Chapter 3 Exercise Solutions

3-1.
$n = 15$; $\bar{x} = 8.2535$ cm; $\sigma = 0.002$ cm

(a)
$\mu_0 = 8.25$, $\alpha = 0.05$
Test H_0: $\mu = 8.25$ vs. H_1: $\mu \neq 8.25$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{8.2535 - 8.25}{0.002/\sqrt{15}} = 6.78$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$
Reject H_0: $\mu = 8.25$, and conclude that the mean bearing ID is not equal to 8.25 cm.

(b)
P-value $= 2[1 - \Phi(Z_0)] = 2[1 - \Phi(6.78)] = 2[1 - 1.00000] = 0$

(c)

$$\bar{x} - Z_{\alpha/2}\left(\sigma/\sqrt{n}\right) \le \mu \le \bar{x} + Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$$

$$8.25 - 1.96\left(0.002/\sqrt{15}\right) \le \mu \le 8.25 + 1.96\left(0.002/\sqrt{15}\right)$$

$$8.249 \le \mu \le 8.251$$

MTB > Stat > Basic Statistics > 1-Sample Z > Summarized data

One-Sample Z

```
Test of mu = 8.2535 vs not = 8.2535
The assumed standard deviation = 0.002
 N     Mean  SE Mean        95% CI            Z      P
15  8.25000  0.00052  (8.24899, 8.25101)  -6.78  0.000
```

3-2.
$n = 8$; $\bar{x} = 127$ psi; $\sigma = 2$ psi

(a)
$\mu_0 = 125$; $\alpha = 0.05$
Test H_0: $\mu = 125$ vs. H_1: $\mu > 125$. Reject H_0 if $Z_0 > Z_\alpha$.

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{127 - 125}{2/\sqrt{8}} = 2.828$$

$Z_\alpha = Z_{0.05} = 1.645$
Reject H_0: $\mu = 125$, and conclude that the mean tensile strength exceeds 125 psi.

3-2 continued

(b)

P-value = $1 - \Phi(Z_0) = 1 - \Phi(2.828) = 1 - 0.99766 = 0.00234$

(c)

In strength tests, we usually are interested in whether some minimum requirement is met, not simply that the mean does not equal the hypothesized value. A one-sided hypothesis test lets us do this.

(d)

$$\bar{x} - Z_\alpha\left(\sigma/\sqrt{n}\right) \le \mu$$
$$127 - 1.645\left(2/\sqrt{8}\right) \le \mu$$
$$125.8 \le \mu$$

MTB > Stat > Basic Statistics > 1-Sample Z > Summarized data

One-Sample Z

```
Test of mu = 125 vs > 125
The assumed standard deviation = 2
                         95%
                       Lower
N     Mean  SE Mean    Bound     Z      P
8  127.000    0.707  125.837  2.83  0.002
```

3-3.

$x \sim N(\mu, \sigma)$; $n = 10$

(a)

$\bar{x} = 26.0$; $s = 1.62$; $\mu_0 = 25$; $\alpha = 0.05$

Test H_0: $\mu = 25$ vs. H_1: $\mu > 25$. Reject H_0 if $t_0 > t_\alpha$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{26.0 - 25}{1.62/\sqrt{10}} = 1.952$$

$t_{\alpha,\, n-1} = t_{0.05,\, 10-1} = 1.833$

Reject H_0: $\mu = 25$, and conclude that the mean life exceeds 25 h.

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-3

```
Test of mu = 25 vs > 25
                                             95%
                                           Lower
Variable   N     Mean   StDev  SE Mean     Bound     T      P
Ex3-3     10  26.0000  1.6248   0.5138   25.0581  1.95  0.042
```

3-3 continued

(b)

$\alpha = 0.10$

$$\bar{x} - t_{\alpha/2,n-1} S/\sqrt{n} \le \mu \le \bar{x} + t_{\alpha/2,n-1} S/\sqrt{n}$$

$$26.0 - 1.833\left(1.62/\sqrt{10}\right) \le \mu \le 26.0 + 1.833\left(1.62/\sqrt{10}\right)$$

$$25.06 \le \mu \le 26.94$$

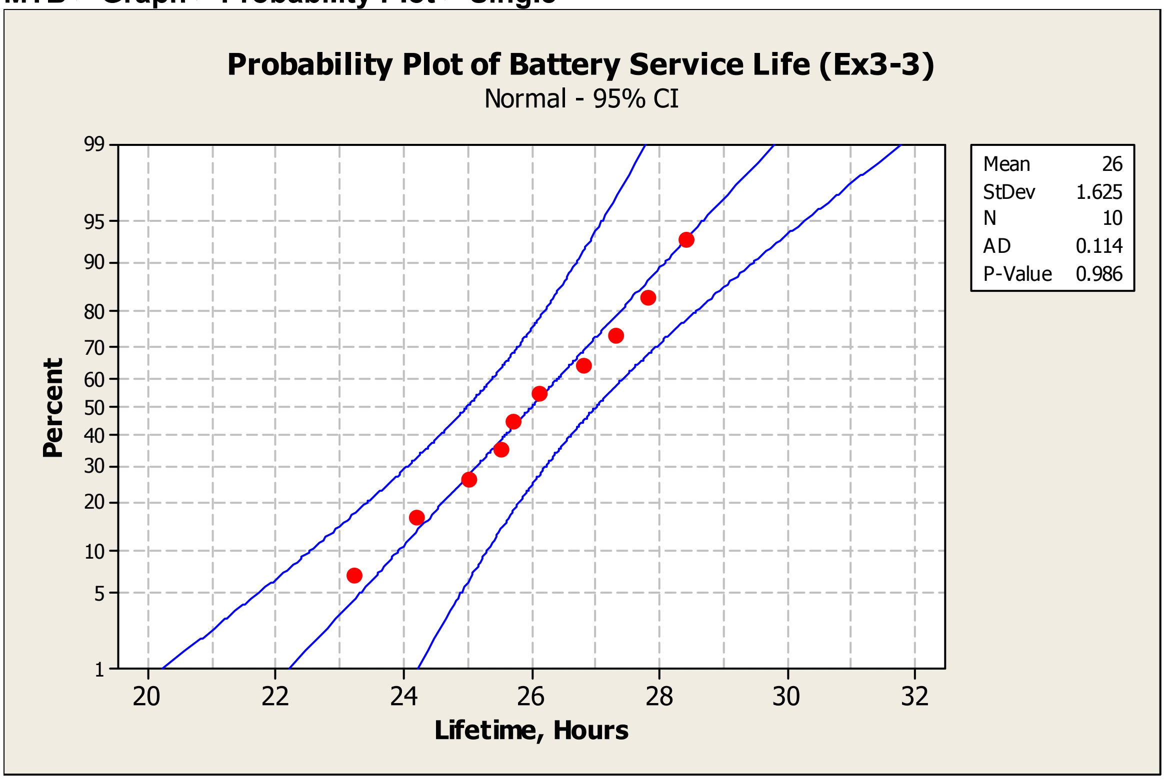
MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-3

```
Test of mu = 25 vs not = 25
Variable    N      Mean    StDev  SE Mean         90% CI              T       P
Ex3-3      10  26.0000  1.6248   0.5138  (25.0581, 26.9419)  1.95  0.083
```

(c)

MTB > Graph > Probability Plot > Single



The plotted points fall approximately along a straight line, so the assumption that battery life is normally distributed is appropriate.

3-4.

$x \sim N(\mu, \sigma)$; $n = 10$; $\bar{x} = 26.0$ h; $s = 1.62$ h; $\alpha = 0.05$; $t_{\alpha,\, n-1} = t_{0.05,9} = 1.833$

$$\bar{x} - t_{\alpha,n-1}\left(S/\sqrt{n}\right) \le \mu$$

$$26.0 - 1.833\left(1.62/\sqrt{10}\right) \le \mu$$

$$25.06 \le \mu$$

The manufacturer might be interested in a lower confidence interval on mean battery life when establishing a warranty policy.

3-5.

(a)

$x \sim N(\mu, \sigma)$, $n = 10$, $\bar{x} = 13.39618 \times 1000$ Å, $s = 0.00391$

$\mu_0 = 13.4 \times 1000$ Å, $\alpha = 0.05$

Test H_0: $\mu = 13.4$ vs. H_1: $\mu \ne 13.4$. Reject H_0 if $|t_0| > t_{\alpha/2}$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{13.39618 - 13.4}{0.00391/\sqrt{10}} = -3.089$$

$t_{\alpha/2,\, n-1} = t_{0.025,\, 9} = 2.262$

Reject H_0: $\mu = 13.4$, and conclude that the mean thickness differs from 13.4×1000 Å.

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-5

```
Test of mu = 13.4 vs not = 13.4
Variable   N     Mean   StDev  SE Mean        95% CI             T      P
Ex3-5     10  13.3962  0.0039   0.0012  (13.3934, 13.3990)  -3.09  0.013
```

(b)

$\alpha = 0.01$

$$\bar{x} - t_{\alpha/2,n-1}\left(S/\sqrt{n}\right) \le \mu \le \bar{x} + t_{\alpha/2,n-1}\left(S/\sqrt{n}\right)$$

$$13.39618 - 3.2498\left(0.00391/\sqrt{10}\right) \le \mu \le 13.39618 + 3.2498\left(0.00391/\sqrt{10}\right)$$

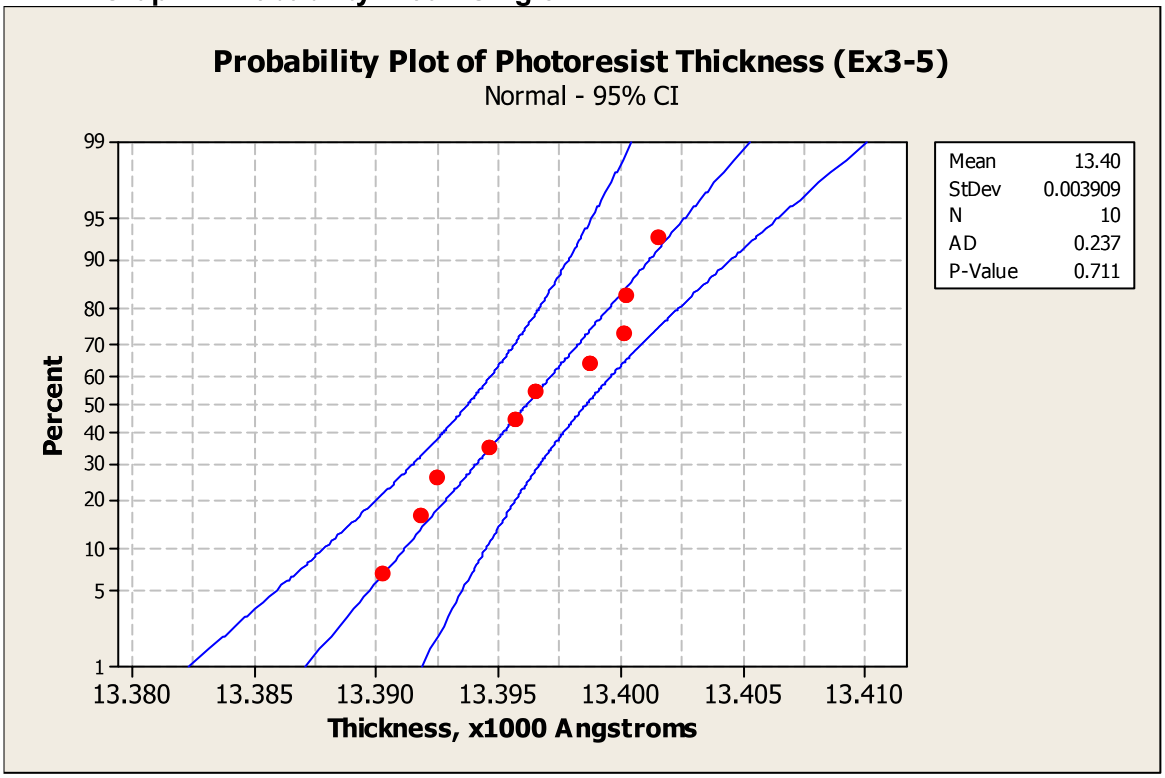
$$13.39216 \le \mu \le 13.40020$$

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-5

```
Test of mu = 13.4 vs not = 13.4
Variable   N     Mean   StDev  SE Mean        99% CI             T      P
Ex3-5     10  13.3962  0.0039   0.0012  (13.3922, 13.4002)  -3.09  0.013
```

3-5 continued
(c)
MTB > Graph > Probability Plot > Single



The plotted points form a reverse-"S" shape, instead of a straight line, so the assumption that battery life is normally distributed is not appropriate.

3-6.
(a)
$x \sim N(\mu, \sigma)$, $\mu_0 = 12$, $\alpha = 0.01$
$n = 10$, $\bar{x} = 12.015$, $s = 0.030$
Test H_0: $\mu = 12$ vs. H_1: $\mu > 12$. Reject H_0 if $t_0 > t_\alpha$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{12.015 - 12}{0.0303/\sqrt{10}} = 1.5655$$

$t_{\alpha/2,\, n-1} = t_{0.005,\, 9} = 3.250$
Do not reject H_0: $\mu = 12$, and conclude that there is not enough evidence that the mean fill volume exceeds 12 oz.

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-6
```
Test of mu = 12 vs > 12
                                              99%
                                            Lower
Variable    N     Mean    StDev  SE Mean    Bound     T      P
Ex3-6      10  12.0150   0.0303   0.0096  11.9880  1.57  0.076
```

3-6 continued
(b)
$\alpha = 0.05$
$t_{\alpha/2,\, n-1} = t_{0.025,\, 9} = 2.262$

$$\bar{x} - t_{\alpha/2,n-1}\left(S/\sqrt{n}\right) \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}\left(S/\sqrt{n}\right)$$

$$12.015 - 2.262\left(S/\sqrt{10}\right) \leq \mu \leq 12.015 + 2.62\left(S/\sqrt{10}\right)$$

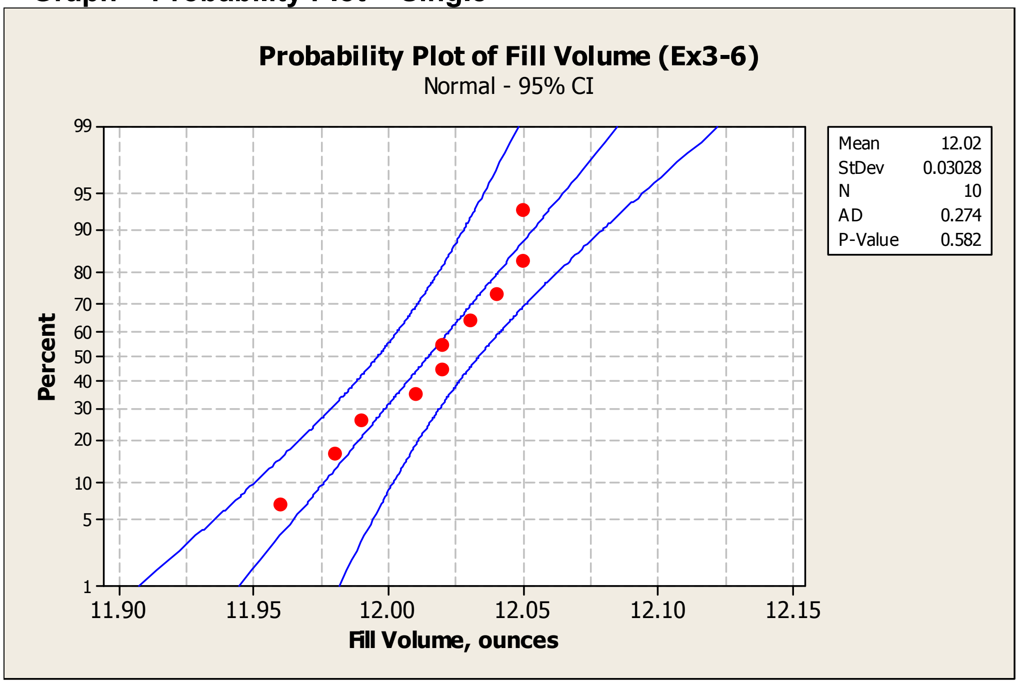
$$11.993 \leq \mu \leq 12.037$$

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-6

```
Test of mu = 12 vs not = 12
Variable    N     Mean   StDev  SE Mean        95% CI            T      P
Ex3-6      10  12.0150  0.0303   0.0096  (11.9933, 12.0367)  1.57  0.152
```

(c)
MTB > Graph > Probability Plot > Single



The plotted points fall approximately along a straight line, so the assumption that fill volume is normally distributed is appropriate.

3-7.
$\sigma = 4$ lb, $\alpha = 0.05$, $Z_{\alpha/2} = Z_{0.025} = 1.9600$, total confidence interval width = 1 lb, find n

$$2\left[Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)\right] = \text{total width}$$

$$2\left[1.9600\left(4/\sqrt{n}\right)\right] = 1$$

$$n = 246$$

3-8.

(a)

$x \sim N(\mu, \sigma)$, $\mu_0 = 0.5025$, $\alpha = 0.05$

$n = 25$, $\bar{x} = 0.5046$ in, $\sigma = 0.0001$ in

Test H_0: $\mu = 0.5025$ vs. H_1: $\mu \neq 0.5025$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{0.5046 - 0.5025}{0.0001/\sqrt{25}} = 105$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$

Reject H_0: $\mu = 0.5025$, and conclude that the mean rod diameter differs from 0.5025.

MTB > Stat > Basic Statistics > 1-Sample Z > Summarized data

One-Sample Z

```
Test of mu = 0.5025 vs not = 0.5025
The assumed standard deviation = 0.0001
 N      Mean   SE Mean         95% CI              Z      P
25  0.504600  0.000020  (0.504561, 0.504639)  105.00  0.000
```

(b)

P-value $= 2[1 - \Phi(Z_0)] = 2[1 - \Phi(105)] = 2[1 - 1] = 0$

(c)

$$\bar{x} - Z_{\alpha/2}\left(\sigma/\sqrt{n}\right) \le \mu \le \bar{x} + Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$$

$$0.5046 - 1.960\left(0.0001/\sqrt{25}\right) \le \mu \le 0.5046 + 1.960\left(0.0001/\sqrt{25}\right)$$

$$0.50456 \le \mu \le 0.50464$$

3-9.

$x \sim N(\mu, \sigma)$, $n = 16$, $\bar{x} = 10.259$ V, $s = 0.999$ V

(a)

$\mu_0 = 12$, $\alpha = 0.05$

Test H_0: $\mu = 12$ vs. H_1: $\mu \neq 12$. Reject H_0 if $|t_0| > t_{\alpha/2}$.

$$t_0 = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{10.259 - 12}{0.999/\sqrt{16}} = -6.971$$

$t_{\alpha/2,\, n-1} = t_{0.025,\, 15} = 2.131$

Reject H_0: $\mu = 12$, and conclude that the mean output voltage differs from 12V.

MTB > Stat > Basic Statistics > 1-Sample t > Samples in columns

One-Sample T: Ex3-9

```
Test of mu = 12 vs not = 12
Variable   N     Mean   StDev  SE Mean         95% CI            T      P
Ex3-9     16  10.2594  0.9990   0.2498  (9.7270, 10.7917)  -6.97  0.000
```

3-9 continued
(b)

$$\bar{x} - t_{\alpha/2,n-1}\left(S/\sqrt{n}\right) \le \mu \le \bar{x} + t_{\alpha/2,n-1}\left(S/\sqrt{n}\right)$$

$$10.259 - 2.131\left(0.999/\sqrt{16}\right) \le \mu \le 10.259 + 2.131\left(0.999/\sqrt{16}\right)$$

$$9.727 \le \mu \le 10.792$$

(c)
$\sigma_0^2 = 1$, $\alpha = 0.05$
Test H_0: $\sigma^2 = 1$ vs. H_1: $\sigma^2 \neq 1$. Reject H_0 if $\chi^2_0 > \chi^2_{\alpha/2,\, n-1}$ or $\chi^2_0 < \chi^2_{1-\alpha/2,\, n-1}$.

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(16-1)0.999^2}{1} = 14.970$$

$\chi^2_{\alpha/2,\, n-1} = \chi^2_{0.025,16-1} = 27.488$
$\chi^2_{1-\alpha/2,\, n-1} = \chi^2_{0.975,16-1} = 6.262$
Do not reject H_0: $\sigma^2 = 1$, and conclude that there is insufficient evidence that the variance differs from 1.

(d)

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{(16-1)0.999^2}{27.488} \le \sigma^2 \le \frac{(16-1)0.999^2}{6.262}$$

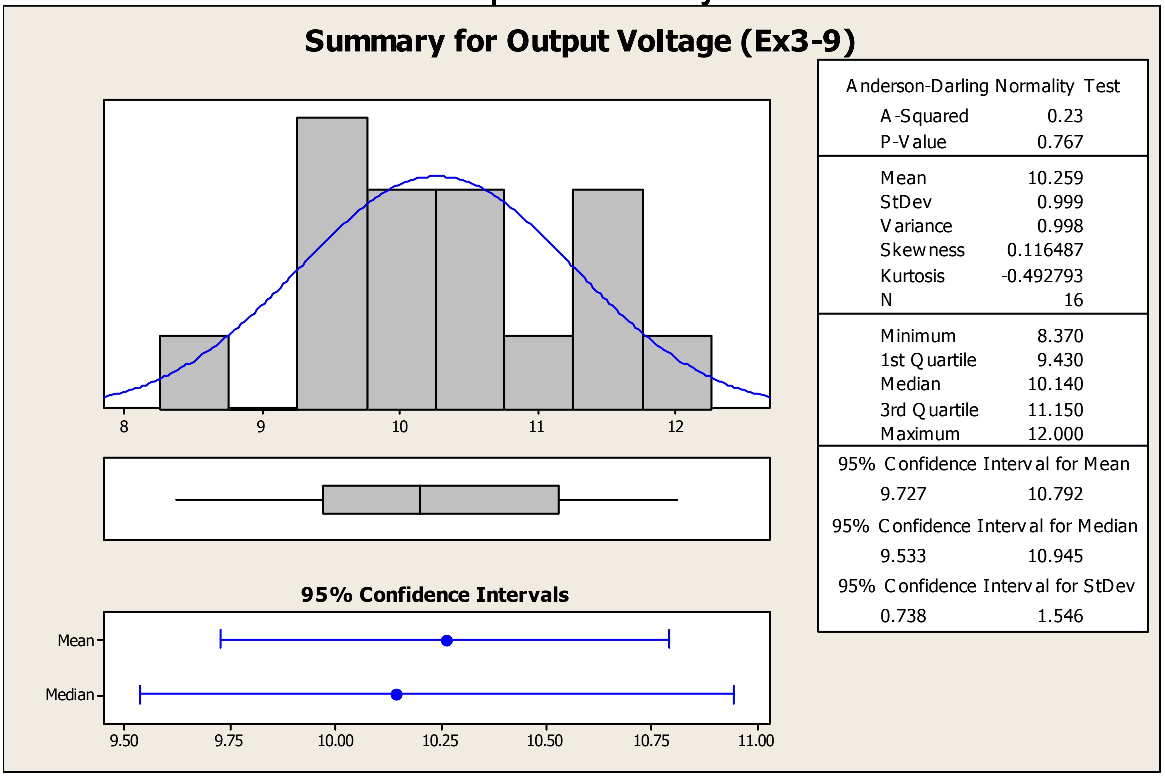
$$0.545 \le \sigma^2 \le 2.391$$

$$0.738 \le \sigma \le 1.546$$

Since the 95% confidence interval on σ contains the hypothesized value, $\sigma_0^2 = 1$, the null hypothesis, H_0: $\sigma^2 = 1$, cannot be rejected.

3-9 (d) continued

MTB > Stat > Basic Statistics > Graphical Summary



(e)

$$\alpha = 0.05; \quad \chi^2_{1-\alpha,n-1} = \chi^2_{0.95,15} = 7.2609$$

$$\sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha,n-1}}$$

$$\sigma^2 \le \frac{(16-1)0.999^2}{7.2609}$$

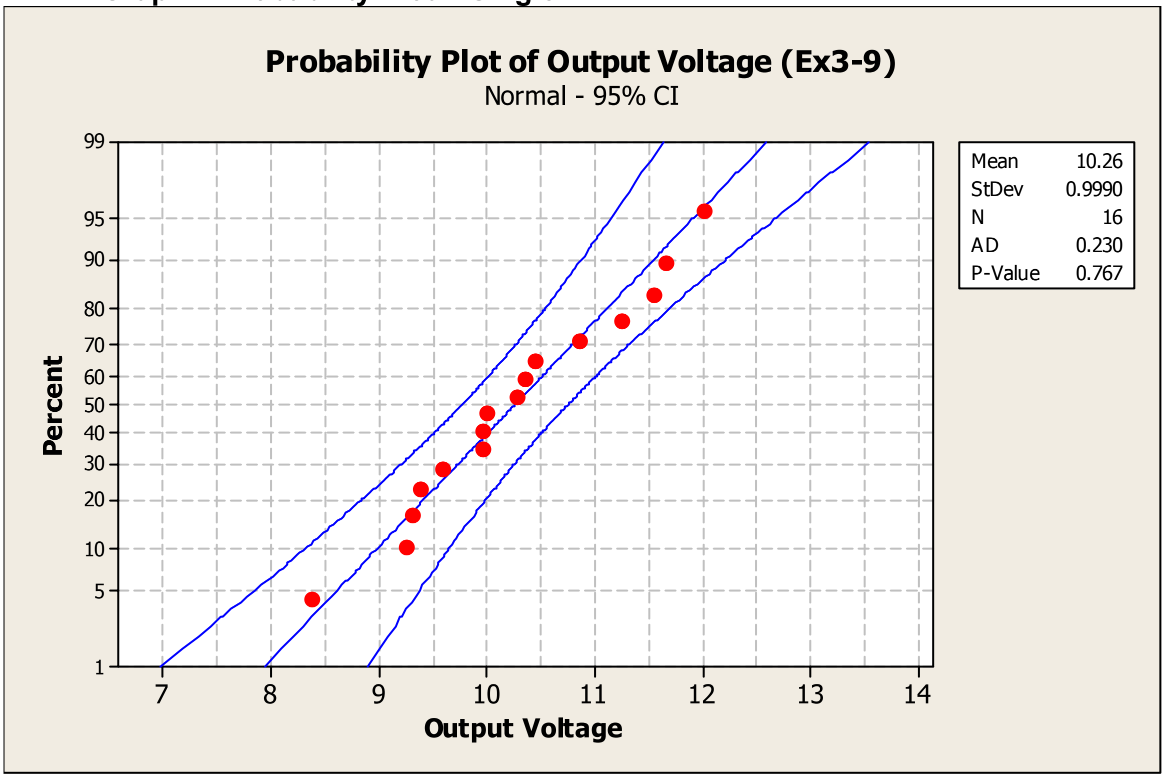
$$\sigma^2 \le 2.062$$

$$\sigma \le 1.436$$

3-9 continued

(f)

MTB > Graph > Probability Plot > Single



From visual examination of the plot, the assumption of a normal distribution for output voltage seems appropriate.

3-10.

$n_1 = 25$, $\bar{x}_1 = 2.04\ l$, $\sigma_1 = 0.010\ l$; $n_2 = 20$, $\bar{x}_2 = 2.07\ l$, $\sigma_2 = 0.015\ l$;

(a)

$\alpha = 0.05$, $\Delta_0 = 0$

Test H_0: $\mu_1 - \mu_2 = 0$ versus H_0: $\mu_1 - \mu_2 \neq 0$. Reject H_0 if $Z_0 > Z_{\alpha/2}$ or $Z_0 < -Z_{\alpha/2}$.

$$Z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} = \frac{(2.04 - 2.07) - 0}{\sqrt{0.010^2/25 + 0.015^2/20}} = -7.682$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96 \quad -Z_{\alpha/2} = -1.96$

Reject H_0: $\mu_1 - \mu_2 = 0$, and conclude that there is a difference in mean net contents between machine 1 and machine 2.

(b)

P-value $= 2[1 - \Phi(Z_0)] = 2[1 - \Phi(-7.682)] = 2[1 - 1.00000] = 0$

3-10 continued
(c)

$$(\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2}\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2} \le (\mu_1 - \mu_2) \le (\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2}\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}$$

$$(2.04 - 2.07) - 1.9600\sqrt{0.010^2/25 + 0.015^2/20} \le (\mu_1 - \mu_2) \le (2.04 - 2.07) + 1.9600\sqrt{0.010^2/25 + 0.015^2/20}$$

$$-0.038 \le (\mu_1 - \mu_2) \le -0.022$$

The confidence interval for the difference does not contain zero. We can conclude that the machines do not fill to the same volume.

3-11.
(a)
MTB > Stat > Basic Statistics > 2-Sample t > Samples in different columns

Two-Sample T-Test and CI: Ex3-11T1, Ex3-11T2

```
Two-sample T for Ex3-11T1 vs Ex3-11T2
          N   Mean  StDev  SE Mean
Ex3-11T1  7  1.383  0.115    0.043
Ex3-11T2  8  1.376  0.125    0.044
Difference = mu (Ex3-11T1) - mu (Ex3-11T2)
Estimate for difference:  0.006607
95% CI for difference:  (-0.127969, 0.141183)
T-Test of difference = 0 (vs not =): T-Value = 0.11  P-Value = 0.917  DF = 13
Both use Pooled StDev = 0.1204
```

Do not reject H_0: $\mu_1 - \mu_2 = 0$, and conclude that there is not sufficient evidence of a difference between measurements obtained by the two technicians.

(b)
The practical implication of this test is that it does not matter which technician measures parts; the readings will be the same. If the null hypothesis had been rejected, we would have been concerned that the technicians obtained different measurements, and an investigation should be undertaken to understand why.

(c)
$n_1 = 7$, $\bar{x}_1 = 1.383$, $S_1 = 0.115$; $n_2 = 8$, $\bar{x}_2 = 1.376$, $S_2 = 0.125$

$\alpha = 0.05$, $t_{\alpha/2,\, n1+n2-2} = t_{0.025,\, 13} = 2.1604$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(7-1)0.115^2 + (8-1)0.125^2}{7+8-2}} = 0.120$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2} S_p\sqrt{1/n_1 + 1/n_2} \le (\mu_1 - \mu_2) \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2} S_p\sqrt{1/n_1 + 1/n_2}$$

$$(1.383 - 1.376) - 2.1604(0.120)\sqrt{1/7 + 1/8} \le (\mu_1 - \mu_2) \le (1.383 - 1.376) + 2.1604(0.120)\sqrt{1/7 + 1/8}$$

$$-0.127 \le (\mu_1 - \mu_2) \le 0.141$$

The confidence interval for the difference contains zero. We can conclude that there is no difference in measurements obtained by the two technicians.

3-11 continued

(d)

$\alpha = 0.05$

Test $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 \neq \sigma_2^2$.

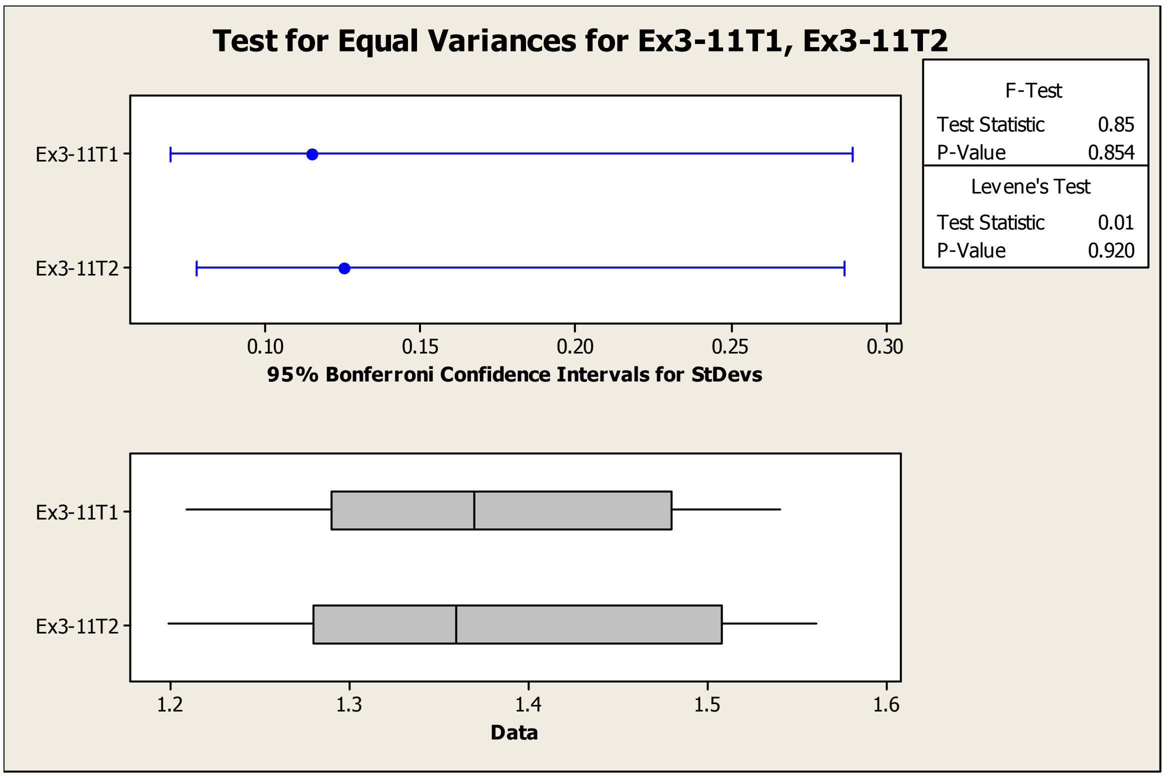
Reject H_0 if $F_0 > F_{\alpha/2, n_1-1, n_2-1}$ or $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$.

$F_0 = S_1^2 / S_2^2 = 0.115^2 / 0.125^2 = 0.8464$

$F_{\alpha/2, n_1-1, n_2-1} = F_{0.05/2, 7-1, 8-1} = F_{0.025, 6, 7} = 5.119$

$F_{1-\alpha/2, n_1-1, n_2-1} = F_{1-0.05/2, 7-1, 8-1} = F_{0.975, 6, 7} = 0.176$

MTB > Stat > Basic Statistics > 2 Variances > Summarized data



Do not reject H_0, and conclude that there is no difference in variability of measurements obtained by the two technicians.

If the null hypothesis is rejected, we would have been concerned about the difference in measurement variability between the technicians, and an investigation should be undertaken to understand why.

3-11 continued

(e)

$\alpha = 0.05 \qquad F_{1-\alpha/2,n_2-1,n_1-1} = F_{0.975,7,6} = 0.1954; \quad F_{\alpha/2,n_2-1,n_1-1} = F_{0.025,7,6} = 5.6955$

$$\frac{S_1^2}{S_2^2} F_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2} F_{\alpha/2,n_2-1,n_1-1}$$

$$\frac{0.115^2}{0.125^2}(0.1954) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{0.115^2}{0.125^2}(5.6955)$$

$$0.165 \le \frac{\sigma_1^2}{\sigma_2^2} \le 4.821$$

(f)

$n_2 = 8; \quad \bar{x}_2 = 1.376; \quad S_2 = 0.125$

$\alpha = 0.05; \quad \chi^2_{\alpha/2,n_2-1} = \chi^2_{0.025,7} = 16.0128; \quad \chi^2_{1-\alpha/2,n_2-1} = \chi^2_{0.975,7} = 1.6899$

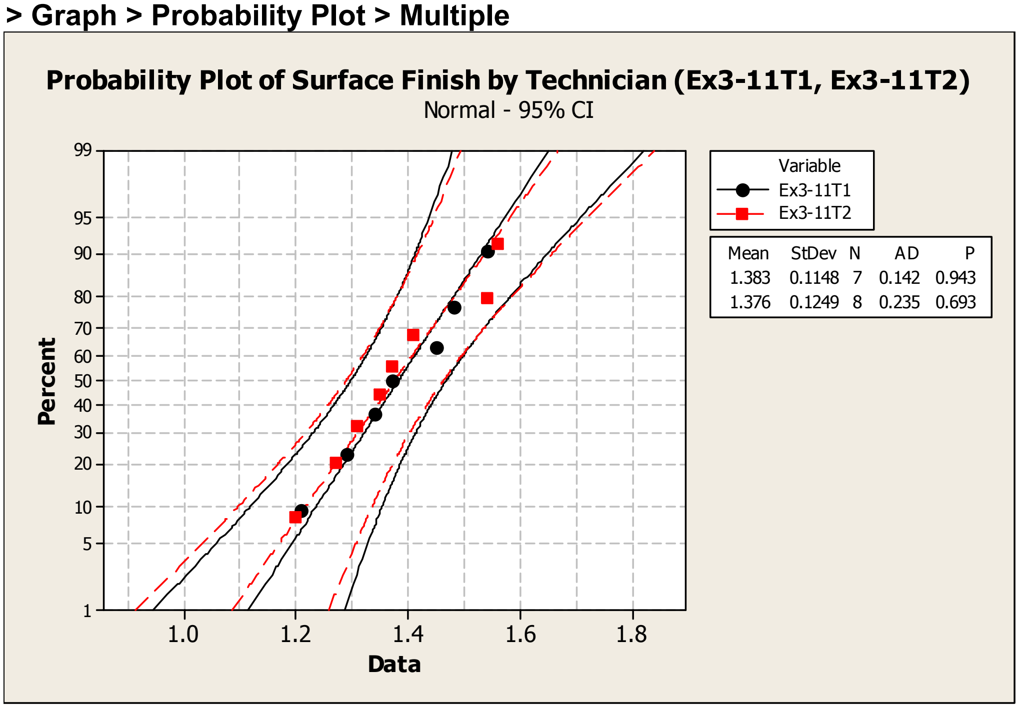
$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{(8-1)0.125^2}{16.0128} \le \sigma^2 \le \frac{(8-1)0.125^2}{1.6899}$$

$$0.007 \le \sigma^2 \le 0.065$$

(g)

MTB > Graph > Probability Plot > Multiple



The normality assumption seems reasonable for these readings.

3-12.

From Eqn. 3-54 and 3-55, for $\sigma_1^2 \neq \sigma_2^2$ and both unknown, the test statistic is

$$t_0^* = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S_1^2/n_1 + S_2^2/n_2}} \text{ with degrees of freedom } \nu = \frac{\left(S_1^2/n_1 + S_2^2/n_2\right)^2}{\frac{\left(S_1^2/n_1\right)^2}{(n_1+1)} + \frac{\left(S_2^2/n_2\right)^2}{(n_2+1)}} - 2$$

A 100(1-α)% confidence interval on the difference in means would be:

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,\nu}\sqrt{S_1^2/n_1 + S_2^2/n_2} \leq (\mu_1 - \mu_2) \leq (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,\nu}\sqrt{S_1^2/n_1 + S_2^2/n_2}$$

3-13.

Saltwater quench: $n_1 = 10$, $\bar{x}_1 = 147.6$, $S_1 = 4.97$

Oil quench: $n_2 = 10$, $\bar{x}_2 = 149.4$, $S_2 = 5.46$

(a)

Assume $\sigma_1^2 = \sigma_2^2$

MTB > Stat > Basic Statistics > 2-Sample t > Samples in different columns

Two-Sample T-Test and CI: Ex3-13SQ, Ex3-13OQ

```
Two-sample T for Ex3-13SQ vs Ex3-13OQ
           N    Mean  StDev  SE Mean
Ex3-13SQ  10  147.60   4.97      1.6
Ex3-13OQ  10  149.40   5.46      1.7
Difference = mu (Ex3-13SQ) - mu (Ex3-13OQ)
Estimate for difference:  -1.80000
95% CI for difference:  (-6.70615, 3.10615)
T-Test of difference = 0 (vs not =): T-Value = -0.77  P-Value = 0.451  DF = 18
Both use Pooled StDev = 5.2217
```

Do not reject H_0, and conclude that there is no difference between the quenching processes.

(b)

$\alpha = 0.05$, $t_{\alpha/2,\, n1+n2-2} = t_{0.025,\, 18} = 2.1009$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} = \sqrt{\frac{(10-1)4.97^2 + (10-1)5.46^2}{10+10-2}} = 5.22$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,n_1+n_2-2}S_p\sqrt{1/n_1 + 1/n_2} \leq (\mu_1 - \mu_2) \leq (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,n_1+n_2-2}S_p\sqrt{1/n_1 + 1/n_2}$$

$$(147.6 - 149.4) - 2.1009(5.22)\sqrt{1/10 + 1/10} \leq (\mu_1 - \mu_2) \leq (147.6 - 149.4) + 2.1009(5.22)\sqrt{1/10 + 1/10}$$

$$-6.7 \leq (\mu_1 - \mu_2) \leq 3.1$$

3-13 continued

(c)

$\alpha = 0.05 \qquad F_{1-\alpha/2,n_2-1,n_1-1} = F_{0.975,9,9} = 0.2484; \;\; F_{\alpha/2,n_2-1,n_1-1} = F_{0.025,9,9} = 4.0260$

$$\frac{S_1^2}{S_2^2} F_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2} F_{\alpha/2,n_2-1,n_1-1}$$

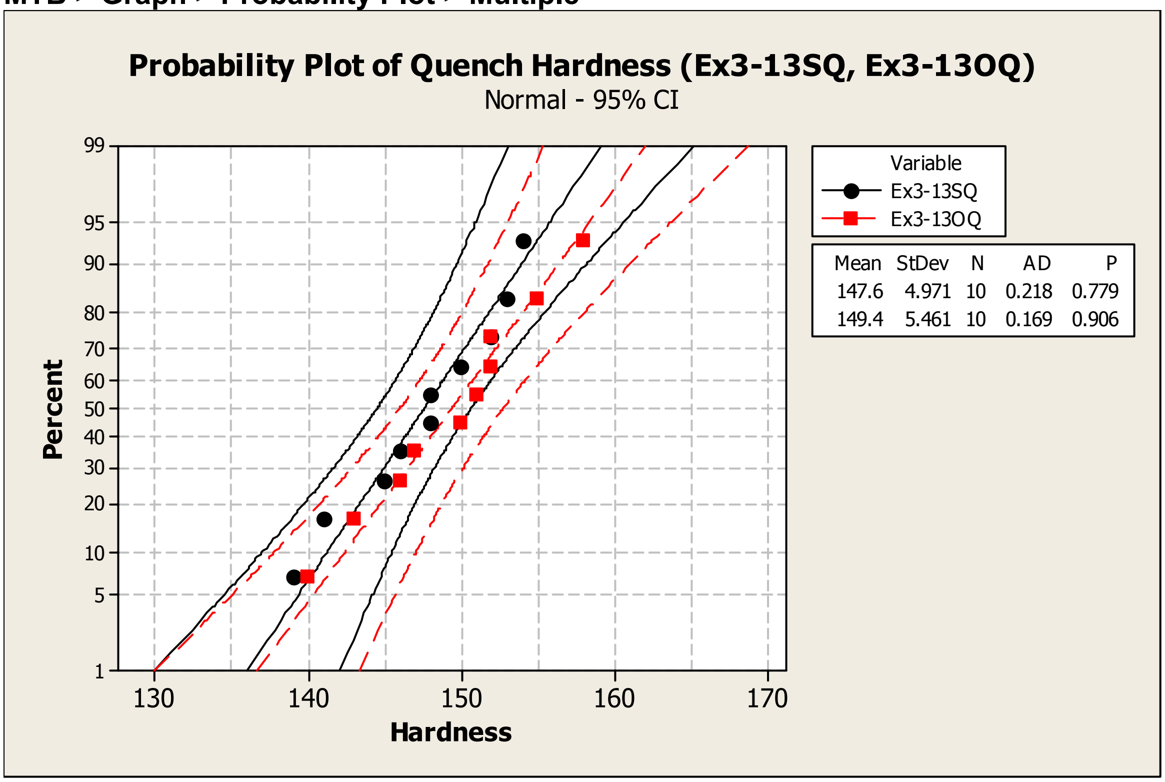
$$\frac{4.97^2}{5.46^2}(0.2484) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{4.97^2}{5.46^2}(4.0260)$$

$$0.21 \le \frac{\sigma_1^2}{\sigma_2^2} \le 3.34$$

Since the confidence interval includes the ratio of 1, the assumption of equal variances seems reasonable.

(d)

MTB > Graph > Probability Plot > Multiple



The normal distribution assumptions for both the saltwater and oil quench methods seem reasonable.

3-14.
$n = 200$, $x = 18$, $\hat{p} = x/n = 18/200 = 0.09$

(a)
$p_0 = 0.10$, $\alpha = 0.05$. Test H_0: $p = 0.10$ versus H_1: $p \neq 0.10$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.
$np_0 = 200(0.10) = 20$

Since $(x = 18) < (np_0 = 20)$, use the normal approximation to the binomial for $x < np_0$.

$$Z_0 = \frac{(x+0.5)-np_0}{\sqrt{np_0(1-p_0)}} = \frac{(18+0.5)-20}{\sqrt{20(1-0.10)}} = -0.3536$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$

Do not reject H_0, and conclude that the sample process fraction nonconforming does not differ from 0.10.

P-value $= 2[1 - \Phi|Z_0|] = 2[1 - \Phi|-0.3536|] = 2[1 - 0.6382] = 0.7236$

MTB > Stat > Basic Statistics > 1 Proportion > Summarized data

Test and CI for One Proportion

```
Test of p = 0.1 vs p not = 0.1
Sample   X    N  Sample p         95% CI          Z-Value  P-Value
1       18  200  0.090000  (0.050338, 0.129662)    -0.47    0.637
```

Note that MINITAB uses an exact method, not an approximation.

(b)
$\alpha = 0.10$, $Z_{\alpha/2} = Z_{0.10/2} = Z_{0.05} = 1.645$

$$\hat{p} - Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n} \leq p \leq \hat{p} + Z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$$

$$0.09 - 1.645\sqrt{0.09(1-0.09)/200} \leq p \leq 0.09 + 1.645\sqrt{0.09(1-0.09)/200}$$

$$0.057 \leq p \leq 0.123$$

3-15.
$n = 500$, $x = 65$, $\hat{p} = x/n = 65/500 = 0.130$

(a)
$p_0 = 0.08$, $\alpha = 0.05$. Test H_0: $p = 0.08$ versus H_1: $p \neq 0.08$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.
$np_0 = 500(0.08) = 40$
Since $(x = 65) > (np_0 = 40)$, use the normal approximation to the binomial for $x > np_0$.

$$Z_0 = \frac{(x-0.5)-np_0}{\sqrt{np_0(1-p_0)}} = \frac{(65-0.5)-40}{\sqrt{40(1-0.08)}} = 4.0387$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$
Reject H_0, and conclude the sample process fraction nonconforming differs from 0.08.

MTB > Stat > Basic Statistics > 1 Proportion > Summarized data

Test and CI for One Proportion

```
Test of p = 0.08 vs p not = 0.08
Sample   X    N  Sample p        95% CI         Z-Value  P-Value
1       65  500  0.130000  (0.100522, 0.159478)     4.12    0.000
```

Note that MINITAB uses an exact method, not an approximation.

(b)
P-value $= 2[1 - \Phi|Z_0|] = 2[1 - \Phi|4.0387|] = 2[1 - 0.99997] = 0.00006$

(c)
$\alpha = 0.05$, $Z_\alpha = Z_{0.05} = 1.645$

$$p \le \hat{p} + Z_\alpha \sqrt{\hat{p}(1-\hat{p})/n}$$

$$p \le 0.13 + 1.645\sqrt{0.13(1-0.13)/500}$$

$$p \le 0.155$$

3-16.
(a)
$n_1 = 200$, $x_1 = 10$, $\hat{p}_1 = x_1/n_1 = 10/200 = 0.05$
$n_2 = 300$, $x_2 = 20$, $\hat{p}_2 = x_2/n_2 = 20/300 = 0.067$

(b)
Use $\alpha = 0.05$.
Test H_0: $p_1 = p_2$ versus H_1: $p_1 \neq p_2$. Reject H_0 if $Z_0 > Z_{\alpha/2}$ or $Z_0 < -Z_{\alpha/2}$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{10+20}{200+300} = 0.06$$

$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(1/n_1 + 1/n_2\right)}} = \frac{0.05 - 0.067}{\sqrt{0.06(1-0.06)\left(1/200 + 1/300\right)}} = -0.7842$$

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96 \quad -Z_{\alpha/2} = -1.96$

Do not reject H_0. Conclude there is no strong evidence to indicate a difference between the fraction nonconforming for the two processes.

MTB > Stat > Basic Statistics > 2 Proportions > Summarized data

Test and CI for Two Proportions
```
Sample   X    N  Sample p
1       10  200  0.050000
2       20  300  0.066667
Difference = p (1) - p (2)
Estimate for difference:  -0.0166667
95% CI for difference:  (-0.0580079, 0.0246745)
Test for difference = 0 (vs not = 0):  Z = -0.77  P-Value = 0.442
```

(c)

$$(\hat{p}_1 - \hat{p}_2) - Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \leq (p_1 - p_2) \leq (\hat{p}_1 - \hat{p}_2) + Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.050 - 0.067) - 1.645\sqrt{\frac{0.05(1-0.05)}{200} + \frac{0.067(1-0.067)}{300}} \leq (p_1 - p_2) \leq (0.05 - 0.067) + 1.645\sqrt{\frac{0.05(1-0.05)}{200} + \frac{0.067(1-0.067)}{300}}$$

$$-0.052 \leq (p_1 - p_2) \leq 0.018$$

3-17.*

before: $n_1 = 10, x_1 = 9.85,\ S_1^2 = 6.79$

after: $n_2 = 8, x_2 = 8.08,\ S_2^2 = 6.18$

(a)

Test $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 \neq \sigma_2^2$, at $\alpha = 0.05$

Reject H_0 if $F_0 > F_{\alpha/2, n_1-1, n_2-2}$ or $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$

$F_{\alpha/2, n_1-1, n_2-2} = F_{0.025,9,7} = 4.8232;\ \ F_{1-\alpha/2, n_1-1, n_2-1} = F_{0.975,9,7} = 0.2383$

$F_0 = S_1^2 \big/ S_2^2 = 6.79/6.18 = 1.0987$

$F_0 = 1.0987 < 4.8232$ and > 0.2383, so do not reject H_0

MTB > Stat > Basic Statistics > 2 Variances > Summarized data

Test for Equal Variances

```
95% Bonferroni confidence intervals for standard deviations
Sample   N    Lower    StDev    Upper
     1  10  1.70449  2.60576  5.24710
     2   8  1.55525  2.48596  5.69405
F-Test (normal distribution)
Test statistic = 1.10, p-value = 0.922
```

The impurity variances before and after installation are the same.

(b)

Test H_0: $\mu_1 = \mu_2$ versus H_1: $\mu_1 > \mu_2$, $\alpha = 0.05$.

Reject H_0 if $t_0 > t_{\alpha, n1+n2-2}$.

$t_{\alpha, n1+n2-2} = t_{0.05,\ 10+8-2} = 1.746$

$$S_P = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(10-1)6.79 + (8-1)6.18}{10+8-2}} = 2.554$$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{S_P\sqrt{1/n_1 + 1/n_2}} = \frac{9.85 - 8.08}{2.554\sqrt{1/10 + 1/8}} = 1.461$$

MTB > Stat > Basic Statistics > 2-Sample t > Summarized data

Two-Sample T-Test and CI

```
Sample   N  Mean  StDev  SE Mean
1       10  9.85   2.61     0.83
2        8  8.08   2.49     0.88
Difference = mu (1) - mu (2)
Estimate for difference:  1.77000
95% lower bound for difference:  -0.34856
T-Test of difference = 0 (vs >): T-Value = 1.46  P-Value = 0.082  DF = 16
Both use Pooled StDev = 2.5582
```

The mean impurity after installation of the new purification unit is not less than before.

3-18.

$n_1 = 16$, $\bar{x}_1 = 175.8$ psi, $n_2 = 16$, $\bar{x}_2 = 181.3$ psi, $\sigma_1 = \sigma_2 = 3.0$ psi

Want to demonstrate that μ_2 is greater than μ_1 by at least 5 psi, so H_1: $\mu_1 + 5 < \mu_2$. So test a difference $\Delta_0 = -5$, test H_0: $\mu_1 - \mu_2 = -5$ versus H_1: $\mu_1 - \mu_2 < -5$.

Reject H_0 if $Z_0 < -Z_\alpha$.

$\Delta_0 = -5 \qquad -Z_\alpha = -Z_{0.05} = -1.645$

$$Z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} = \frac{(175.8 - 181.3) - (-5)}{\sqrt{3^2/16 + 3^2/16}} = -0.4714$$

$(Z_0 = -0.4714) > -1.645$, so do not reject H_0.

The mean strength of Design 2 does not exceed Design 1 by 5 psi.

P-value $= \Phi(Z_0) = \Phi(-0.4714) = 0.3187$

MTB > Stat > Basic Statistics > 2-Sample t > Summarized data

Two-Sample T-Test and CI

```
Sample   N    Mean  StDev  SE Mean
1       16  175.80   3.00     0.75
2       16  181.30   3.00     0.75
Difference = mu (1) - mu (2)
Estimate for difference:  -5.50000
95% upper bound for difference:  -3.69978
T-Test of difference = -5 (vs <): T-Value = -0.47  P-Value = 0.320  DF = 30
Both use Pooled StDev = 3.0000
```

Note: For equal variances and sample sizes, the Z-value is the same as the t-value. The P-values are close due to the sample sizes.

3-19.

Test H_0: $\mu_d = 0$ versus H_1: $\mu_d \neq 0$. Reject H_0 if $|t_0| > t_{\alpha/2,\, n1+n2-2}$.

$t_{\alpha/2,\, n1+n2-2} = t_{0.005,22} = 2.8188$

$$\bar{d} = \frac{1}{n}\sum_{j=1}^{n}\left(x_{\text{Micrometer},j} - x_{\text{Vernier},j}\right) = \frac{1}{12}\left[(0.150-0.151)+\cdots+(0.151-0.152)\right] = -0.000417$$

$$S_d^2 = \frac{\sum_{j=1}^{n} d_j^2 - \left(\sum_{j=1}^{n} d_j\right)^2 \Big/ n}{(n-1)} = 0.001311^2$$

$$t_0 = \bar{d}\Big/\left(S_d/\sqrt{n}\right) = -0.000417\Big/\left(0.001311/\sqrt{12}\right) = -1.10$$

($|t_0| = 1.10$) < 2.8188, so do not reject H_0. There is no strong evidence to indicate that the two calipers differ in their mean measurements.

MTB > Stat > Basic Statistics > Paired t > Samples in Columns

Paired T-Test and CI: Ex3-19MC, Ex3-19VC

```
Paired T for Ex3-19MC - Ex3-19VC
             N       Mean     StDev   SE Mean
Ex3-19MC    12   0.151167  0.000835  0.000241
Ex3-19VC    12   0.151583  0.001621  0.000468
Difference  12  -0.000417  0.001311  0.000379
95% CI for mean difference: (-0.001250, 0.000417)
T-Test of mean difference = 0 (vs not = 0): T-Value = -1.10  P-Value = 0.295
```

3-20.

(a)

The alternative hypothesis H_1: μ > 150 is preferable to H_1: μ < 150 we desire a true mean weld strength greater than 150 psi. In order to achieve this result, H_0 must be rejected in favor of the alternative H_1, μ > 150.

(b)

$n = 20$, $\bar{x} = 153.7$, $s = 11.5$, $\alpha = 0.05$

Test H_0: $\mu = 150$ versus H_1: $\mu > 150$. Reject H_0 if $t_0 > t_{\alpha,\, n-1}$. $t_{\alpha,\, n-1} = t_{0.05,19} = 1.7291$.

$$t_0 = (\bar{x} - \mu)\Big/\left(S/\sqrt{n}\right) = (153.7 - 150)\Big/\left(11.5/\sqrt{20}\right) = 1.4389$$

($t_0 = 1.4389$) < 1.7291, so do not reject H_0. There is insufficient evidence to indicate that the mean strength is greater than 150 psi.

MTB > Stat > Basic Statistics > 1-Sample t > Summarized data

One-Sample T

```
Test of mu = 150 vs > 150
                                  95%
                                 Lower
 N     Mean   StDev  SE Mean     Bound     T      P
20  153.700  11.500    2.571   149.254  1.44  0.083
```

3-21.

$n = 20$, $\bar{x} = 752.6$ ml, $s = 1.5$, $\alpha = 0.05$

(a)

Test H_0: $\sigma^2 = 1$ versus H_1: $\sigma^2 < 1$. Reject H_0 if $\chi^2_0 < \chi^2_{1-\alpha,\, n-1}$.

$\chi^2_{1-\alpha,\, n-1} = \chi^2_{0.95,19} = 10.1170$

$$\chi^2_0 = \left[(n-1)S^2\right]\Big/\sigma^2_0 = \left[(20-1)1.5^2\right]\Big/1 = 42.75$$

$\chi^2_0 = 42.75 > 10.1170$, so do not reject H_0. The standard deviation of the fill volume is not less than 1ml.

(b)

$\chi^2_{\alpha/2,\, n-1} = \chi^2_{0.025,19} = 32.85$. $\chi^2_{1-\alpha/2,\, n-1} = \chi^2_{0.975,19} = 8.91$.

$$(n-1)S^2\Big/\chi^2_{\alpha/2,n-1} \le \sigma^2 \le (n-1)S^2\Big/\chi^2_{1-\alpha/2,n-1}$$

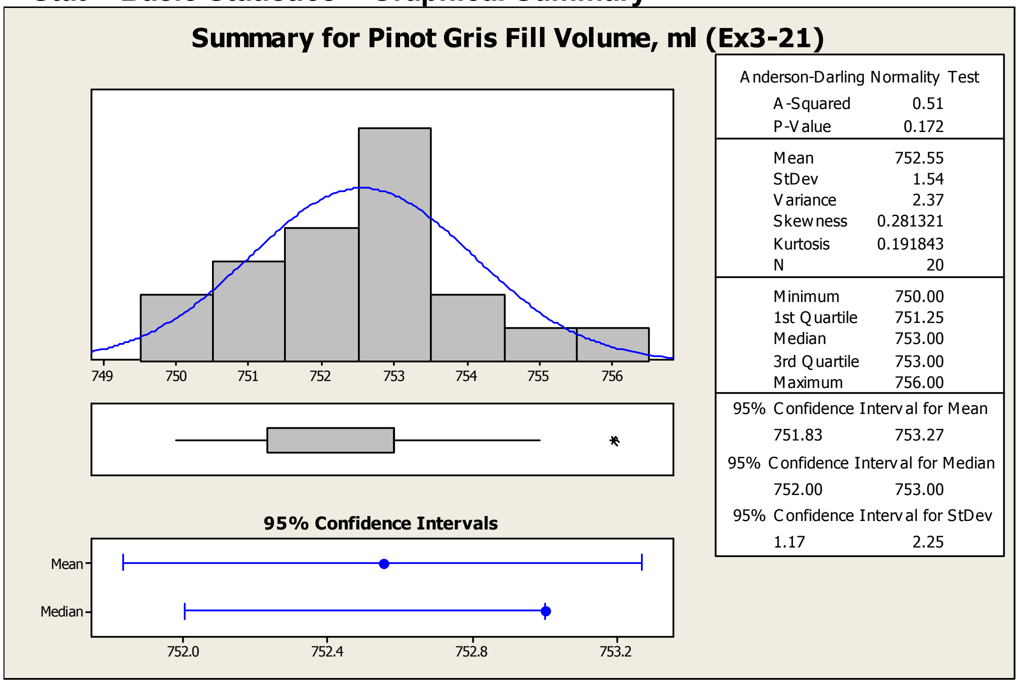
$$(20-1)1.5^2\Big/32.85 \le \sigma^2 \le (20-1)1.5^2\Big/8.91$$

$$1.30 \le \sigma^2 \le 4.80$$

$$1.14 \le \sigma \le 2.19$$

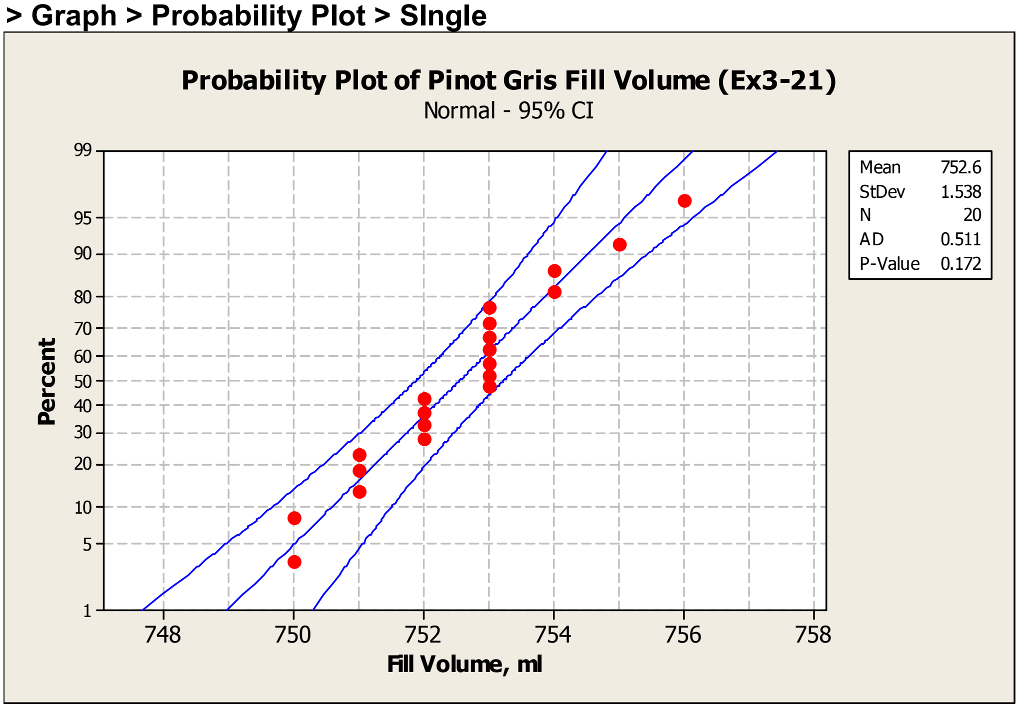
3-21 (b) continued

MTB > Stat > Basic Statistics > Graphical Summary



(c)

MTB > Graph > Probability Plot > SIngle



The plotted points do not fall approximately along a straight line, so the assumption that battery life is normally distributed is not appropriate.

3-22.

$\mu_0 = 15$, $\sigma^2 = 9.0$, $\mu_1 = 20$, $\alpha = 0.05$. Test H_0: $\mu = 15$ versus H_1: $\mu \neq 15$.

What n is needed such that the Type II error, β, is less than or equal to 0.10?

$\delta = \mu_1 - \mu_2 = 20 - 15 = 5 \qquad d = |\delta|/\sigma = 5/\sqrt{9} = 1.6667$

From Figure 3-7, the operating characteristic curve for two-sided at $\alpha = 0.05$, $n = 4$.

Check:

$$\beta = \Phi\left(Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right) - \Phi\left(-Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right) = \Phi\left(1.96 - 5\sqrt{4}/3\right) - \Phi\left(-1.96 - 5\sqrt{4}/3\right)$$

$$= \Phi(-1.3733) - \Phi(-5.2933) = 0.0848 - 0.0000 = 0.0848$$

MTB > Stat > Power and Sample Size > 1-Sample Z

Power and Sample Size

```
1-Sample Z Test
Testing mean = null (versus not = null)
Calculating power for mean = null + difference
Alpha = 0.05  Assumed standard deviation = 3
            Sample  Target
Difference    Size   Power  Actual Power
         5       4     0.9      0.915181
```

3-23.

Let $\mu_1 = \mu_0 + \delta$. From Eqn. 3-46, $\beta = \Phi\left(Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right) - \Phi\left(-Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right)$

If $\delta > 0$, then $\Phi\left(-Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right)$ is likely to be small compared with β. So,

$$\beta \approx \Phi\left(Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right)$$

$$\Phi(\beta) \approx \Phi^{-1}\left(Z_{\alpha/2} - \delta\sqrt{n}/\sigma\right)$$

$$-Z_\beta \approx Z_{\alpha/2} - \delta\sqrt{n}/\sigma$$

$$n \approx \left[(Z_{\alpha/2} + Z_\beta)\sigma/\delta\right]^2$$

3-24.

Maximize: $Z_0 = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$ Subject to: $n_1 + n_2 = N$.

Since $(\bar{x}_1 - \bar{x}_2)$ is fixed, an equivalent statement is

Minimize: $L = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2} = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{N - n_1}$

$$\frac{dL}{dn_1}\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{N - n_1}\right) = \frac{dL}{dn_1}\left[n_1^{-1}\sigma_1^2 + \left(N - n_1\right)^{-1}\sigma_2^2\right]$$

$$= -1n_1^{-2}\sigma_1^2 + (-1)(-1)\left(N - n_1\right)^{-2}\sigma_2^2 = 0$$

$$= -\frac{\sigma_1^2}{n_1^2} + \frac{\sigma_2^2}{\left(N - n_1\right)^2} = 0$$

$$\frac{n_1}{n_2} = \frac{\sigma_1}{\sigma_2}$$

Allocate N between n_1 and n_2 according to the ratio of the standard deviations.

3-25.

Given $x \sim N,\ n_1,\ \bar{x}_1,\ n_2,\ \bar{x}_2,\ x_1$ independent of x_2 .

Assume $\mu_1 = 2\mu_2$ and let $Q = (\bar{x}_1 - \bar{x}_2)$.

$$E(Q) = E(\bar{x}_1 - 2\bar{x}_2) = \mu_1 - 2\mu_2 = 0$$

$$\text{var}(Q) = \text{var}(\bar{x}_1 - 2\bar{x}_2) = \text{var}(\bar{x}_1) + \text{var}(2\bar{x}_2) = \text{var}(\bar{x}_1) + 2^2\ \text{var}(\bar{x}_2) = \frac{\text{var}(x_1)}{n_1} + 4\frac{\text{var}(x_2)}{n_2}$$

$$Z_0 = \frac{Q - 0}{SD(Q)} = \frac{\bar{x}_1 - 2\bar{x}_2}{\sqrt{\sigma_1^2/n_1 + 4\sigma_2^2/n_2}}$$

And, reject H_0 if $|Z_0| > Z_{\alpha/2}$

3-26.
(a)
Wish to test H_0: $\lambda = \lambda_0$ versus H_1: $\lambda \neq \lambda_0$.

Select random sample of n observations $x_1, x_2, \ldots, x_n$. Each $x_i \sim \text{POI}(\lambda)$. $\sum_{i=1}^{n} x_i \sim \text{POI}(n\lambda)$.

Using the normal approximation to the Poisson, if n is large, $\bar{x} = x/n = \sim \text{N}(\lambda, \lambda/n)$.
$Z_0 = (\bar{x} - \lambda) \big/ \sqrt{\lambda_0 / n}$. Reject H_0: $\lambda = \lambda_0$ if $|Z_0| > Z_{\alpha/2}$

(b)
$x \sim \text{Poi}(\lambda)$, $n = 100$, $x = 11$, $\bar{x} = x/N = 11/100 = 0.110$
Test H_0: $\lambda = 0.15$ versus H_1: $\lambda \neq 0.15$, at $\alpha = 0.01$. Reject H_0 if $|Z_0| > Z_{\alpha/2}$.
$Z_{\alpha/2} = Z_{0.005} = 2.5758$
$Z_0 = (\bar{x} - \lambda_0) \big/ \sqrt{\lambda_0/n} = (0.110 - 0.15) \big/ \sqrt{0.15/100} = -1.0328$
($|Z_0| = 1.0328$) < 2.5758, so do not reject H_0.

3-27.
$x \sim \text{Poi}(\lambda)$, $n = 5$, $x = 3$, $\bar{x} = x/N = 3/5 = 0.6$
Test H_0: $\lambda = 0.5$ versus H_1: $\lambda > 0.5$, at $\alpha = 0.05$. Reject H_0 if $Z_0 > Z_\alpha$.
$Z_\alpha = Z_{0.05} = 1.645$
$Z_0 = (\bar{x} - \lambda_0) \big/ \sqrt{\lambda_0/n} = (0.6 - 0.5) \big/ \sqrt{0.5/5} = 0.3162$
($Z_0 = 0.3162$) < 1.645, so do not reject H_0.

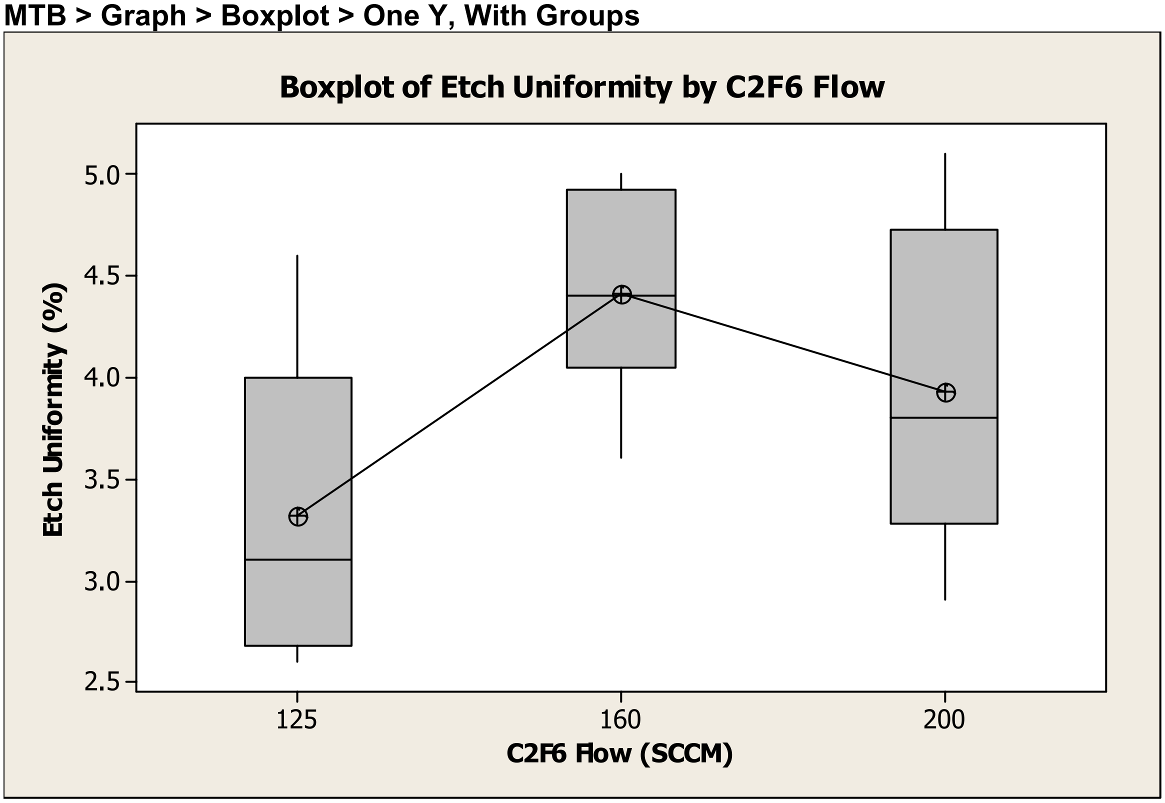
3-28.
$x \sim \text{Poi}(\lambda)$, $n = 1000$, $x = 688$, $\bar{x} = x/N = 688/1000 = 0.688$
Test H_0: $\lambda = 1$ versus H_1: $\lambda \neq 1$, at $\alpha = 0.05$. Reject H_0 if $|Z_0| > Z_\alpha$.
$Z_{\alpha/2} = Z_{0.025} = 1.96$
$Z_0 = (\bar{x} - \lambda_0) \big/ \sqrt{\lambda_0/n} = (0.688 - 1) \big/ \sqrt{1/1000} = -9.8663$
($|Z_0| = 9.8663$) > 1.96, so reject H_0.

3-29.
(a)
MTB > Stat > ANOVA > One-Way

```
One-way ANOVA: Ex3-29Obs versus Ex3-29Flow
Source      DF      SS     MS     F      P
Ex3-29Flow   2   3.648  1.824  3.59  0.053
Error       15   7.630  0.509
Total       17  11.278
S = 0.7132   R-Sq = 32.34%   R-Sq(adj) = 23.32%
                          Individual 95% CIs For Mean Based on
                          Pooled StDev
Level  N    Mean   StDev  -----+---------+---------+---------+----
125    6  3.3167  0.7600  (---------*----------)
160    6  4.4167  0.5231                    (----------*---------)
200    6  3.9333  0.8214           (----------*---------)
                          -----+---------+---------+---------+----
                             3.00      3.60      4.20      4.80
Pooled StDev = 0.7132
```

$(F_{0.05,2,15} = 3.6823) > (F_0 = 3.59)$, so flow rate does not affect etch uniformity at a significance level $\alpha = 0.05$. However, the P-value is just slightly greater than 0.05, so there is some evidence that gas flow rate affects the etch uniformity.

(b)
MTB > Stat > ANOVA > One-Way > Graphs, Boxplots of data
MTB > Graph > Boxplot > One Y, With Groups

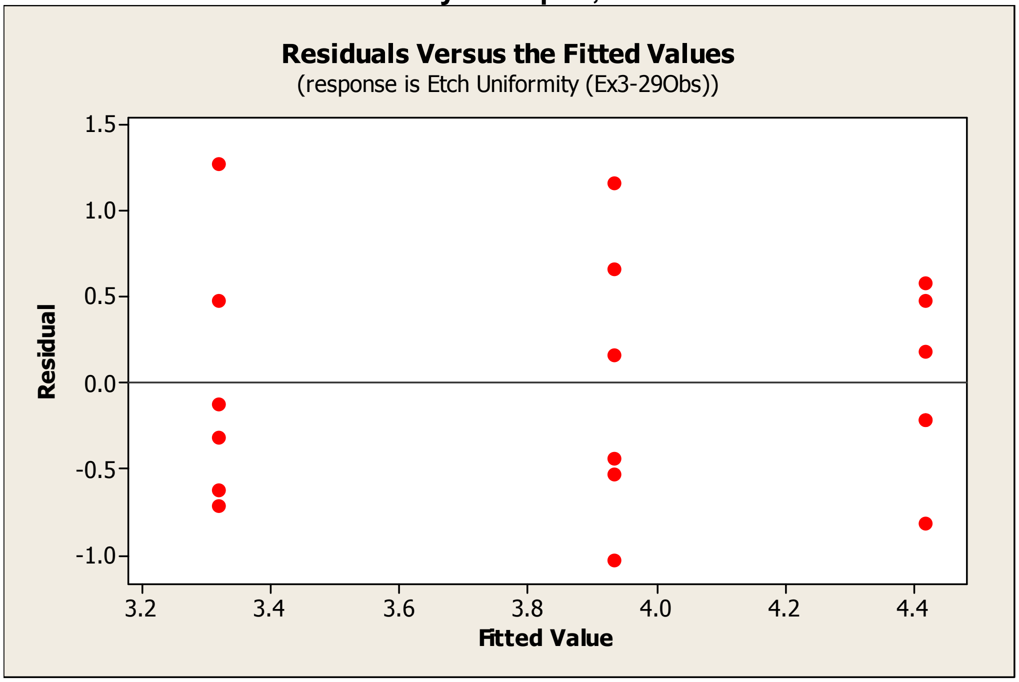


Gas flow rate of 125 SCCM gives smallest mean percentage uniformity.

3-29 continued

(c)

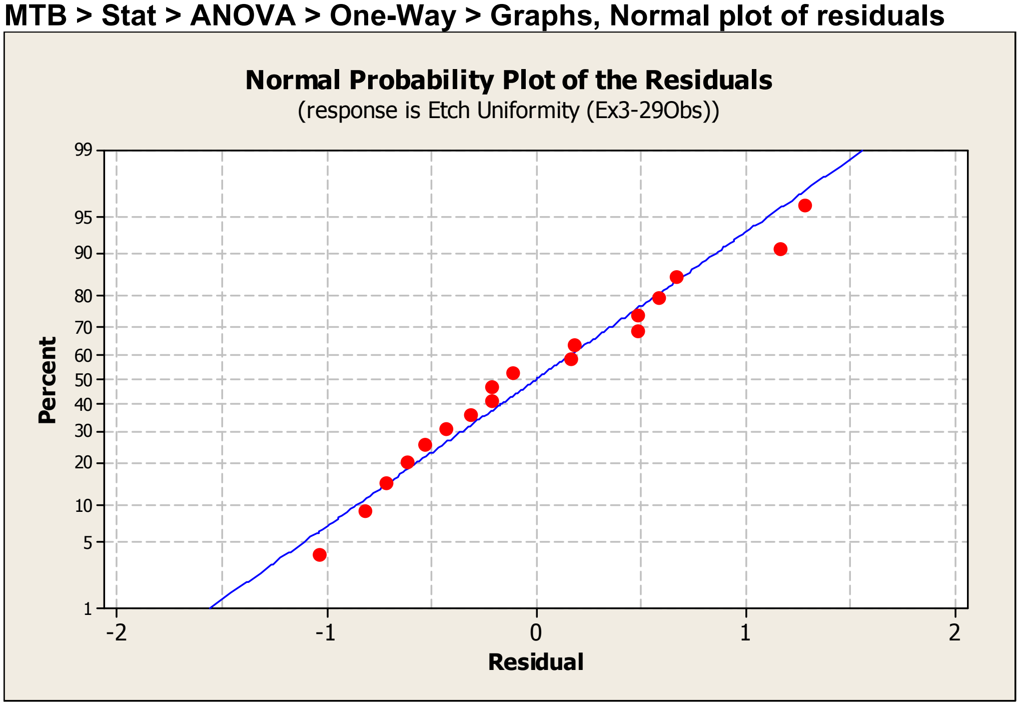
MTB > Stat > ANOVA > One-Way > Graphs, Residuals versus fits



Residuals are satisfactory.

(d)

MTB > Stat > ANOVA > One-Way > Graphs, Normal plot of residuals

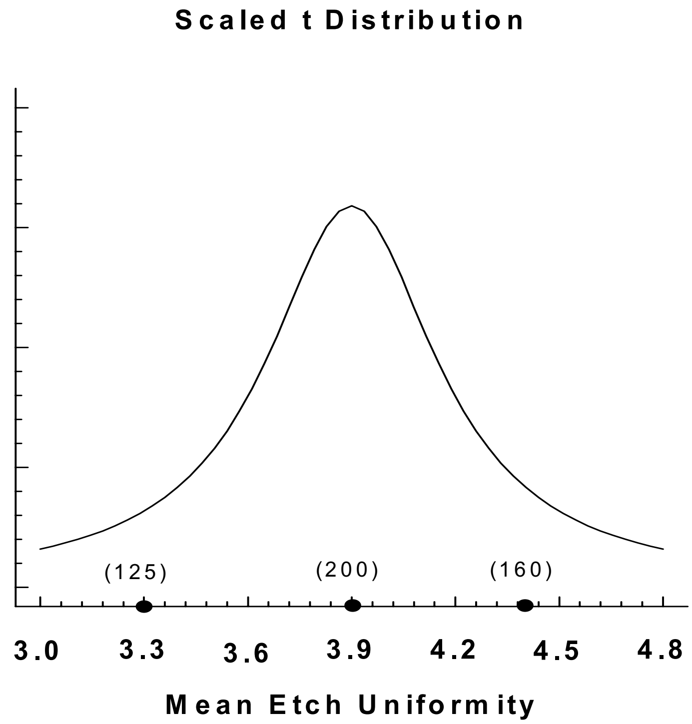


The normality assumption is reasonable.

3-30.

Flow Rate	Mean Etch Uniformity
125	3.3%
160	4.4%
200	3.9%

scale factor $= \sqrt{MS_E/n} = \sqrt{0.5087/6} = 0.3$



The graph does not indicate a large difference between the mean etch uniformity of the three different flow rates. The statistically significant difference between the mean uniformities can be seen by centering the t distribution between, say, 125 and 200, and noting that 160 would fall beyond the tail of the curve.

3-31.

(a)

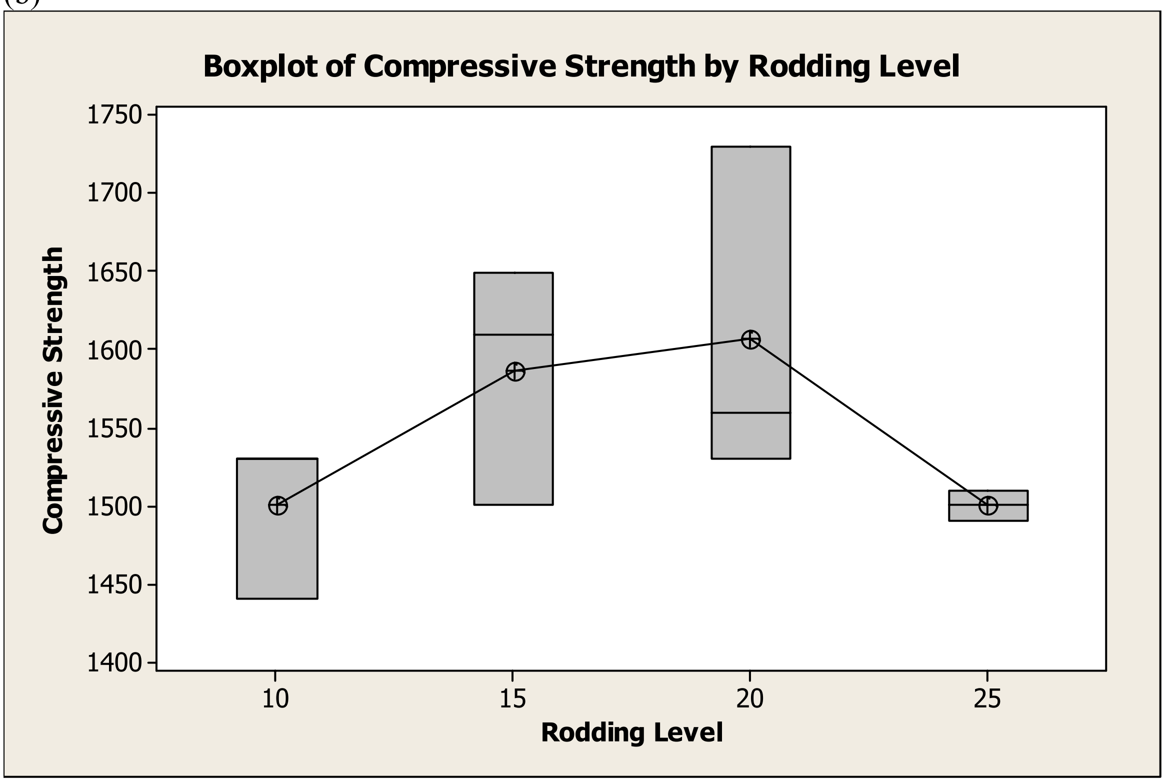
MTB > Stat > ANOVA > One-Way > Graphs> Boxplots of data, Normal plot of residuals

One-way ANOVA: Ex3-31Str versus Ex3-31Rod

```
Source     DF     SS    MS     F      P
Ex3-31Rod   3  28633  9544  1.87  0.214
Error       8  40933  5117
Total      11  69567
S = 71.53   R-Sq = 41.16%   R-Sq(adj) = 19.09%
                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level  N    Mean  StDev  ----+---------+---------+---------+-----
10     3  1500.0   52.0  (-----------*----------)
15     3  1586.7   77.7            (-----------*-----------)
20     3  1606.7  107.9               (-----------*-----------)
25     3  1500.0   10.0  (-----------*----------)
                         ----+---------+---------+---------+-----
                          1440      1520      1600      1680
Pooled StDev = 71.5
```

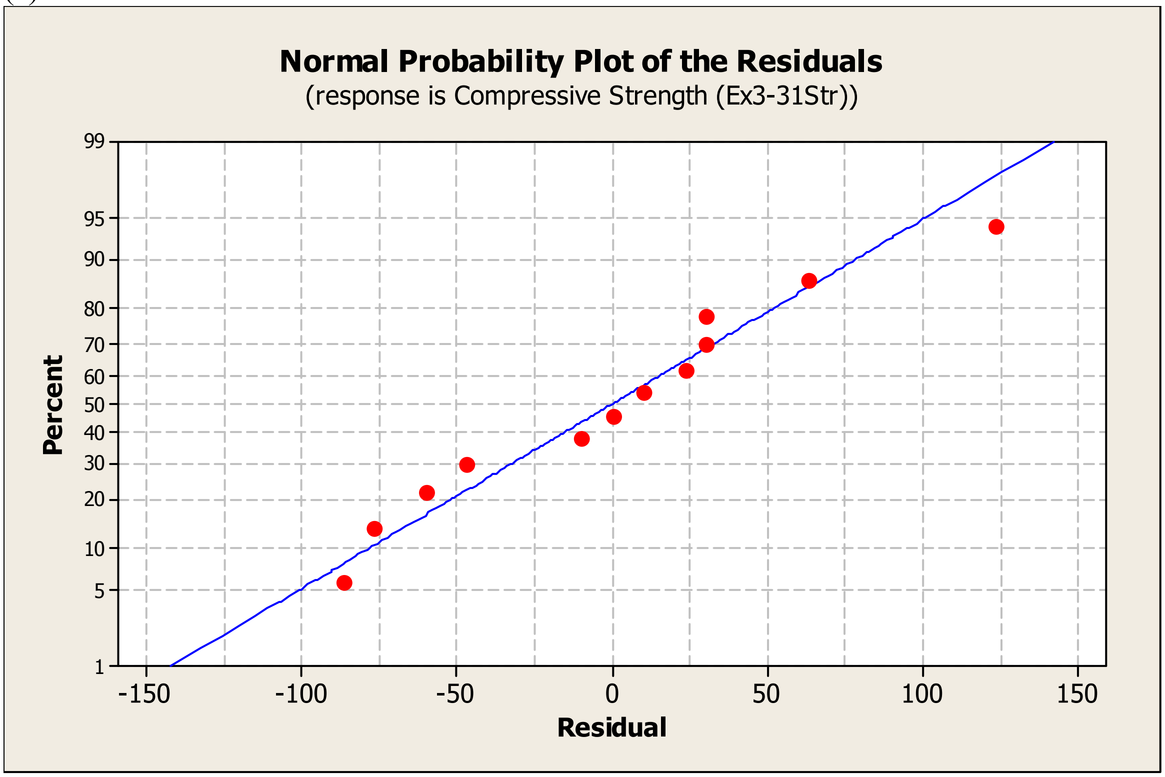
No difference due to rodding level at $\alpha = 0.05$.

(b)



Level 25 exhibits considerably less variability than the other three levels.

3-31 continued
(c)

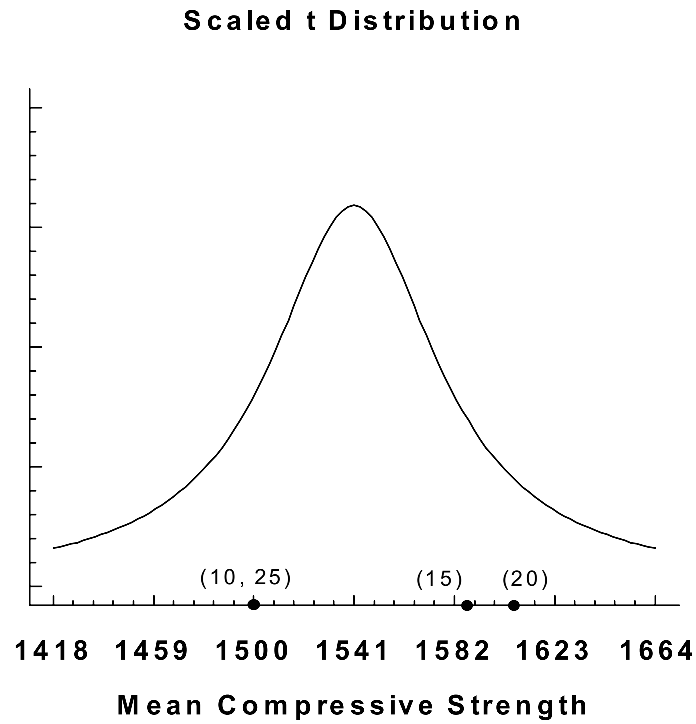


The normal distribution assumption for compressive strength is reasonable.

3-32.

Rodding Level	Mean Compressive Strength
10	1500
15	1587
20	1607
25	1500

scale factor = $\sqrt{MS_E/n} = \sqrt{5117/3} = 41$



There is no difference due to rodding level.

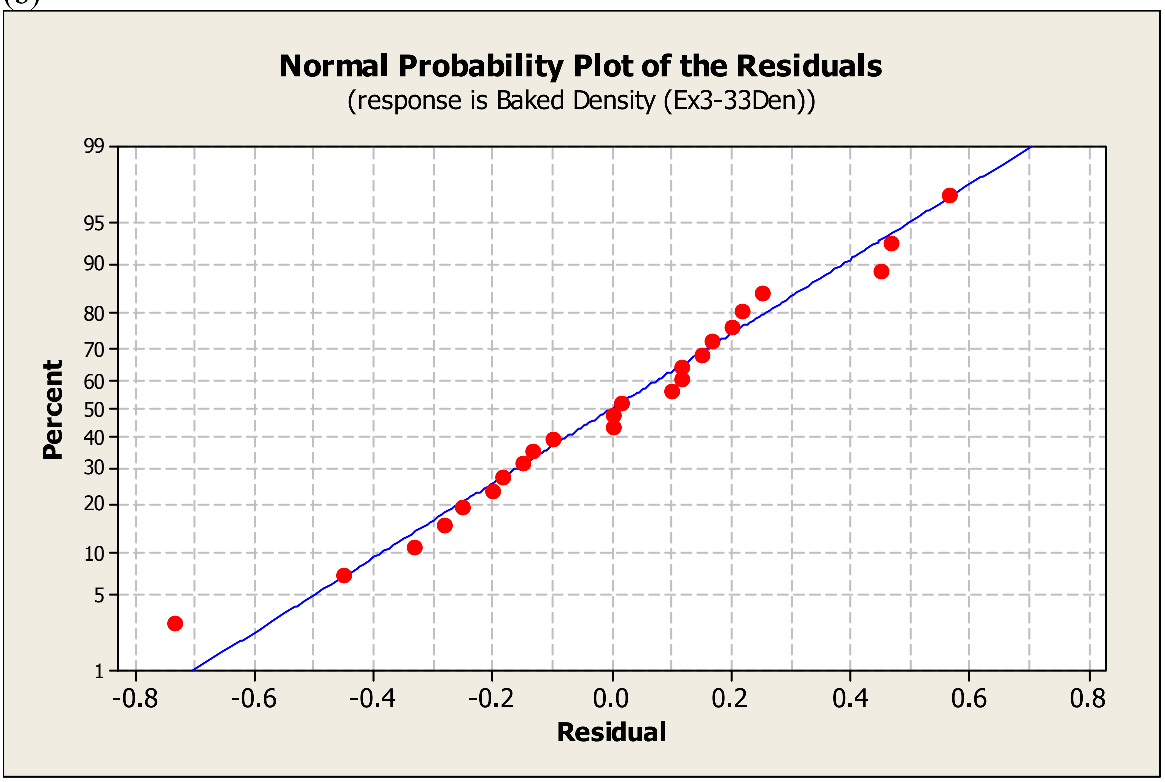
3-33.
(a)
MTB > Stat > ANOVA > One-Way > Graphs> Boxplots of data, Normal plot of residuals

One-way ANOVA: Ex3-33Den versus Ex3-33T

```
Source    DF      SS     MS     F      P
Ex3-33T    3   0.457  0.152  1.45  0.258
Error     20   2.097  0.105
Total     23   2.553
S = 0.3238    R-Sq = 17.89%    R-Sq(adj) = 5.57%
                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level   N    Mean  StDev  --------+---------+---------+---------+-
500     6  41.700  0.141                (----------*----------)
525     6  41.583  0.194           (----------*----------)
550     6  41.450  0.339       (----------*----------)
575     6  41.333  0.497  (----------*----------)
                          --------+---------+---------+---------+-
                              41.25     41.50     41.75     42.00
Pooled StDev = 0.324
```

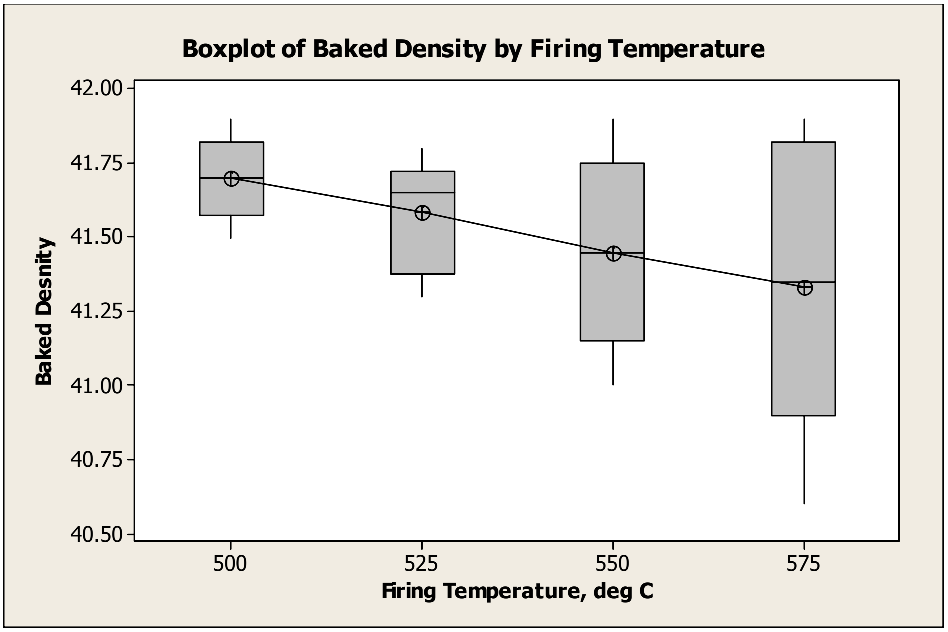
Temperature level does not significantly affect mean baked anode density.

(b)



Normality assumption is reasonable.

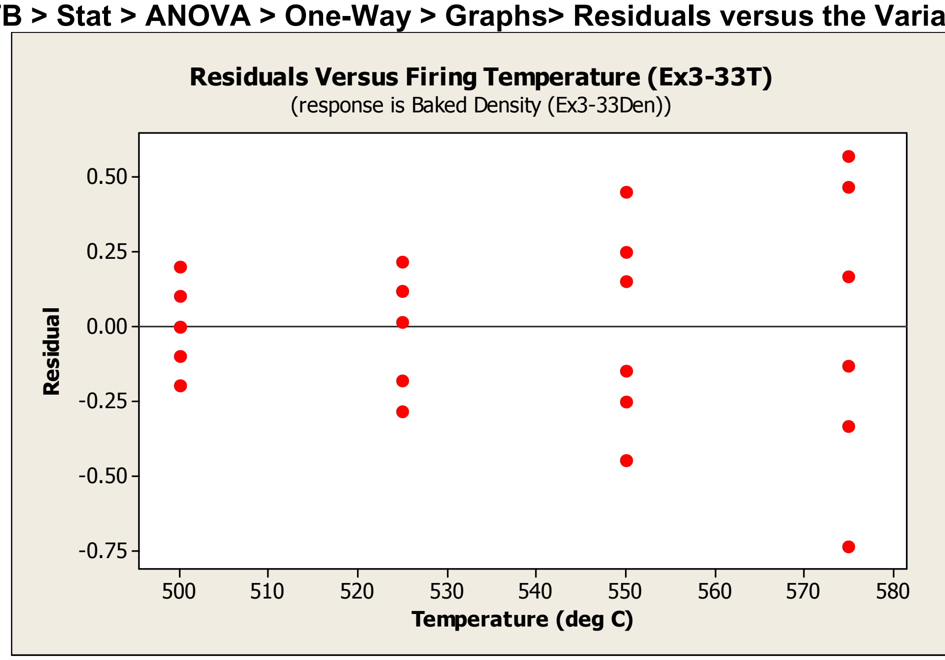
3-33 continued
(c)



Since statistically there is no evidence to indicate that the means are different, select the temperature with the smallest variance, 500°C (see Boxplot), which probably also incurs the smallest cost (lowest temperature).

3-34.

MTB > Stat > ANOVA > One-Way > Graphs> Residuals versus the Variables

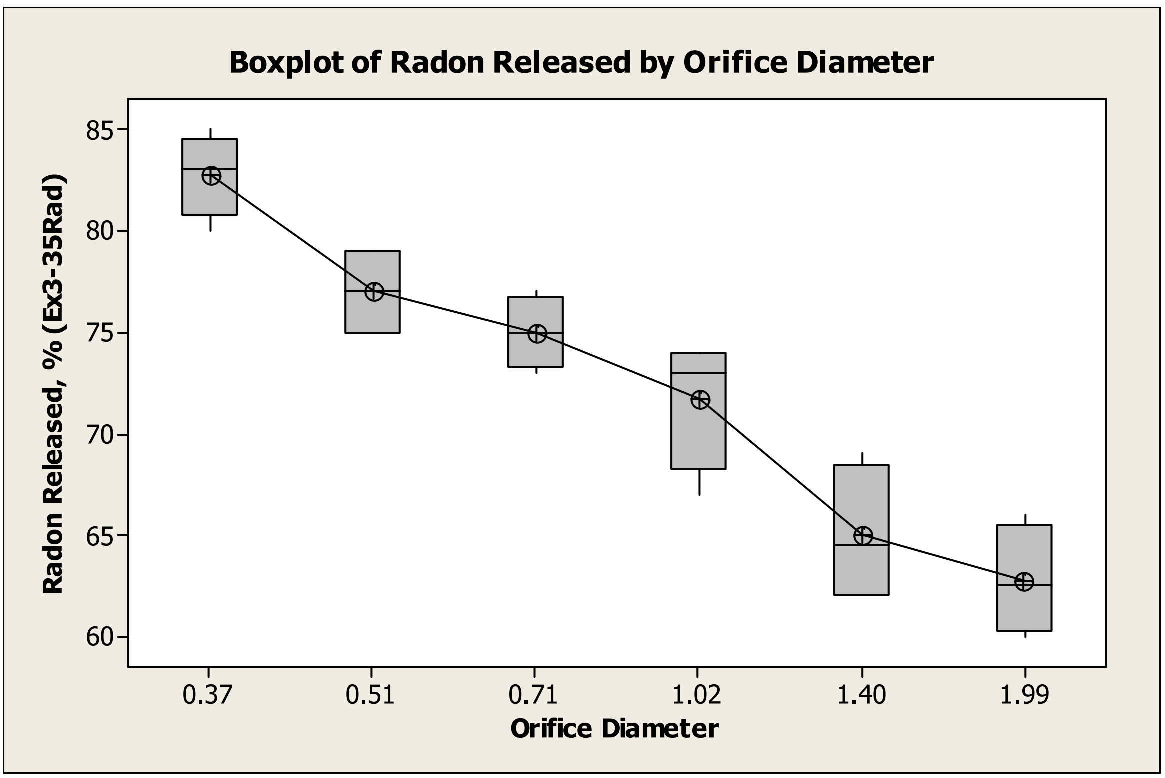


As firing temperature increases, so does variability. More uniform anodes are produced at lower temperatures. Recommend 500°C for smallest variability.

3-35.
(a)
MTB > Stat > ANOVA > One-Way > Graphs> Boxplots of data

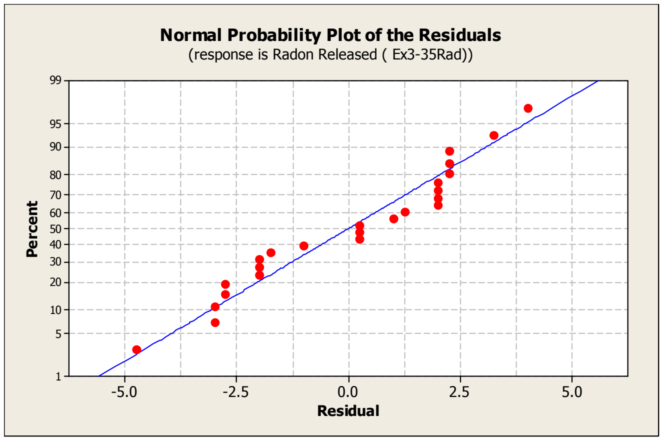
```
One-way ANOVA: Ex3-35Rad versus Ex3-35Dia
Source     DF       SS      MS      F      P
Ex3-35Dia   5  1133.38  226.68  30.85  0.000
Error      18   132.25    7.35
Total      23  1265.63
S = 2.711   R-Sq = 89.55%   R-Sq(adj) = 86.65%
                         Individual 95% CIs For Mean Based on
                         Pooled StDev
Level  N    Mean  StDev  ----+---------+---------+---------+-----
0.37   4  82.750  2.062                             (---*---)
0.51   4  77.000  2.309                     (---*---)
0.71   4  75.000  1.826                  (---*---)
1.02   4  71.750  3.304             (----*---)
1.40   4  65.000  3.559     (---*---)
1.99   4  62.750  2.754  (---*---)
                         ----+---------+---------+---------+-----
                          63.0      70.0      77.0      84.0
Pooled StDev = 2.711
```

Orifice size does affect mean % radon release, at α = 0.05.

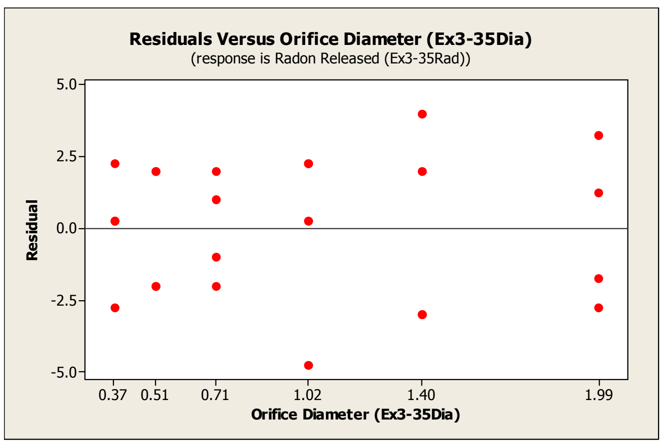


Smallest % radon released at 1.99 and 1.4 orifice diameters.

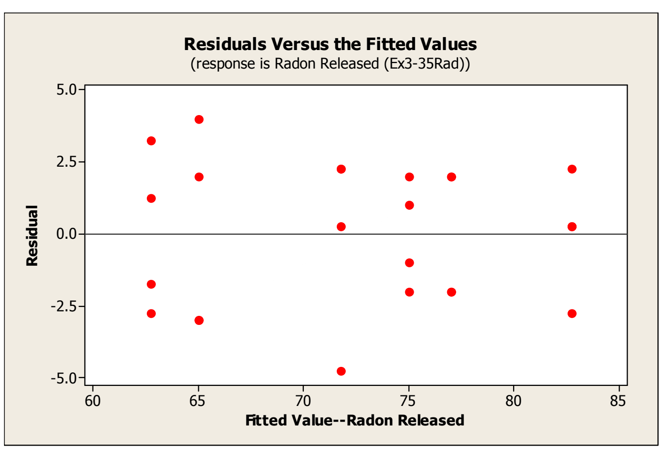
3-35 continued
(b)
MTB > Stat > ANOVA > One-Way > Graphs> Normal plot of residuals, Residuals versus fits, Residuals versus the Variables



Residuals violate the normality distribution.



The assumption of equal variance at each factor level appears to be violated, with larger variances at the larger diameters (1.02, 1.40, 1.99).

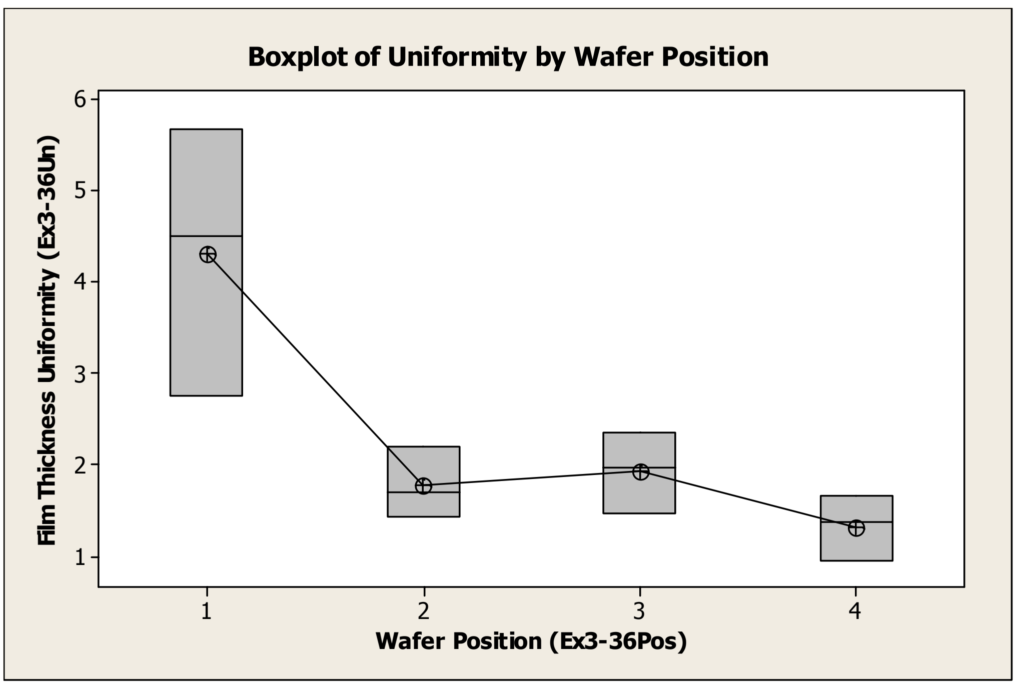


Variability in residuals does not appear to depend on the magnitude of predicted (or fitted) values.

3-36.
(a)
MTB > Stat > ANOVA > One-Way > Graphs, Boxplots of data

```
One-way ANOVA: Ex3-36Un versus Ex3-36Pos
Source     DF      SS     MS     F      P
Ex3-36Pos   3  16.220  5.407  8.29  0.008
Error       8   5.217  0.652
Total      11  21.437
S = 0.8076   R-Sq = 75.66%   R-Sq(adj) = 66.53%
                          Individual 95% CIs For Mean Based on
                          Pooled StDev
Level  N    Mean   StDev  --------+---------+---------+---------+-
1      3  4.3067  1.4636                      (------*------)
2      3  1.7733  0.3853     (------*------)
3      3  1.9267  0.4366      (------*------)
4      3  1.3167  0.3570  (------*------)
                          --------+---------+---------+---------+-
                                1.5       3.0       4.5       6.0
Pooled StDev = 0.8076
```

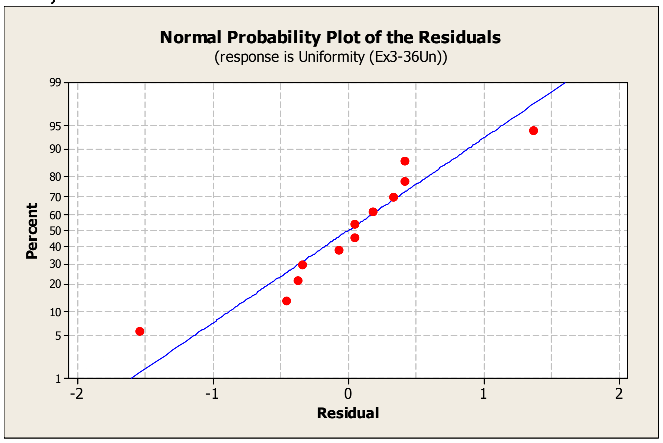
There is a statistically significant difference in wafer position, 1 is different from 2, 3, and 4.



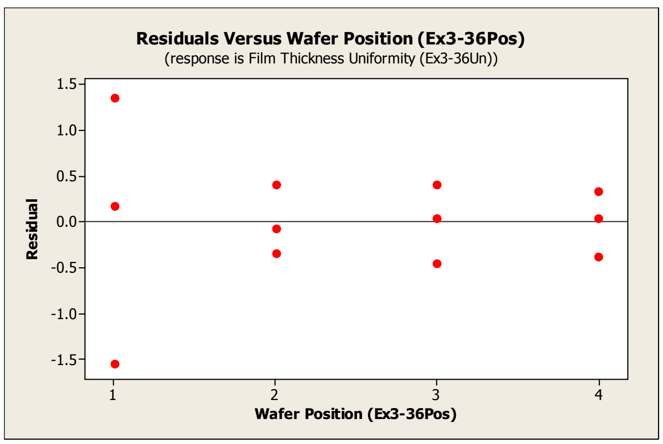
(b)
$$\hat{\sigma}_\tau^2 = \frac{\text{MS}_{\text{factor}} - \text{MS}_E}{n} = \frac{5.4066 - 0.6522}{12} = 0.3962$$

(c)
$$\hat{\sigma}^2 = \text{MS}_E = 0.6522$$
$$\hat{\sigma}_{\text{uniformity}}^2 = \hat{\sigma}_\tau^2 + \hat{\sigma}^2 = 0.3962 + 0.6522 = 1.0484$$

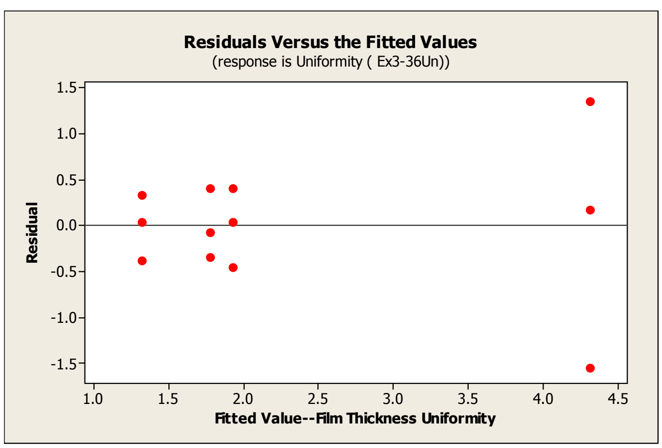
3-36 continued
(d) **MTB > Stat > ANOVA > One-Way > Graphs> Normal plot of residuals, Residuals versus fits, Residuals versus the Variables**



Normality assumption is probably not unreasonable, but there are two very unusual observations – the outliers at either end of the plot – therefore model adequacy is questionable.



Both outlier residuals are from wafer position 1.



The variability in residuals does appear to depend on the magnitude of predicted (or fitted) values.