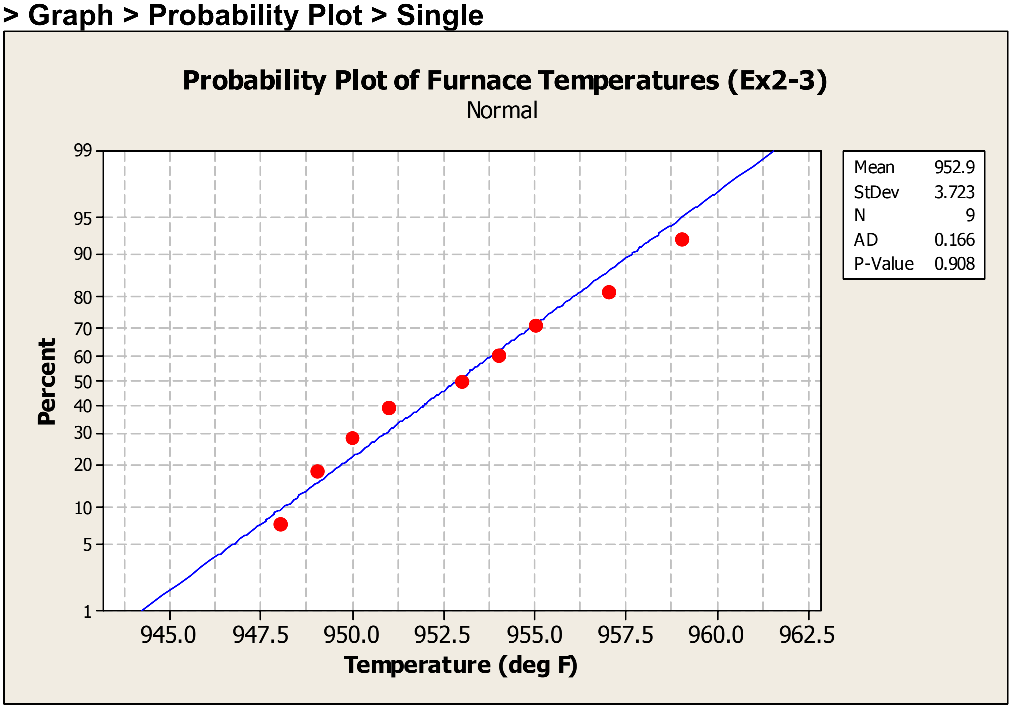
$x_{0.90}$ = (16.4 + 16.1)/2 = 16.25

2-9 ☺.
**MTB > Graph > Probability Plot > Single**




When plotted on a normal probability plot, the data points tend to fall along a straight line, indicating that a normal distribution adequately describes the volume of detergent.

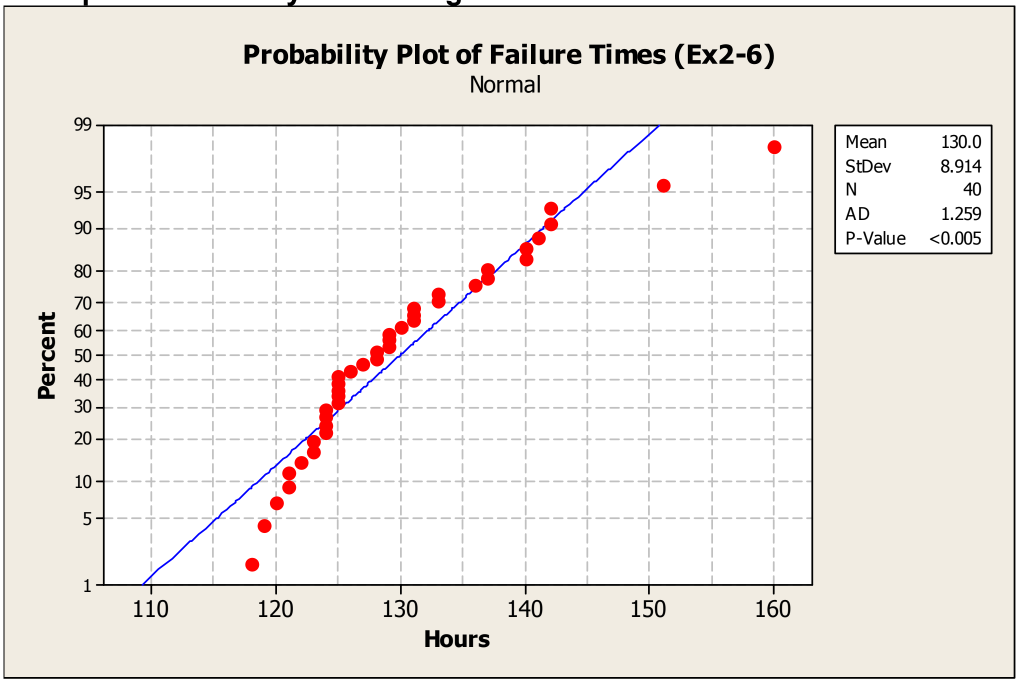
2-10 ☺.
**MTB > Graph > Probability Plot > Single**




When plotted on a normal probability plot, the data points tend to fall along a straight line, indicating that a normal distribution adequately describes the furnace temperatures.

2-11 ☺.

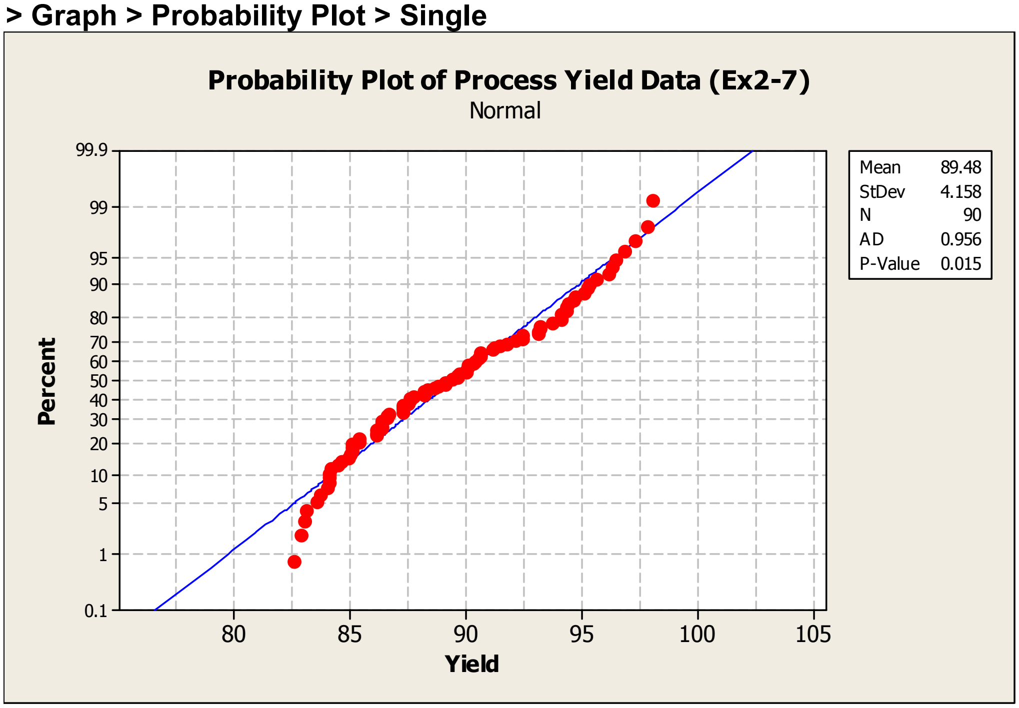
**MTB > Graph > Probability Plot > Single**




When plotted on a normal probability plot, the data points do not fall along a straight line, indicating that the normal distribution does not reasonably describe the failure times.

2-12 ☺.

**MTB > Graph > Probability Plot > Single**

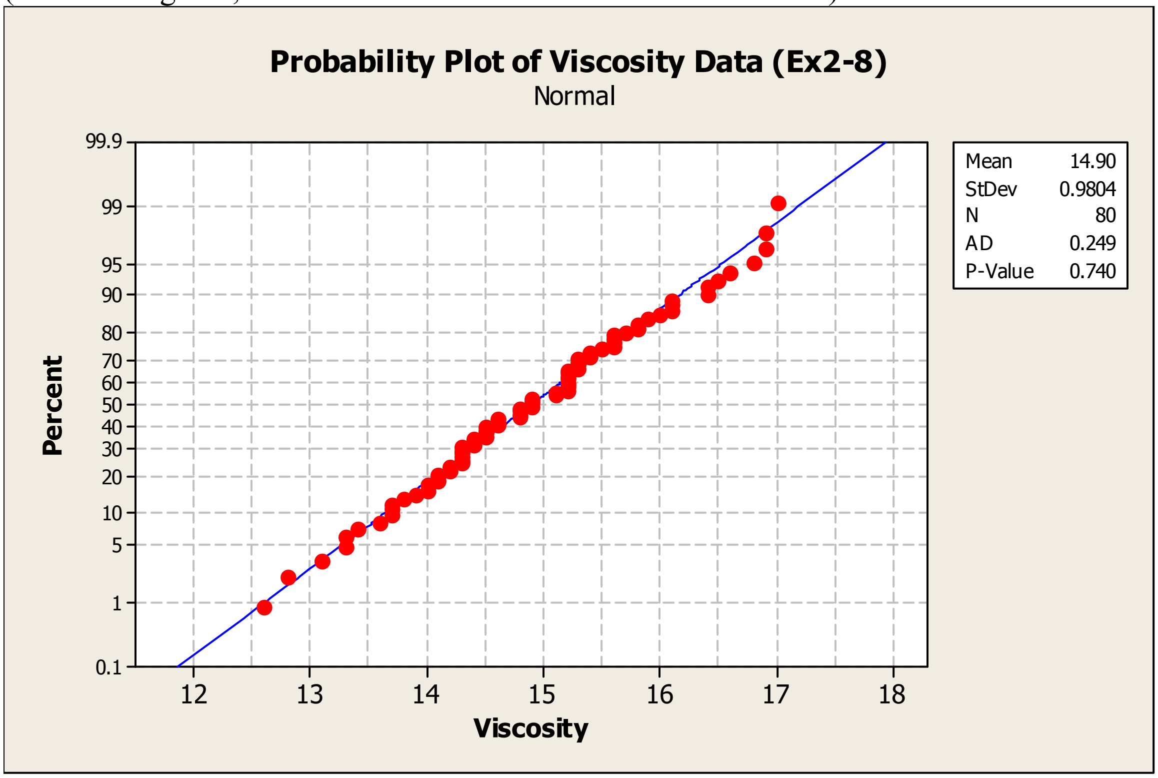


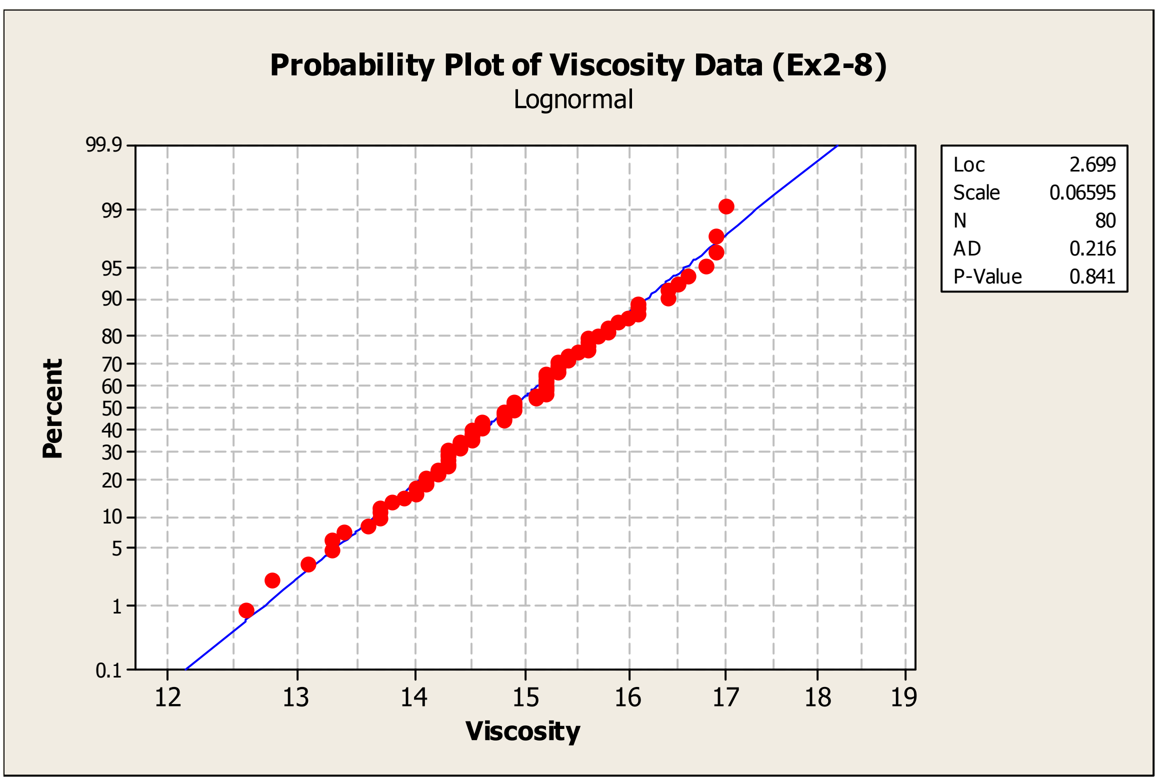

When plotted on a normal probability plot, the data points do not fall along a straight line, indicating that the normal distribution does not reasonably describe process yield.

2-13 ☺.

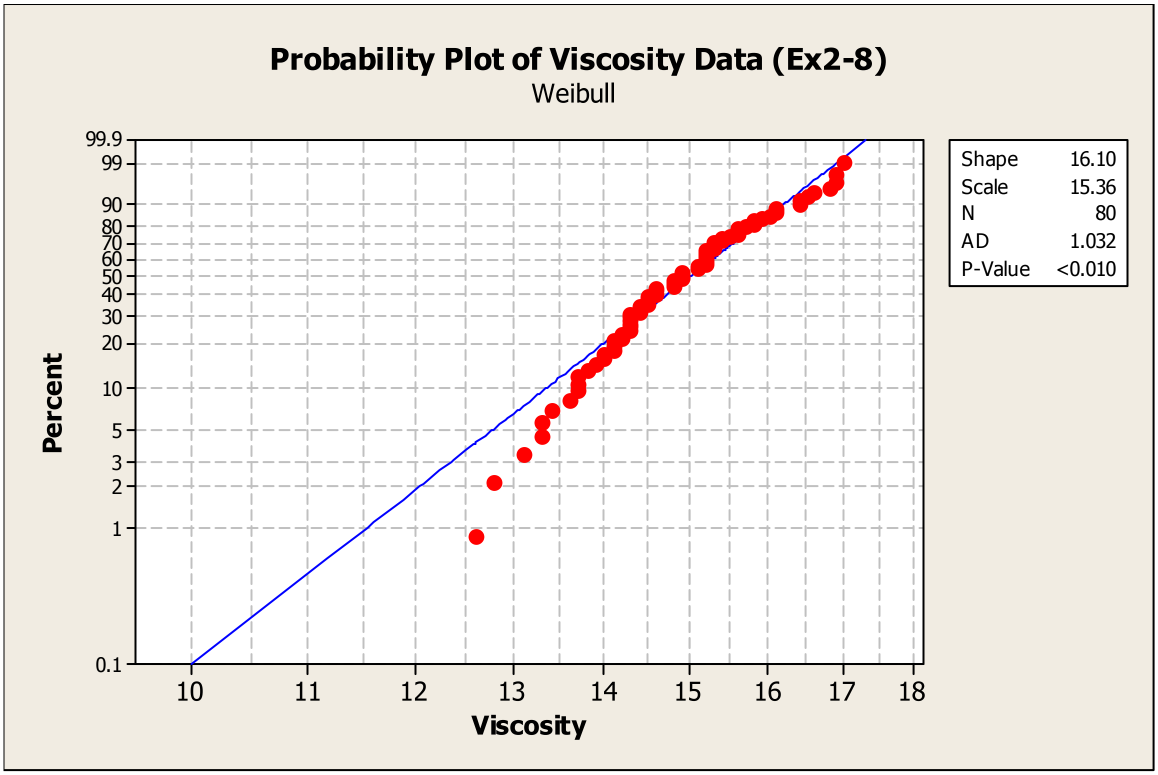
**MTB > Graph > Probability Plot > Single**

(In the dialog box, select Distribution to choose the distributions)



2-13 continued

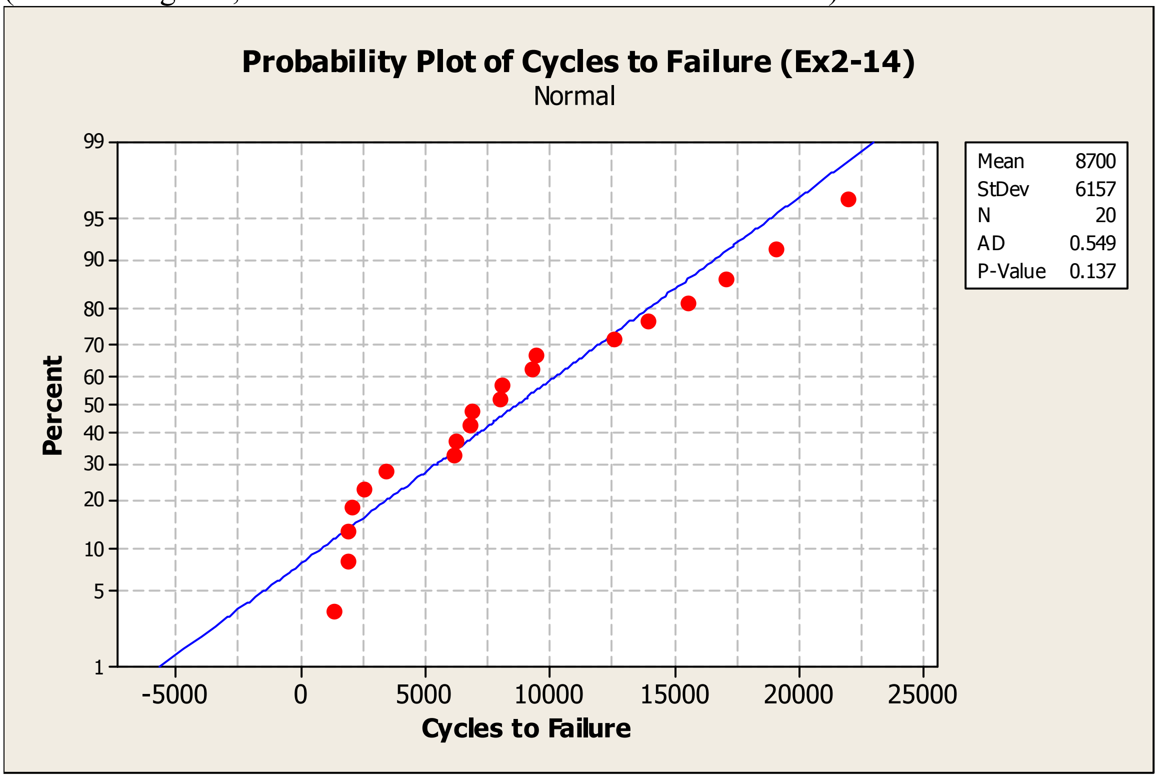


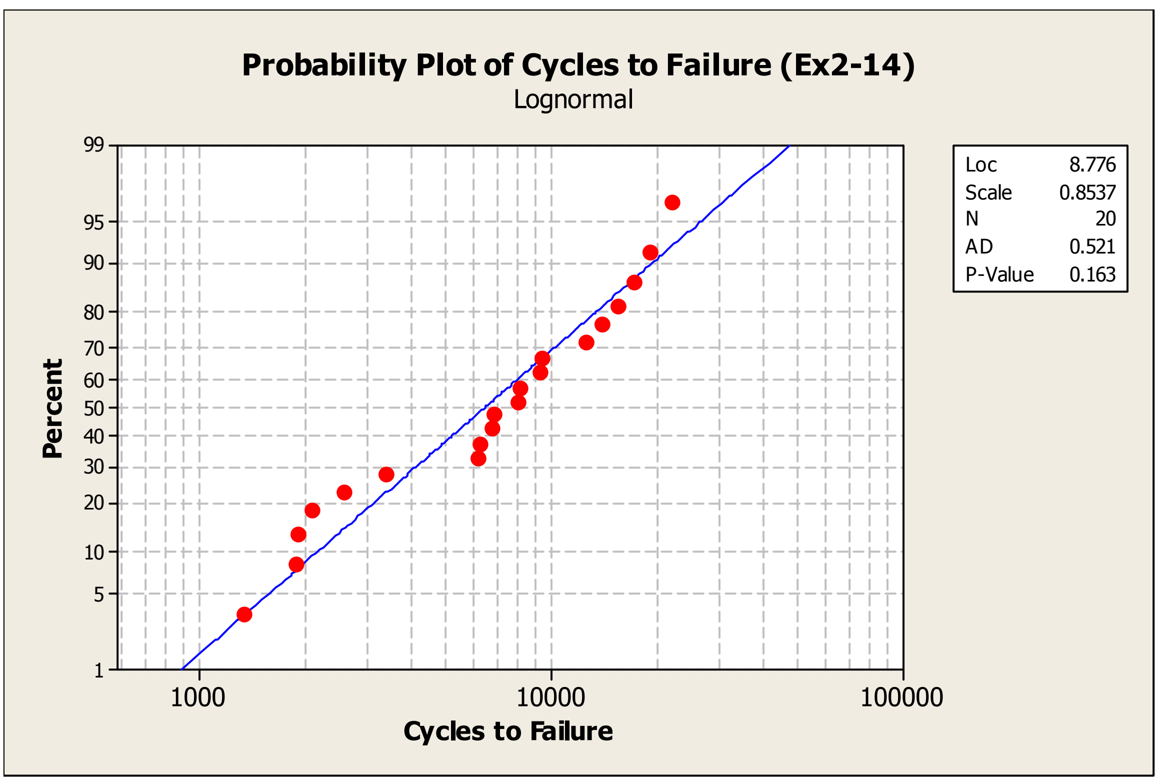

Both the normal and lognormal distributions appear to be reasonable models for the data; the plot points tend to fall along a straight line, with no bends or curves. However, the plot points on the Weibull probability plot are not straight—particularly in the tails—indicating it is not a reasonable model.

2-14 ☺.

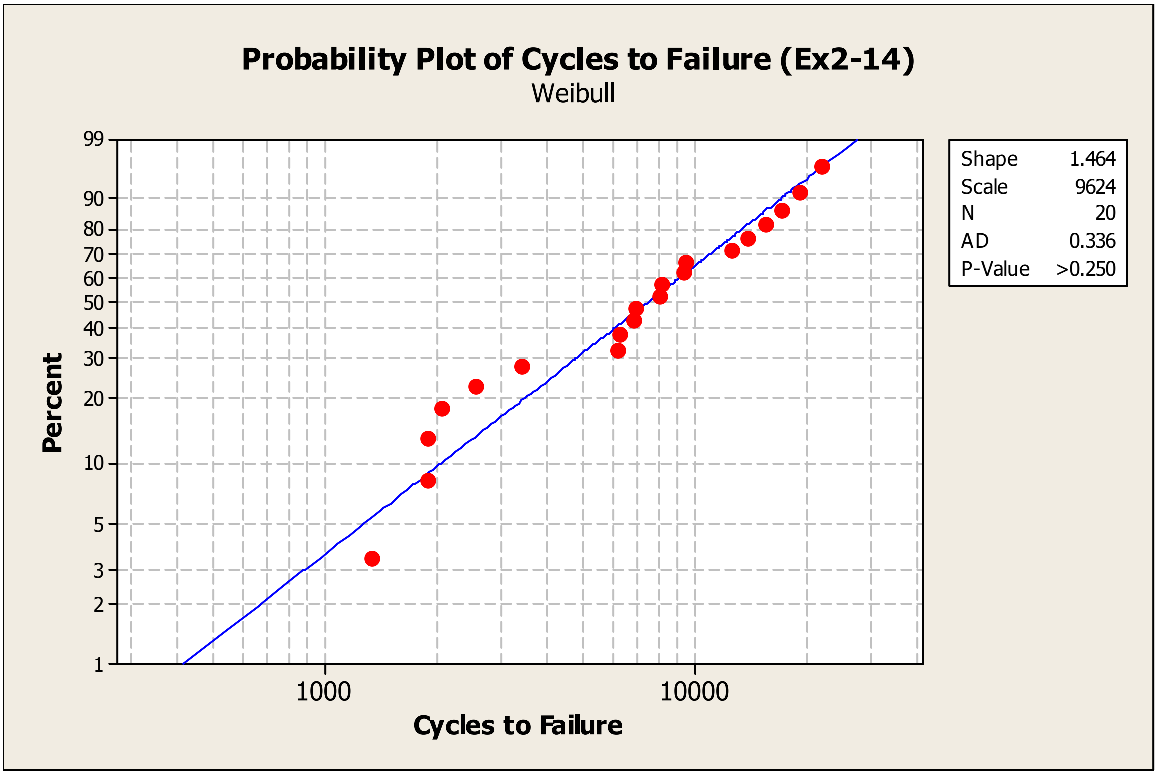
**MTB > Graph > Probability Plot > Single**

(In the dialog box, select Distribution to choose the distributions)



2-14 continued

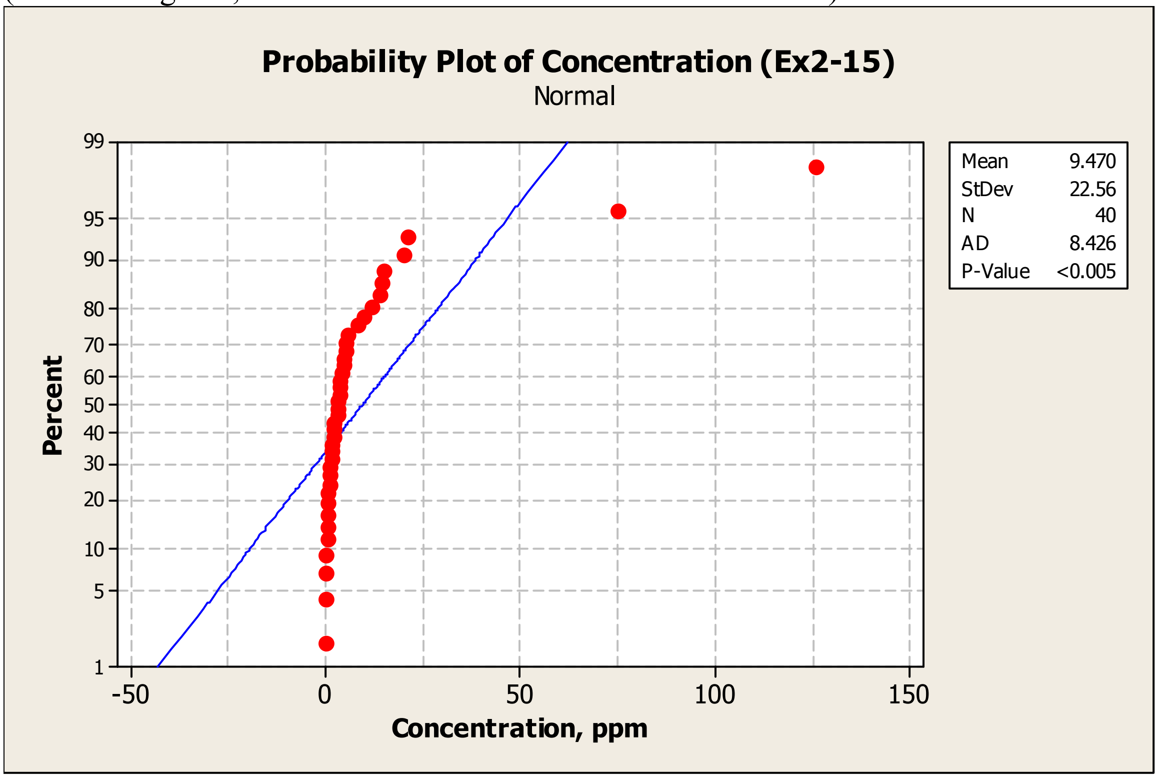


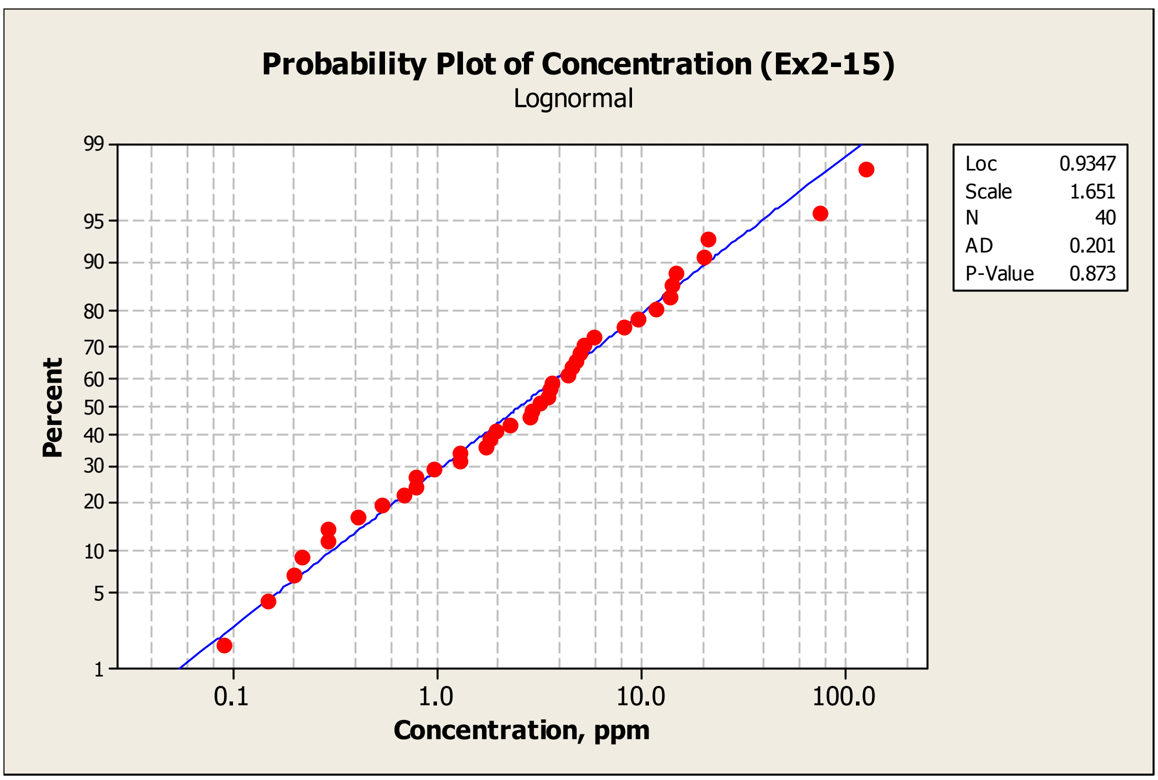

Plotted points do not tend to fall on a straight line on any of the probability plots, though the Weibull distribution appears to best fit the data in the tails.

2-15 ☺.

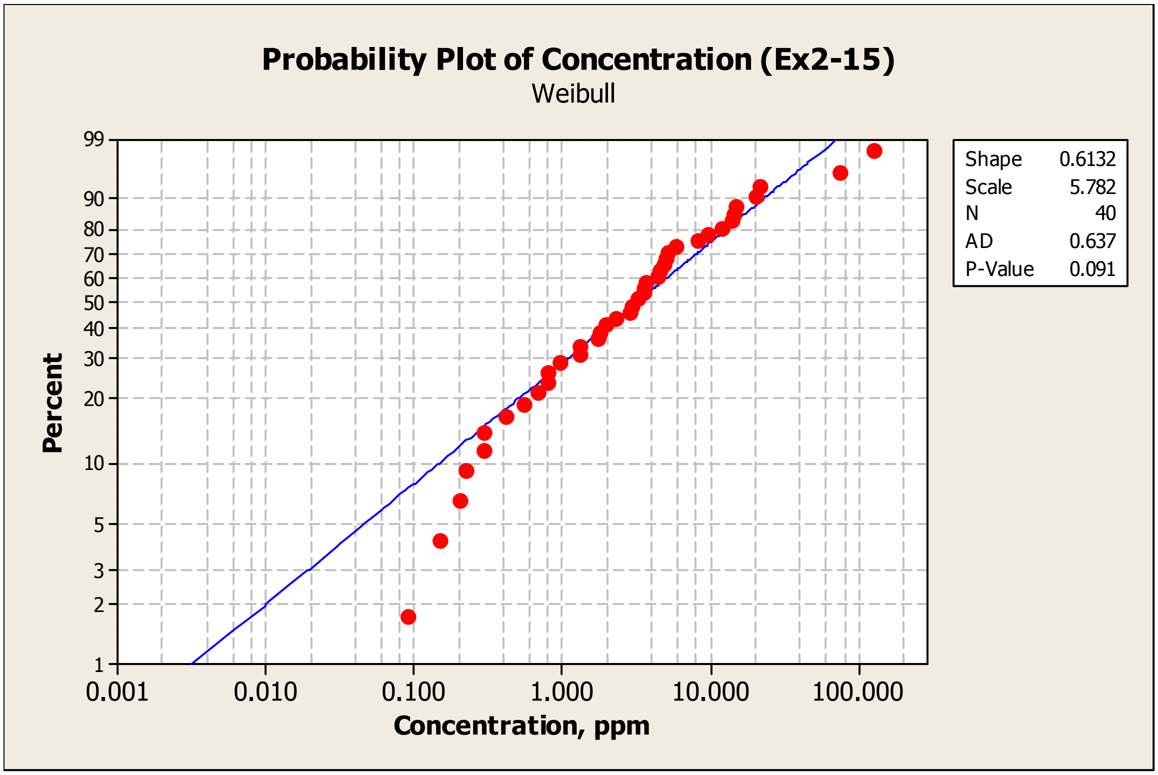
**MTB > Graph > Probability Plot > Single**

(In the dialog box, select Distribution to choose the distributions)



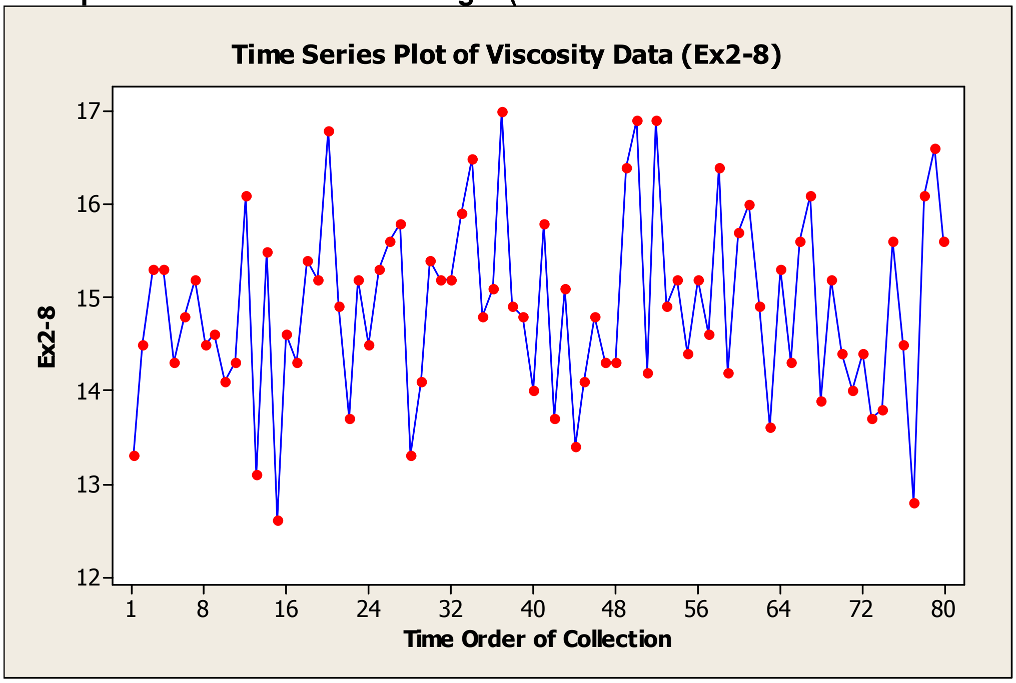
2-15 continued




The lognormal distribution appears to be a reasonable model for the concentration data. Plotted points on the normal and Weibull probability plots tend to fall off a straight line.

2-16* (2-9).

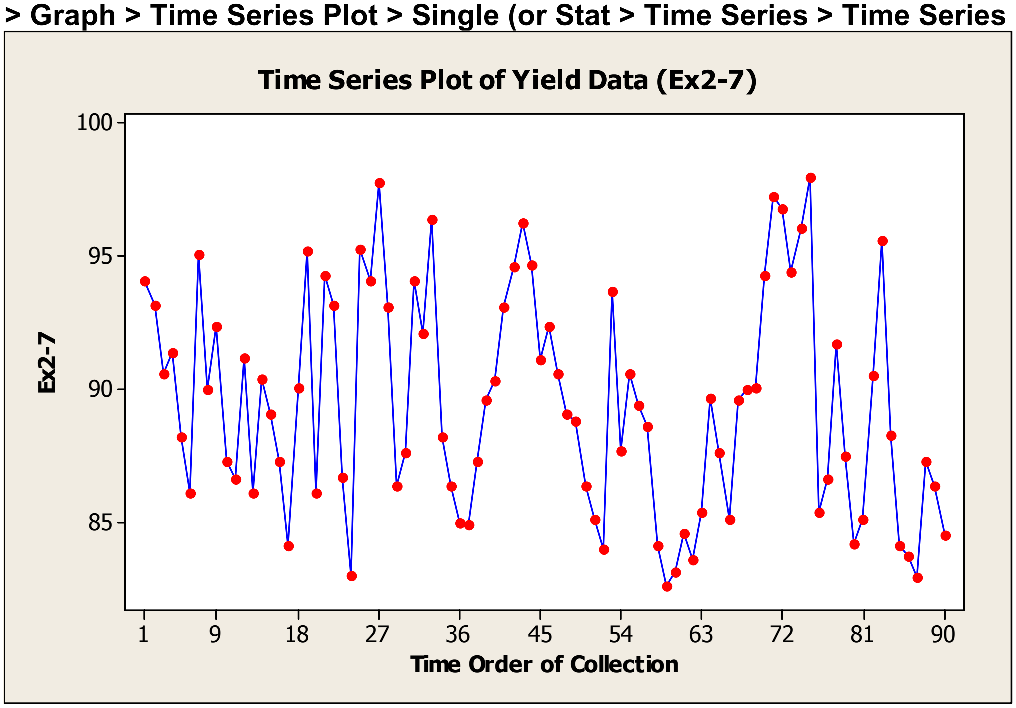
**MTB > Graph > Time Series Plot > Single (or Stat > Time Series > Time Series Plot)**




From visual examination, there are no trends, shifts or obvious patterns in the data, indicating that time is not an important source of variability.

2-17* (2-10).

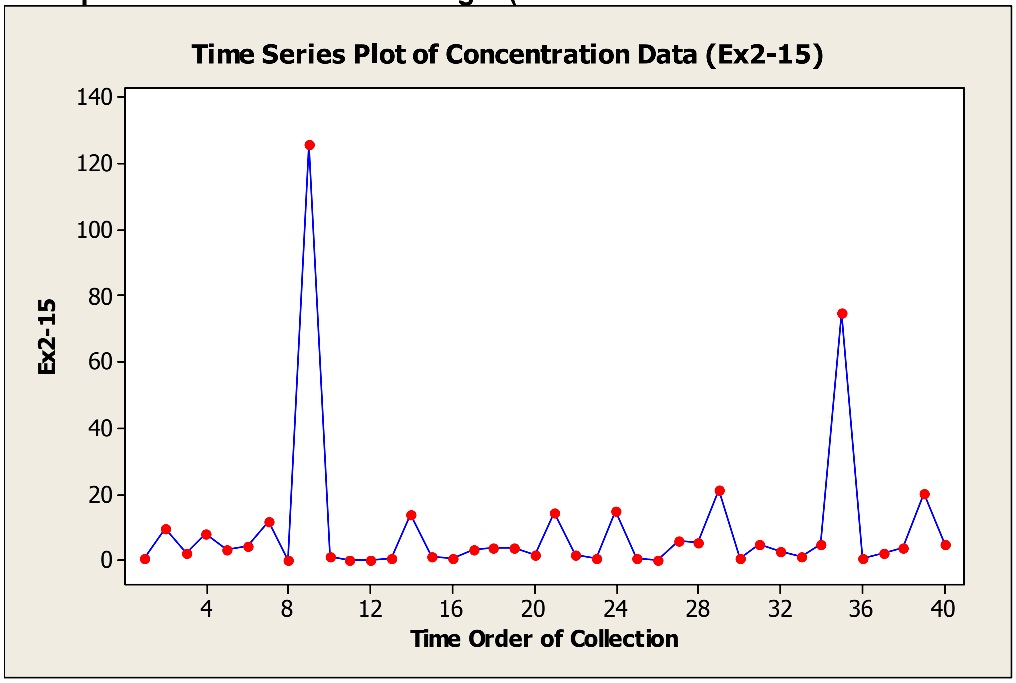
**MTB > Graph > Time Series Plot > Single (or Stat > Time Series > Time Series Plot)**




Time may be an important source of variability, as evidenced by potentially cyclic behavior.

2-18 ☺.

**MTB > Graph > Time Series Plot > Single (or Stat > Time Series > Time Series Plot)**




Although most of the readings are between 0 and 20, there are two unusually large readings (9, 35), as well as occasional "spikes" around 20. The order in which the data were collected may be an important source of variability.

2-19 (2-11).

**MTB > Stat > Basic Statistics > Display Descriptive Statistics**

**Descriptive Statistics: Ex2-7**

| Variable | N | N* | Mean | SE Mean | StDev | Minimum | Q1 | Median | Q3 |
|---|---|---|---|---|---|---|---|---|---|
| Ex2-7 | 90 | 0 | 89.476 | 0.438 | 4.158 | 82.600 | 86.100 | 89.250 | 93.125 |

| Variable | Maximum |
|---|---|
| Ex2-7 | 98.000 |

2-20 (2-12).

**MTB > Graph > Stem-and-Leaf**

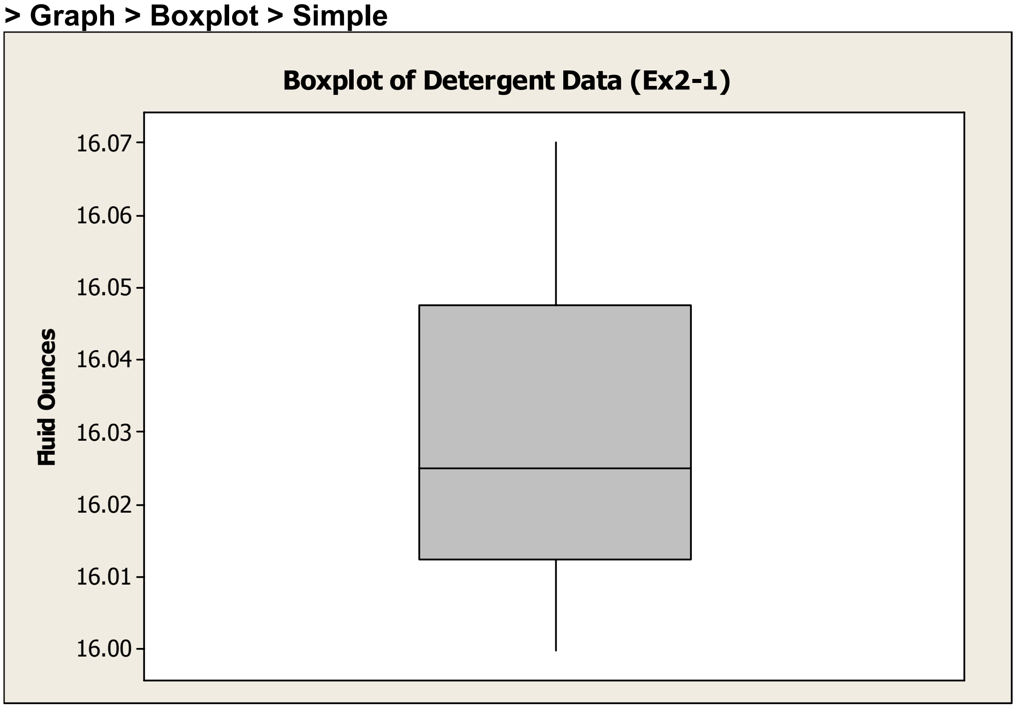
**Stem-and-Leaf Display: Ex2-7**

```
Stem-and-leaf of Ex2-7  N  = 90
Leaf Unit = 0.10
 2   82  69
 6   83  0167
 14  84  01112569
 20  85  011144
 30  86  1114444667
 38  87  33335667
 43  88  22368
(6)  89  114667
 41  90  0011345666
 31  91  1247
 27  92  144
 24  93  11227
 19  94  11133467
 11  95  1236
 7   96  1348
 3   97  38
 1   98  0
```

Neither the stem-and-leaf plot nor the frequency histogram reveals much about an underlying distribution or a central tendency in the data. The data appear to be fairly well scattered. The stem-and-leaf plot suggests that certain values may occur more frequently than others; for example, those ending in 1, 4, 6, and 7.

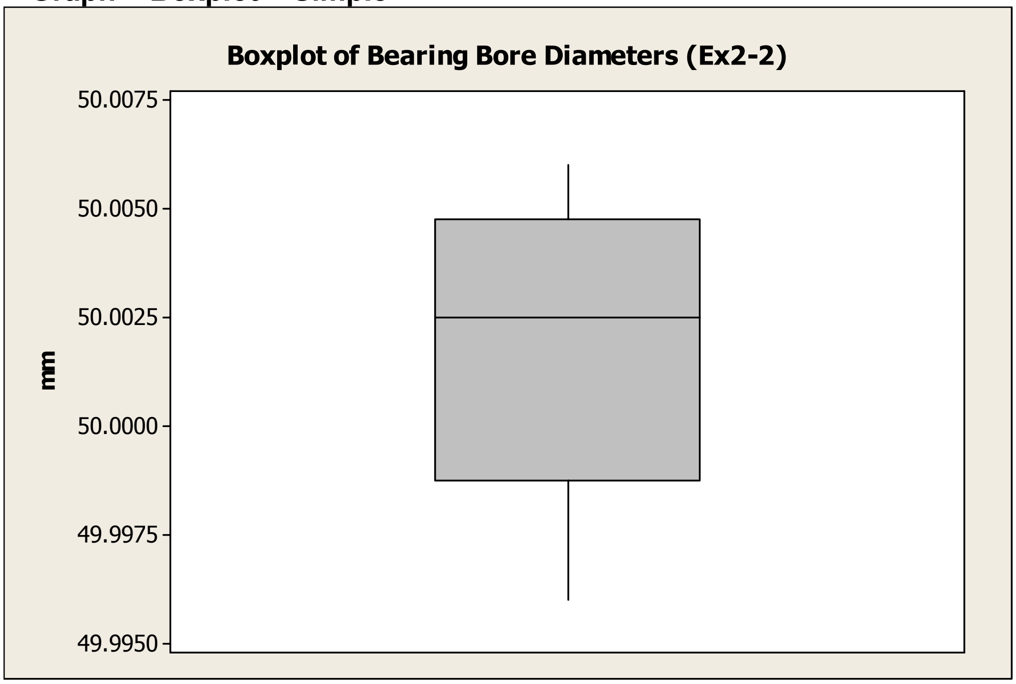
2-21 (2-13).

**MTB > Graph > Boxplot > Simple**



2-22 (2-14).

**MTB > Graph > Boxplot > Simple**




2-23 (2-15).

*x*: {the sum of two up dice faces}

*sample space*: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

$$\Pr\{x=2\}=\Pr\{1,1\}=\tfrac{1}{6}\times\tfrac{1}{6}=\tfrac{1}{36}$$

$$\Pr\{x=3\}=\Pr\{1,2\}+\Pr\{2,1\}=\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)=\tfrac{2}{36}$$

$$\Pr\{x=4\}=\Pr\{1,3\}+\Pr\{2,2\}+\Pr\{3,1\}=\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)+\left(\tfrac{1}{6}\times\tfrac{1}{6}\right)=\tfrac{3}{36}$$

. . .

$$p(x)=\begin{cases}1/36;x=2 \quad 2/36;x=3 \quad 3/36;x=4 \quad 4/36;x=5 \quad 5/36;x=6 \quad 6/36;x=7\\ 5/36;x=8 \quad 4/36;x=9 \quad 3/36;x=10 \quad 2/36;x=11 \quad 1/36;x=12 \quad 0;\ \text{otherwise}\end{cases}$$

2-24 (2-16).

$$\bar{x}=\sum_{i=1}^{11}x_i p(x_i)=2\left(1/36\right)+3\left(2/36\right)+\cdots+12\left(1/36\right)=7$$

$$S=\sqrt{\frac{\sum_{i=1}^{n}x_i p(x_i)-\left[\sum_{i=1}^{n}x_i p(x_i)\right]^2\Big/n}{n-1}}=\sqrt{\frac{5.92-7^2/11}{10}}=0.38$$

2-25 (2-17).
This is a Poisson distribution with parameter λ = 0.02, $x$ ~ POI(0.02).

(a)

$$\Pr\{x=1\} = p(1) = \frac{e^{-0.02}(0.02)^1}{1!} = 0.0196$$

(b)

$$\Pr\{x \geq 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.02}(0.02)^0}{0!} = 1 - 0.9802 = 0.0198$$

(c)
This is a Poisson distribution with parameter λ = 0.01, $x$ ~ POI(0.01).

$$\Pr\{x \geq 1\} = 1 - \Pr\{x=0\} = 1 - p(0) = 1 - \frac{e^{-0.01}(0.01)^0}{0!} = 1 - 0.9900 = 0.0100$$

Cutting the rate at which defects occur reduces the probability of one or more defects by approximately one-half, from 0.0198 to 0.0100.

2-26 (2-18).
For $f(x)$ to be a probability distribution, $\int_{-\infty}^{+\infty} f(x)dx$ must equal unity.

$$\int_0^{\infty} ke^{-x}dx = [-ke^{-x}]_0^{\infty} = -k[0-1] = k \Rightarrow 1$$

This is an exponential distribution with parameter λ=1.
$\mu = 1/\lambda = 1$ (Eqn. 2-32)
$\sigma^2 = 1/\lambda^2 = 1$ (Eqn. 2-33)

2-27 (2-19).

$$p(x) = \begin{cases} (1+3k)/3; \;\; x=1 \quad (1+2k)/3; \;\; x=2 \\ (0.5+5k)/3; \;\; x=3 \quad 0; \;\; \text{otherwise} \end{cases}$$

(a)
To solve for $k$, use $F(x) = \sum_{i=1}^{\infty} p(x_i) = 1$

$$\frac{(1+3k)+(1+2k)+(0.5+5k)}{3} = 1$$

$$10k = 0.5$$

$$k = 0.05$$

2-27 continued
(b)

$$\mu = \sum_{i=1}^{3} x_i p(x_i) = 1\times\left[\frac{1+3(0.05)}{3}\right] + 2\times\left[\frac{1+2(0.05)}{3}\right] + 3\times\left[\frac{0.5+5(0.05)}{3}\right] = 1.867$$

$$\sigma^2 = \sum_{i=1}^{3} x_i^2 p(x_i) - \mu^2 = 1^2(0.383) + 2^2(0.367) + 3^2(0.250) - 1.867^2 = 0.615$$

(c)

$$F(x) = \begin{cases} \dfrac{1.15}{3} = 0.383; x = 1 \\ \dfrac{1.15+1.1}{3} = 0.750; x = 2 \\ \dfrac{1.15+1.1+0.75}{3} = 1.000; x = 3 \end{cases}$$

2-28 (2-20).

$$p(x) = kr^x; \quad 0 < r < 1; \quad x = 0,1,2,\ldots$$

$$F(x) = \sum_{i=0}^{\infty} kr^x = 1 \text{ by definition}$$

$$k\left[1/(1-r)\right] = 1$$

$$k = 1 - r$$

2-29 (2-21).
(a)
This is an exponential distribution with parameter $\lambda = 0.125$:
$\Pr\{x \le 1\} = F(1) = 1 - e^{-0.125(1)} = 0.118$
Approximately 11.8% will fail during the first year.

(b)
Mfg. cost = \$50/calculator
Sale profit = \$25/calculator
Net profit = \$[-50(1 + 0.118) + 75]/calculator = \$19.10/calculator.
The effect of warranty replacements is to decrease profit by \$5.90/calculator.

2-30 (2-22).

$$\Pr\{x < 12\} = F(12) = \int_{-\infty}^{12} f(x)dx = \int_{11.75}^{12} 4(x - 11.75)dx = \frac{4x^2}{2}\Big|_{11.75}^{12} - 47x\Big|_{11.75}^{12} = 11.875 - 11.75 = 0.125$$

2-31* (2-23).

This is a binomial distribution with parameter $p = 0.01$ and $n = 25$. The process is stopped if $x \geq 1$.

$$\Pr\{x \geq 1\} = 1 - \Pr\{x < 1\} = 1 - \Pr\{x = 0\} = 1 - \binom{25}{0}(0.01)^0(1-0.01)^{25} = 1 - 0.78 = 0.22$$

This decision rule means that 22% of the samples will have one or more nonconforming units, and the process will be stopped to look for a cause. This is a somewhat difficult operating situation.

This exercise may also be solved using Excel or MINITAB:

(1) **Excel Function BINOMDIST(x, n, p, TRUE)**

(2) **MTB > Calc > Probability Distributions > Binomial**

**Cumulative Distribution Function**

```
Binomial with n = 25 and p = 0.01
x  P( X <= x )
0     0.777821
```

2-32* (2-24).

$x \sim$ BIN(25, 0.04) Stop process if $x \geq 1$.

$$\Pr\{x \geq 1\} = 1 - \Pr\{x < 1\} = 1 - \Pr\{x = 0\} = 1 - \binom{25}{0}(0.04)^0(1-0.04)^{25} = 1 - 0.36 = 0.64$$

2-33* (2-25).

This is a binomial distribution with parameter $p = 0.02$ and $n = 50$.

$$\Pr\{\hat{p} \leq 0.04\} = \Pr\{x \leq 2\} = \sum_{x=0}^{4}\binom{50}{x}(0.02)^x(1-0.02)^{(50-x)}$$

$$= \binom{50}{0}(0.02)^0(1-0.02)^{50} + \binom{50}{1}(0.02)^1(1-0.02)^{49} + \cdots + \binom{50}{4}(0.02)^4(1-0.02)^{46} = 0.921$$

2-34* (2-26).
This is a binomial distribution with parameter $p$ = 0.01 and $n$ = 100.
$\sigma = \sqrt{0.01(1-0.01)/100} = 0.0100$

$$\Pr\{\hat{p} > k\sigma + p\} = 1 - \Pr\{\hat{p} \le k\sigma + p\} = 1 - \Pr\{x \le n(k\sigma + p)\}$$

$k = 1$

$$1 - \Pr\{x \le n(k\sigma + p)\} = 1 - \Pr\{x \le 100(1(0.0100) + 0.01)\} = 1 - \Pr\{x \le 2\}$$

$$= 1 - \sum_{x=0}^{2} \binom{100}{x} (0.01)^x (1-0.01)^{100-x}$$

$$= 1 - \left[ \binom{100}{0} (0.01)^0 (0.99)^{100} + \binom{100}{1} (0.01)^1 (0.99)^{99} + \binom{100}{2} (0.01)^2 (0.99)^{98} \right]$$

$$= 1 - [0.921] = 0.079$$

$k = 2$

$$1 - \Pr\{x \le n(k\sigma + p)\} = 1 - \Pr\{x \le 100(2(0.0100) + 0.01)\} = 1 - \Pr\{x \le 3\}$$

$$= 1 - \sum_{x=0}^{3} \binom{100}{x} (0.01)^x (0.99)^{100-x} = 1 - \left[ 0.921 + \binom{100}{3} (0.01)^3 (0.99)^{97} \right]$$

$$= 1 - [0.982] = 0.018$$

$k = 3$

$$1 - \Pr\{x \le n(k\sigma + p)\} = 1 - \Pr\{x \le 100(3(0.0100) + 0.01)\} = 1 - \Pr\{x \le 4\}$$

$$= 1 - \sum_{x=0}^{4} \binom{100}{x} (0.01)^x (0.99)^{100-x} = 1 - \left[ 0.982 + \binom{100}{4} (0.01)^4 (0.99)^{96} \right]$$

$$= 1 - [0.992] = 0.003$$

2-35* (2-27).
This is a hypergeometric distribution with $N = 25$ and $n = 5$, without replacement.

(a)
Given $D = 2$ and $x = 0$:

$$\Pr\{\text{Acceptance}\} = p(0) = \frac{\binom{2}{0}\binom{25-2}{5-0}}{\binom{25}{5}} = \frac{(1)(33,649)}{(53,130)} = 0.633$$

This exercise may also be solved using Excel or MINITAB:
(1) **Excel Function HYPGEOMDIST(x, n, D, N)**
(2) **MTB > Calc > Probability Distributions > Hypergeometric**

**Cumulative Distribution Function**
```
Hypergeometric with N = 25, M = 2, and n = 5
x  P( X <= x )
0     0.633333
```

(b)
For the binomial approximation to the hypergeometric, $p = D/N = 2/25 = 0.08$ and $n = 5$.

$$\Pr\{\text{acceptance}\} = p(0) = \binom{5}{0}(0.08)^0(1-0.08)^5 = 0.659$$

This approximation, though close to the exact solution for $x = 0$, violates the rule-of-thumb that $n/N = 5/25 = 0.20$ be less than the suggested 0.1. The binomial approximation is not satisfactory in this case.

(c)
For $N = 150$, $n/N = 5/150 = 0.033 \leq 0.1$, so the binomial approximation would be a satisfactory approximation the hypergeometric in this case.

2-35 continued
(d)
Find $n$ to satisfy $\Pr\{x \geq 1 \mid D \geq 5\} \geq 0.95$, or equivalently $\Pr\{x = 0 \mid D = 5\} < 0.05$.

$$p(0) = \frac{\binom{5}{0}\binom{25-5}{n-0}}{\binom{25}{n}} = \frac{\binom{5}{0}\binom{20}{n}}{\binom{25}{n}}$$

try $n = 10$

$$p(0) = \frac{\binom{5}{0}\binom{20}{10}}{\binom{25}{10}} = \frac{(1)(184,756)}{(3,268,760)} = 0.057$$

try $n = 11$

$$p(0) = \frac{\binom{5}{0}\binom{20}{11}}{\binom{25}{11}} = \frac{(1)(167,960)}{(4,457,400)} = 0.038$$

Let sample size $n = 11$.

2-36 (2-28).
This is a hypergeometric distribution with $N = 30$, $n = 5$, and $D = 3$.

$$\Pr\{x = 1\} = p(1) = \frac{\binom{3}{1}\binom{30-3}{5-1}}{\binom{30}{5}} = \frac{(3)(17,550)}{(142,506)} = 0.369$$

$$\Pr\{x \geq 1\} = 1 - \Pr\{x = 0\} = 1 - p(0) = 1 - \frac{\binom{3}{0}\binom{27}{5}}{\binom{30}{5}} = 1 - 0.567 = 0.433$$

2-37 (2-29).
This is a hypergeometric distribution with $N$ = 500 pages, $n$ = 50 pages, and $D$ = 10 errors. Checking $n/N$ = 50/500 = 0.1 ≤ 0.1, the binomial distribution can be used to approximate the hypergeometric, with $p = D/N$ = 10/500 = 0.020.

$$\Pr\{x=0\} = p(0) = \binom{50}{0}(0.020)^0(1-0.020)^{50-0} = (1)(1)(0.364) = 0.364$$

$$\Pr\{x \geq 2\} = 1-\Pr\{x \leq 1\} = 1-[\Pr\{x=0\}+\Pr\{x=1\}] = 1-p(0)-p(1)$$

$$= 1-0.364-\binom{50}{1}(0.020)^1(1-0.020)^{50-1} = 1-0.364-0.372 = 0.264$$

2-38 (2-30).
This is a Poisson distribution with λ = 0.1 defects/unit.

$$\Pr\{x \geq 1\} = 1-\Pr\{x=0\} = 1-p(0) = 1-\frac{e^{-0.1}(0.1)^0}{0!} = 1-0.905 = 0.095$$

This exercise may also be solved using Excel or MINITAB:
(1) **Excel Function POISSON(λ, x, TRUE)**
(2) **MTB > Calc > Probability Distributions > Poisson**

**Cumulative Distribution Function**
```
Poisson with mean = 0.1
x  P( X <= x )
0     0.904837
```

2-39 (2-31).
This is a Poisson distribution with λ = 0.00001 stones/bottle.

$$\Pr\{x \geq 1\} = 1-\Pr\{x=0\} = 1-\frac{e^{-0.00001}(0.00001)^0}{0!} = 1-0.99999 = 0.00001$$

2-40 (2-32).
This is a Poisson distribution with λ = 0.01 errors/bill.

$$\Pr\{x=1\} = p(1) = \frac{e^{-0.01}(0.01)^1}{1} = 0.0099$$

2-41 (2-33).

$$\Pr(t) = p(1-p)^{t-1};\ \ t = 1, 2, 3, \ldots$$

$$\mu = \sum_{t=1}^{\infty} t\left[p(1-p)^{t-1}\right] = p\frac{d}{dq}\left[\sum_{t=1}^{\infty} q^t\right] = \frac{1}{p}$$

2-42 (2-34).

This is a Pascal distribution with Pr{defective weld} = $p$ = 0.01, $r$ = 3 welds, and $x$ = 1 + (5000/100) = 51.

$$\Pr\{x = 51\} = p(51) = \binom{51-1}{3-1}(0.01)^3(1-0.01)^{51-3} = (1225)(0.000001)(0.617290) = 0.0008$$

$$\Pr\{x > 51\} = \Pr\{r = 0\} + \Pr\{r = 1\} + \Pr\{r = 2\}$$

$$= \binom{50}{0}0.01^0 0.99^{50} + \binom{50}{1}0.01^1 0.99^{49}\binom{50}{2}0.01^2 0.99^{48} = 0.9862$$

2-43* (2-35).

$x \sim N(40, 5^2)$; $n$ = 50,000

How many fail the minimum specification, LSL = 35 lb.?

$$\Pr\{x \le 35\} = \Pr\left\{z \le \frac{35-40}{5}\right\} = \Pr\{z \le -1\} = \Phi(-1) = 0.159$$

So, the number that fail the minimum specification are (50,000) × (0.159) = 7950.

This exercise may also be solved using Excel or MINITAB:

(1) **Excel Function NORMDIST(X, μ, σ, TRUE)**

(2) **MTB > Calc > Probability Distributions > Normal**

**Cumulative Distribution Function**

```
Normal with mean = 40 and standard deviation = 5
 x  P( X <= x )
35     0.158655
```

How many exceed 48 lb.?

$$\Pr\{x > 48\} = 1 - \Pr\{x \le 48\} = 1 - \Pr\left\{z \le \frac{48-40}{5}\right\} = 1 - \Pr\{z \le 1.6\}$$

$$= 1 - \Phi(1.6) = 1 - 0.945 = 0.055$$

So, the number that exceed 48 lb. is (50,000) × (0.055) = 2750.

2-44* (2-36).
$x \sim N(5, 0.02^2)$; LSL = 4.95 V; USL = 5.05 V

$$\Pr\{\text{Conformance}\} = \Pr\{\text{LSL} \le x \le \text{USL}\} = \Pr\{x \le \text{USL}\} - \Pr\{x \le \text{LSL}\}$$
$$= \Phi\left(\frac{5.05-5}{0.02}\right) - \Phi\left(\frac{4.95-5}{0.02}\right) = \Phi(2.5) - \Phi(-2.5) = 0.99379 - 0.00621 = 0.98758$$

2-45* (2-37).
The process, with mean 5 V, is currently centered between the specification limits (target = 5 V). Shifting the process mean in either direction would increase the number of nonconformities produced.

Desire Pr{Conformance} = 1 / 1000 = 0.001. Assume that the process remains centered between the specification limits at 5 V. Need $\Pr\{x \le \text{LSL}\} = 0.001 / 2 = 0.0005$.

$$\Phi(z) = 0.0005$$
$$z = \Phi^{-1}(0.0005) = -3.29$$

$$z = \frac{\text{LSL} - \mu}{\sigma}, \quad \text{so } \sigma = \frac{\text{LSL} - \mu}{z} = \frac{4.95 - 5}{-3.29} = 0.015$$

Process variance must be reduced to $0.015^2$ to have at least 999 of 1000 conform to specification.

2-46 (2-38).
$x \sim N(\mu, 4^2)$. Find $\mu$ such that $\Pr\{x < 32\} = 0.0228$.

$$\Phi^{-1}(0.0228) = -1.9991$$
$$\frac{32 - \mu}{4} = -1.9991$$
$$\mu = -4(-1.9991) + 32 = 40.0$$

2-47 (2-39).
$x \sim N(900, 35^2)$

$$\Pr\{x > 1000\} = 1 - \Pr\{x \le 1000\}$$
$$= 1 - \Pr\left\{x \le \frac{1000 - 900}{35}\right\}$$
$$= 1 - \Phi(2.8571)$$
$$= 1 - 0.9979$$
$$= 0.0021$$

2-48 (2-40).

$x \sim N(5000, 50^2)$. Find LSL such that $\Pr\{x < \text{LSL}\} = 0.005$

$$\Phi^{-1}(0.005) = -2.5758$$

$$\frac{\text{LSL} - 5000}{50} = -2.5758$$

$$\text{LSL} = 50(-2.5758) + 5000 = 4871$$

2-49 (2-41).

$x_1 \sim N(7500, \sigma_1^2 = 1000^2)$; $x_2 \sim N(7500, \sigma_2^2 = 500^2)$; LSL = 5,000 h; USL = 10,000 h

sales = \$10/unit, defect = \$5/unit, profit = \$10 × Pr{good} + \$5 × Pr{bad} – $c$

<u>For Process 1</u>

$$\begin{aligned}\text{proportion defective} = p_1 &= 1 - \Pr\{\text{LSL} \le x_1 \le \text{USL}\} = 1 - \Pr\{x_1 \le \text{USL}\} + \Pr\{x_1 \le \text{LSL}\} \\ &= 1 - \Pr\left\{z_1 \le \frac{10,000 - 7,500}{1,000}\right\} + \Pr\left\{z_1 \le \frac{5,000 - 7,500}{1,000}\right\} \\ &= 1 - \Phi(2.5) + \Phi(-2.5) = 1 - 0.9938 + 0.0062 = 0.0124\end{aligned}$$

profit for process 1 = 10 (1 – 0.0124) + 5 (0.0124) – $c_1$ = 9.9380 – $c_1$

<u>For Process 2</u>

$$\begin{aligned}\text{proportion defective} = p_2 &= 1 - \Pr\{\text{LSL} \le x_2 \le \text{USL}\} = 1 - \Pr\{x_2 \le \text{USL}\} + \Pr\{x_2 \le \text{LSL}\} \\ &= 1 - \Pr\left\{z_2 \le \frac{10,000 - 7,500}{500}\right\} + \Pr\left\{z_2 \le \frac{5,000 - 7,500}{500}\right\} \\ &= 1 - \Phi(5) + \Phi(-5) = 1 - 1.0000 + 0.0000 = 0.0000\end{aligned}$$

profit for process 2 = 10 (1 – 0.0000) + 5 (0.0000) – $c_2$ = 10 – $c_2$

If $c_2 > c_1 + 0.0620$, then choose process 1

2-50 (2-42).

Proportion less than lower specification:

$$p_l = \Pr\{x < 6\} = \Pr\left\{z \le \frac{6-\mu}{1}\right\} = \Phi(6-\mu)$$

Proportion greater than upper specification:

$$p_u = \Pr\{x > 8\} = 1 - \Pr\{x \le 8\} = 1 - \Pr\left\{z \le \frac{8-\mu}{1}\right\} = 1 - \Phi(8-\mu)$$

$$\begin{aligned}\text{Profit} &= +C_0 p_{\text{within}} - C_1 p_l - C_2 p_u \\ &= C_0[\Phi(8-\mu) - \Phi(6-\mu)] - C_1[\Phi(6-\mu)] - C_2[1-\Phi(8-\mu)] \\ &= (C_0 + C_2)[\Phi(8-\mu)] - (C_0 + C_1)[\Phi(6-\mu)] - C_2\end{aligned}$$

$$\frac{d}{d\mu}[\Phi(8-\mu)] = \frac{d}{d\mu}\left[\int_{-\infty}^{8-\mu} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)dt\right]$$

Set $s = 8 - \mu$ and use chain rule

$$\frac{d}{d\mu}[\Phi(8-\mu)] = \frac{d}{ds}\left[\int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)dt\right]\frac{ds}{d\mu} = -\frac{1}{\sqrt{2\pi}} \exp\left(-1/2 \times (8-\mu)^2\right)$$

$$\frac{d(\text{Profit})}{d\mu} = -(C_0 + C_2)\left[\frac{1}{\sqrt{2\pi}} \exp\left(-1/2 \times (8-\mu)^2\right)\right] + (C_0 + C_1)\left[\frac{1}{\sqrt{2\pi}} \exp\left(-1/2 \times (6-\mu)^2\right)\right]$$

Setting equal to zero

$$\frac{C_0 + C_1}{C_0 + C_2} = \frac{\exp\left(-1/2 \times (8-\mu)^2\right)}{\exp\left(-1/2 \times (8-\mu)^2\right)} = \exp(2\mu - 14)$$

So $\mu = \frac{1}{2}\left[\ln\left(\frac{C_0 + C_1}{C_0 + C_2}\right) + 14\right]$ maximizes the expected profit.

2-51 (2-43).

For the binomial distribution, $p(x) = \binom{n}{x} p^x (1-p)^{n-x}; \quad x = 0,1,\ldots,n$

$$\mu = E(x) = \sum_{i=1}^{\infty} x_i p(x_i) = \sum_{x=0}^{n} x \left[ \binom{n}{x} p^x (1-p)^{n-x} \right] = n\left[ p + (1-p) \right]^{n-1} p = np$$

$$\sigma^2 = E[(x-\mu)^2] = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum_{i=1}^{\infty} x_i^2 p(x_i) = \sum_{x=0}^{n} x^2 \left[ \binom{n}{x} p^x (1-p)^{n-x} \right] = np + (np)^2 - np^2$$

$$\sigma^2 = \left[ np + (np)^2 - np^2 \right] - \left[ np \right]^2 = np(1-p)$$

2-52 (2-44).

For the Poisson distribution, $p(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}; x = 0,1,\ldots$

$$\mu = E[x] = \sum_{i=1}^{\infty} x_i p(x_i) = \sum_{x=0}^{\infty} x \left( \frac{e^{-\lambda}\lambda^x}{x!} \right) = e^{-\lambda}\lambda \sum_{x=0}^{\infty} \frac{\lambda^{(x-1)}}{(x-1)!} = e^{-\lambda}\lambda\left(e^{\lambda}\right) = \lambda$$

$$\sigma^2 = E[(x-\mu)^2] = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum_{i=1}^{\infty} x_i^2 p(x_i) = \sum_{x=0}^{\infty} x^2 \left( \frac{e^{-\lambda}\lambda^x}{x!} \right) = \lambda^2 + \lambda$$

$$\sigma^2 = (\lambda^2 + \lambda) - \left[ \lambda \right]^2 = \lambda$$

2-53 (2-45).

For the exponential distribution, $f(x)=\lambda e^{-\lambda x}; x \ge 0$

For the mean:

$$\mu=\int_{0}^{+\infty} xf(x)dx=\int_{0}^{+\infty} x\left(\lambda e^{-\lambda x}\right)dx$$

Integrate by parts, setting $u=x$ and $dv=\lambda\exp(-\lambda x)$

$$uv-\int vdu=\left[-x\exp\left(-\lambda x\right)\right]_{0}^{+\infty}+\int_{0}^{+\infty}\exp\left(-\lambda x\right)dx=0+\frac{1}{\lambda}=\frac{1}{\lambda}$$

For the variance:

$$\sigma^2=E[(x-\mu)^2]=E(x^2)-[E(x)^2]=E(x^2)-\left(\frac{1}{\lambda}\right)^2$$

$$E(x^2)=\int_{-\infty}^{+\infty} x^2 f(x)dx=\int_{0}^{+\infty} x^2\lambda\exp(-\lambda x)dx$$

Integrate by parts, setting $u=x^2$ and $dv=\lambda\exp(-\lambda x)$

$$uv-\int vdu=\left[x^2\exp(-\lambda x)\right]_{0}^{+\infty}+2\int_{0}^{+\infty} x\exp(-\lambda x)dx=(0-0)+\frac{2}{\lambda^2}$$

$$\sigma^2=\frac{2}{\lambda^2}-\frac{1}{\lambda^2}=\frac{1}{\lambda^2}$$