

Chapter 5 Exercise Solutions

Notes:

- Several exercises in this chapter differ from those in the 4th edition. An “*” indicates that the description has changed. A second exercise number in parentheses indicates that the exercise number has changed. New exercises are denoted with a “☺”.
- The MINITAB convention for determining whether a point is out of control is: (1) if a plot point is within the control limits, it is in control, or (2) if a plot point is on or beyond the limits, it is out of control.
- MINITAB uses pooled standard deviation to estimate standard deviation for control chart limits and capability estimates. This can be changed in dialog boxes or under Tools>Options>Control Charts and Quality Tools>Estimating Standard Deviation.
- MINITAB defines some sensitizing rules for control charts differently than the standard rules. In particular, a run of n consecutive points on one side of the center line is defined as 9 points, not 8. This can be changed under Tools > Options > Control Charts and Quality Tools > Define Tests.

5-1.

(a) for $n = 5, A_2 = 0.577, D_4 = 2.114, D_3 = 0$

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \dots + \bar{x}_m}{m} = \frac{34.5 + 34.2 + \dots + 34.2}{24} = 34.00$$

$$\bar{R} = \frac{R_1 + R_2 + \dots + R_m}{m} = \frac{3 + 4 + \dots + 2}{24} = 4.71$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R} = 34.00 + 0.577(4.71) = 36.72$$

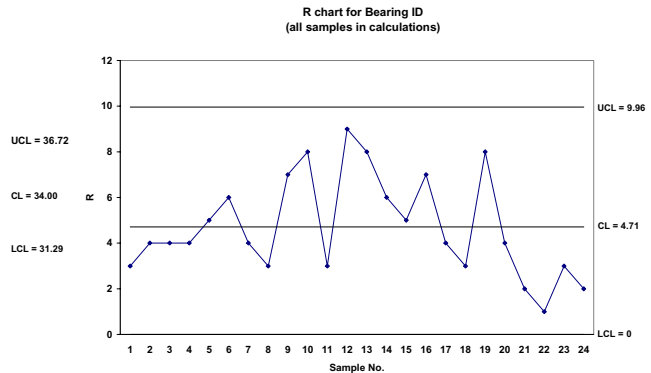
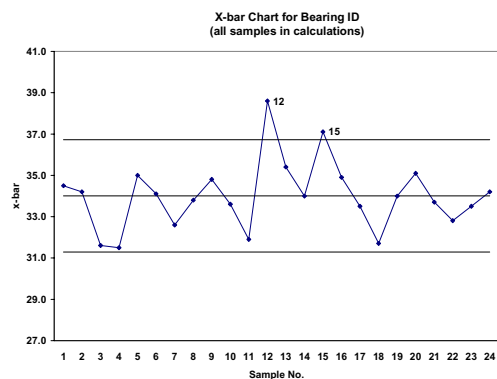
$$CL_{\bar{x}} = \bar{\bar{x}} = 34.00$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R} = 34.00 - 0.577(4.71) = 31.29$$

$$UCL_R = D_4 \bar{R} = 2.115(4.71) = 9.96$$

$$CL_R = \bar{R} = 4.71$$

$$LCL_R = D_3 \bar{R} = 0(4.71) = 0.00$$



Chapter 5 Exercise Solutions

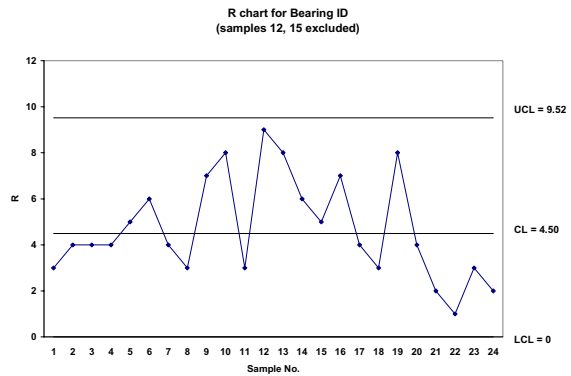
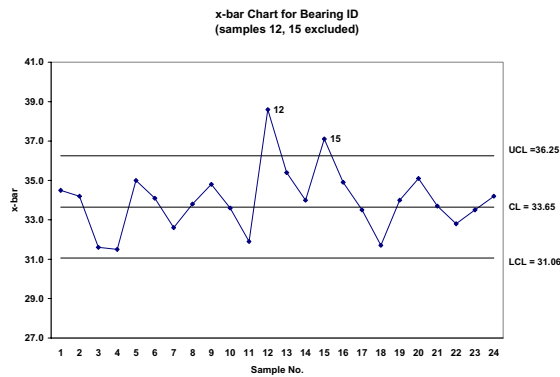
5-1 (a) continued

The process is not in statistical control; \bar{x} is beyond the upper control limit for both Sample No. 12 and Sample No. 15. Assuming an assignable cause is found for these two out-of-control points, the two samples can be excluded from the control limit calculations. The new process parameter estimates are:

$$\bar{\bar{x}} = 33.65; \bar{R} = 4.5; \hat{\sigma}_x = \bar{R} / d_2 = 4.5 / 2.326 = 1.93$$

$$UCL_{\bar{x}} = 36.25; CL_{\bar{x}} = 33.65; LCL_{\bar{x}} = 31.06$$

$$UCL_R = 9.52; CL_R = 4.5; LCL_R = 0.00$$



(b)

$$\hat{p} = \Pr\{x < LSL\} + \Pr\{x > USL\} = \Pr\{x < 20\} + \Pr\{x > 40\} = \Pr\{x < 20\} + [1 - \Pr\{x < 40\}]$$

$$= \Phi\left(\frac{20 - 33.65}{1.93}\right) + \left[1 - \Phi\left(\frac{40 - 33.65}{1.93}\right)\right]$$

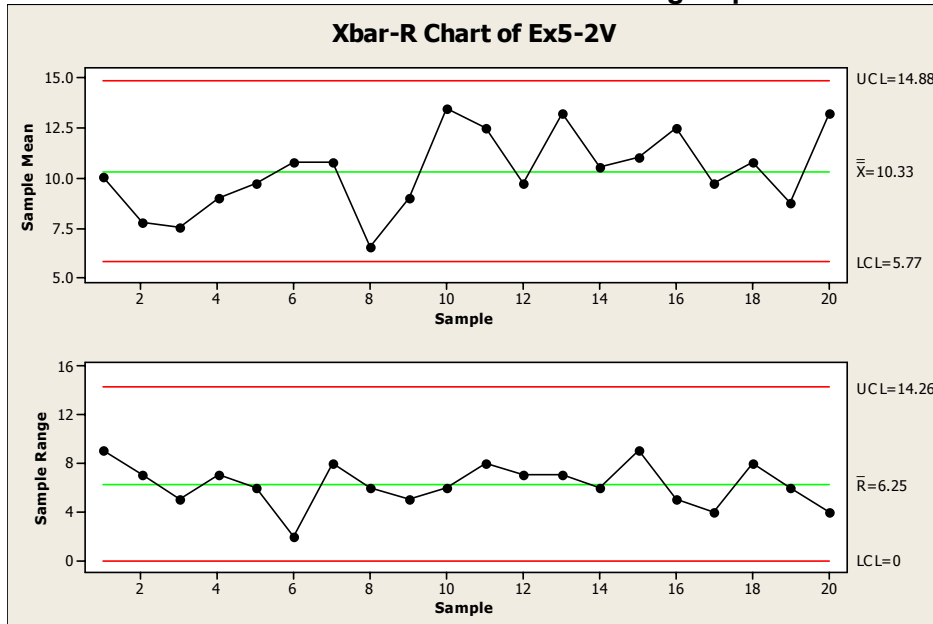
$$= \Phi(-7.07) + 1 - \Phi(3.29) = 0 + 1 - 0.99950 = 0.00050$$

Chapter 5 Exercise Solutions

5-2.

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

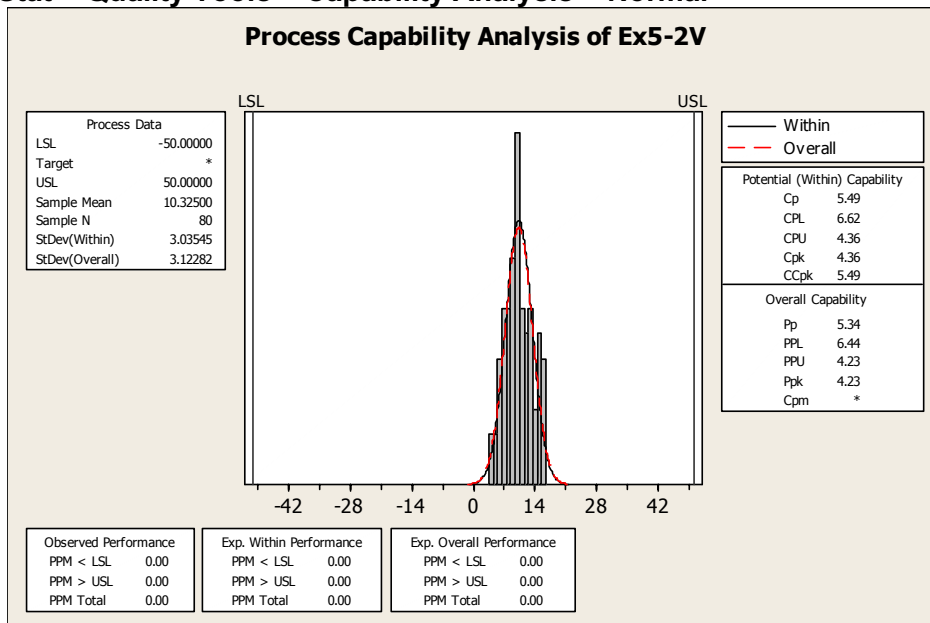
(b)

$n = 4$, $\bar{\bar{x}} = 10.33$, $\bar{R} = 6.25$, $\hat{\sigma}_x = \bar{R} / d_2 = 6.25 / 2.059 = 3.035$. Actual specs are 350 ± 5 V.

With $x_i = (\text{observed voltage on unit } i - 350) \times 10$: $USL_T = +50$, $LSL_T = -50$

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{+50 - (-50)}{6(3.035)} = 5.49, \text{ so the process is capable.}$$

MTB > Stat > Quality Tools > Capability Analysis > Normal

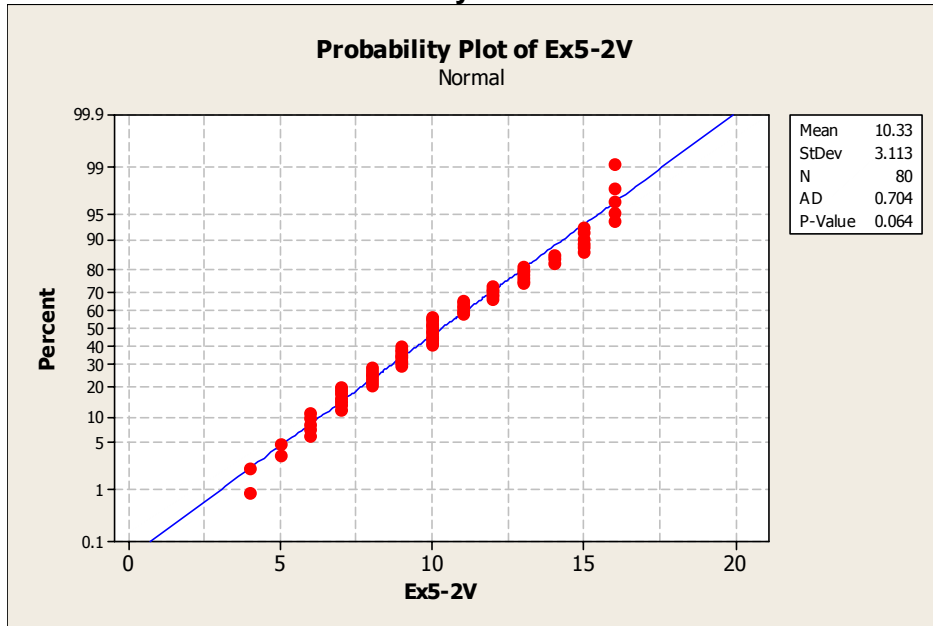


Chapter 5 Exercise Solutions

5-2 continued

(c)

MTB > Stat > Basic Statistics > Normality Test

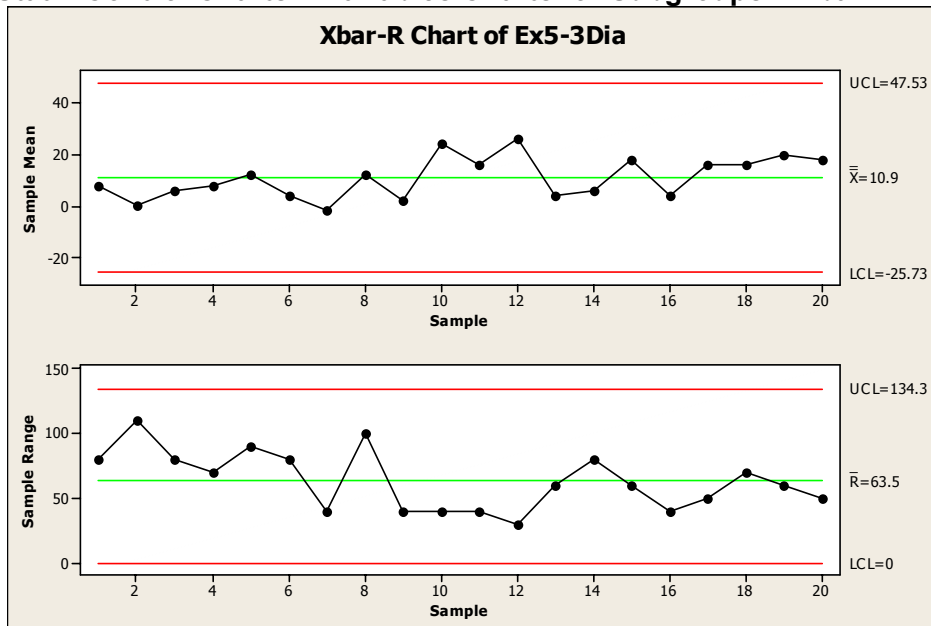


A normal probability plot of the transformed output voltage shows the distribution is close to normal.

5-3.

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

Chapter 5 Exercise Solutions

5-3 continued

(b)

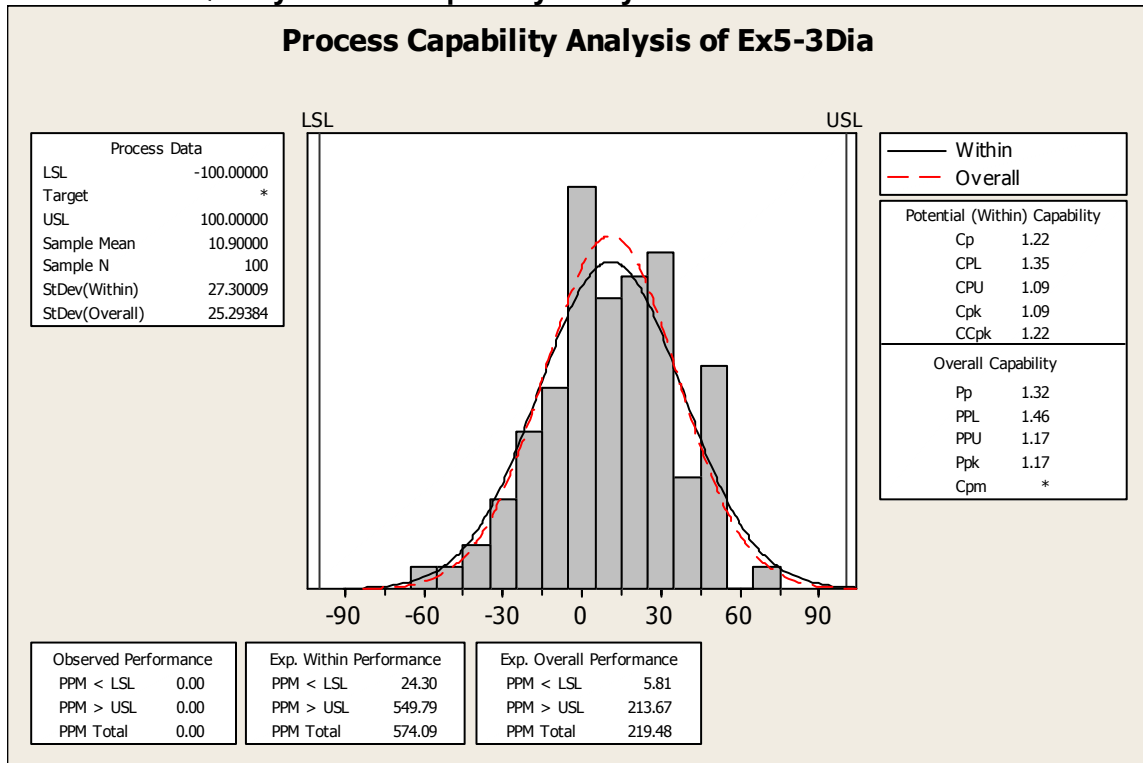
$$\hat{\sigma}_x = \bar{R} / d_2 = 63.5 / 2.326 = 27.3$$

(c)

USL = +100, LSL = -100

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+100 - (-100)}{6(27.3)} = 1.22, \text{ so the process is capable.}$$

MTB > Stat > Quality Tools > Capability Analysis > Normal

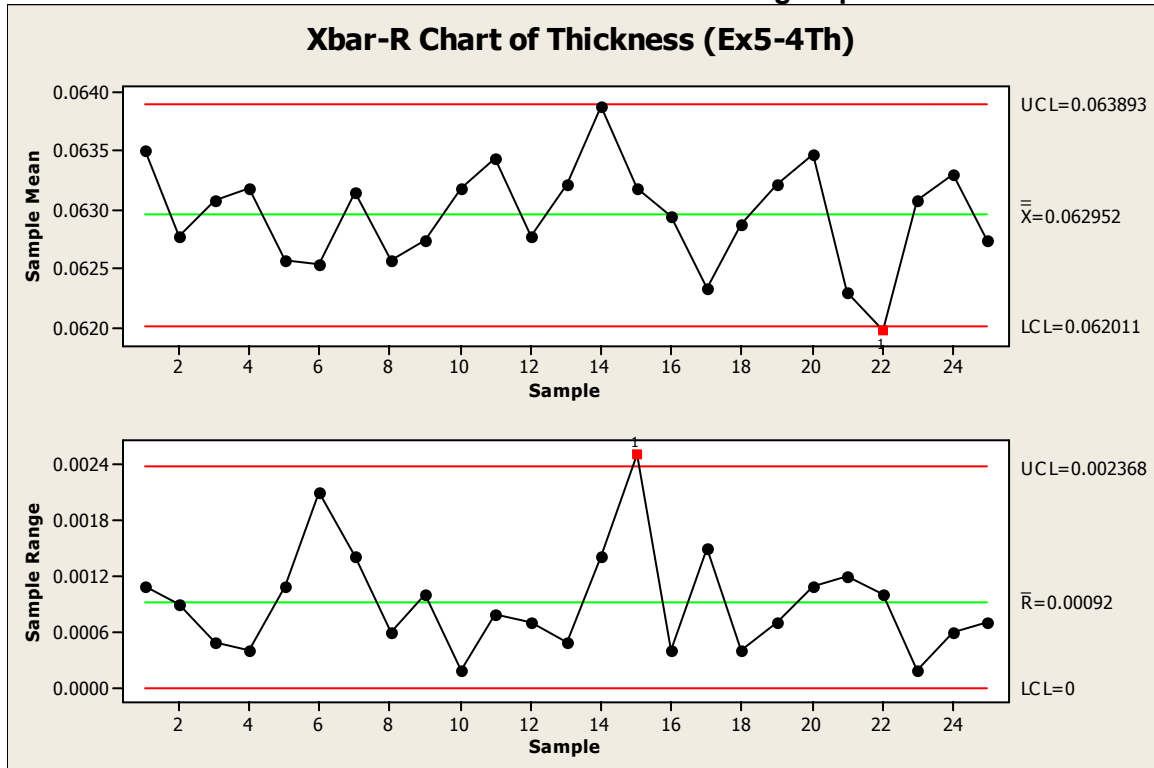


Chapter 5 Exercise Solutions

5-4.

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



Test Results for Xbar Chart of Ex5-4Th

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 22

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 22

Test Results for R Chart of Ex5-4Th

TEST 1. One point more than 3.00 standard deviations from center line.

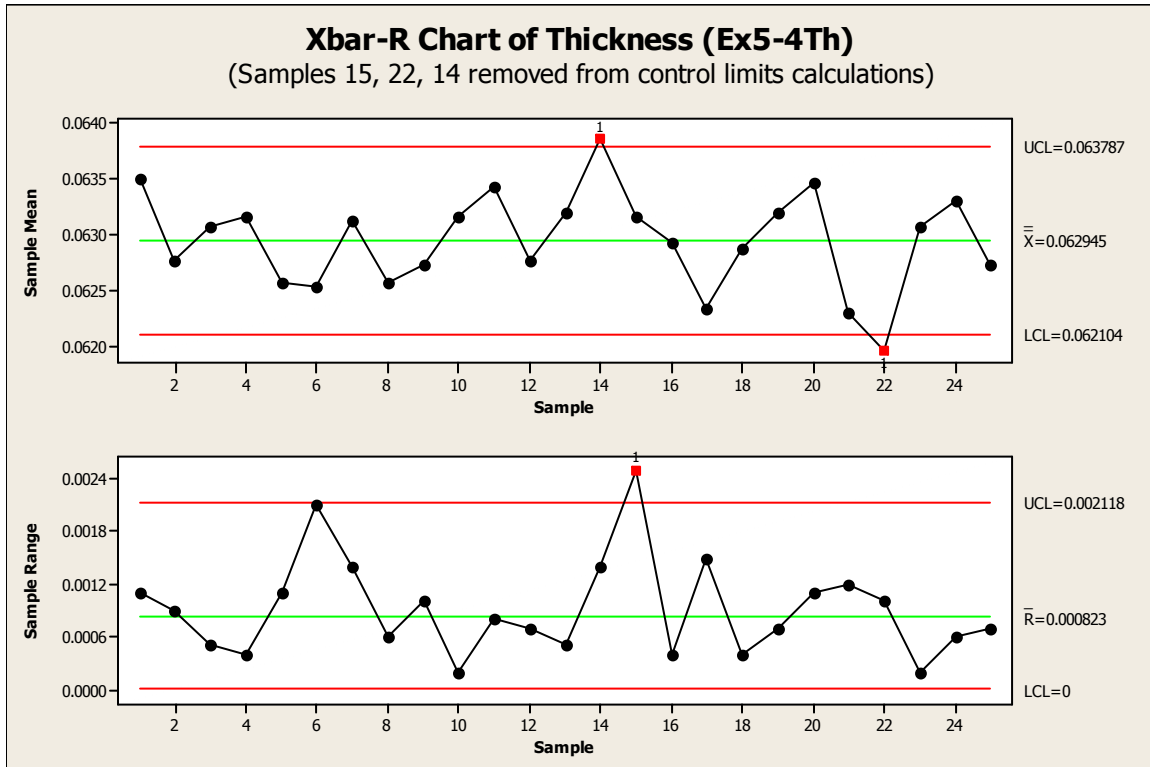
Test Failed at points: 15

* WARNING * If graph is updated with new data, the results above may no longer be correct.

Chapter 5 Exercise Solutions

5-4 continued

The process is out-of-control, failing tests on both the \bar{x} and the R charts. Assuming assignable causes are found, remove the out-of-control points (samples 15, 22) and re-calculate control limits. With the revised limits, sample 14 is also out-of-control on the \bar{x} chart. Removing all three samples from calculation, the new control limits are:



(b)

$$\hat{\sigma}_x = \bar{R} / d_2 = 0.000823 / 1.693 = 0.000486$$

(c)

Natural tolerance limits are: $\bar{\bar{x}} \pm 3\hat{\sigma}_x = 0.06295 \pm 3(0.000486) = [0.061492, 0.064408]$

Chapter 5 Exercise Solutions

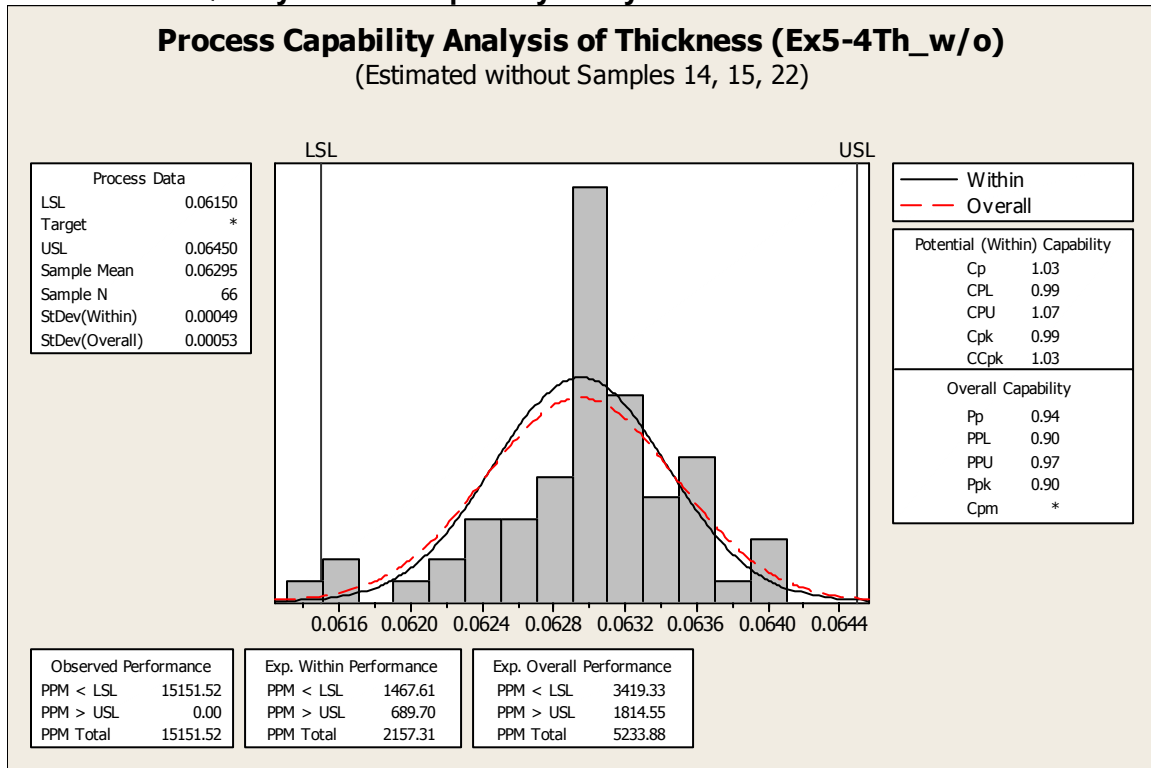
5-4 continued

(d)

Assuming that printed circuit board thickness is normally distributed, and excluding samples 14, 15, and 22 from the process capability estimation:

$$\hat{C}_P = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+0.0015 - (-0.0015)}{6(0.000486)} = 1.028$$

MTB > Stat > Quality Tools > Capability Analysis > Normal



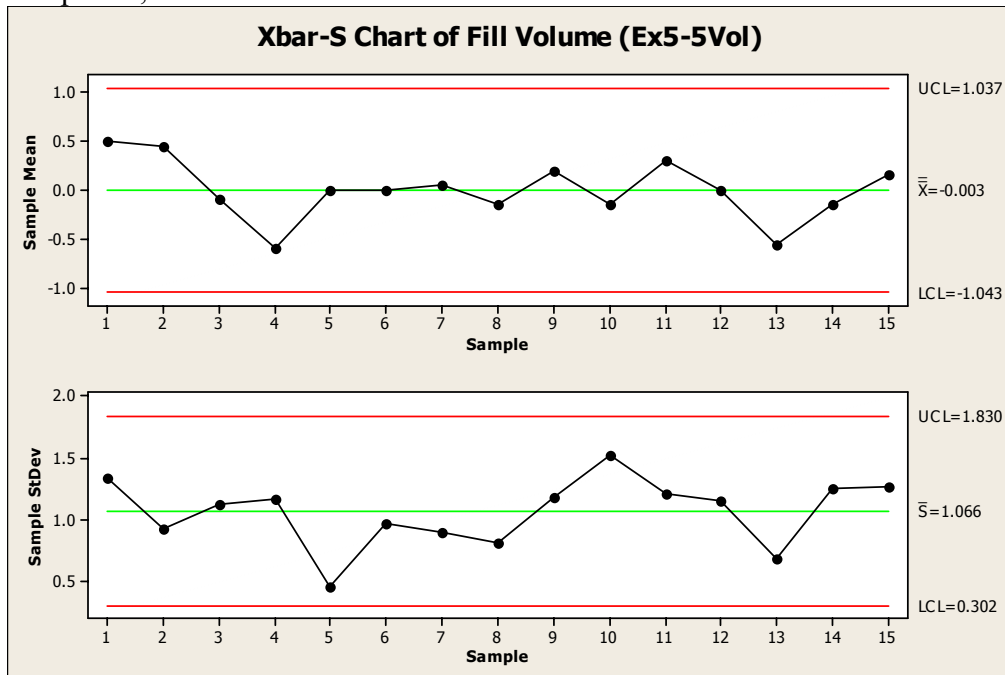
Chapter 5 Exercise Solutions

5-5.

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S (Ex5-5Vol)

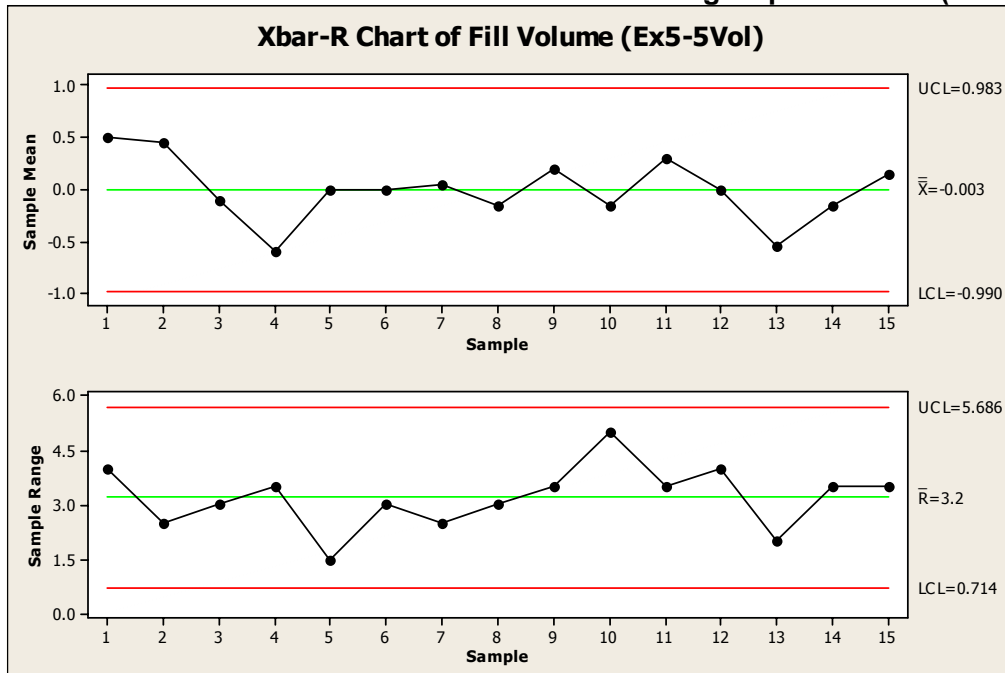
Under “Options, Estimate” select Sbar as method to estimate standard deviation.



The process is in statistical control, with no out-of-control signals, runs, trends, or cycles.

(b)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R (Ex5-5Vol)



The process is in statistical control, with no out-of-control signals, runs, trends, or cycles.

There is no difference in interpretation from the $\bar{x} - s$ chart.

Chapter 5 Exercise Solutions

5-5 continued

(c)

Let $\alpha = 0.010$. $n = 15$, $\bar{s} = 1.066$.

$$CL = \bar{s}^2 = 1.066^2 = 1.136$$

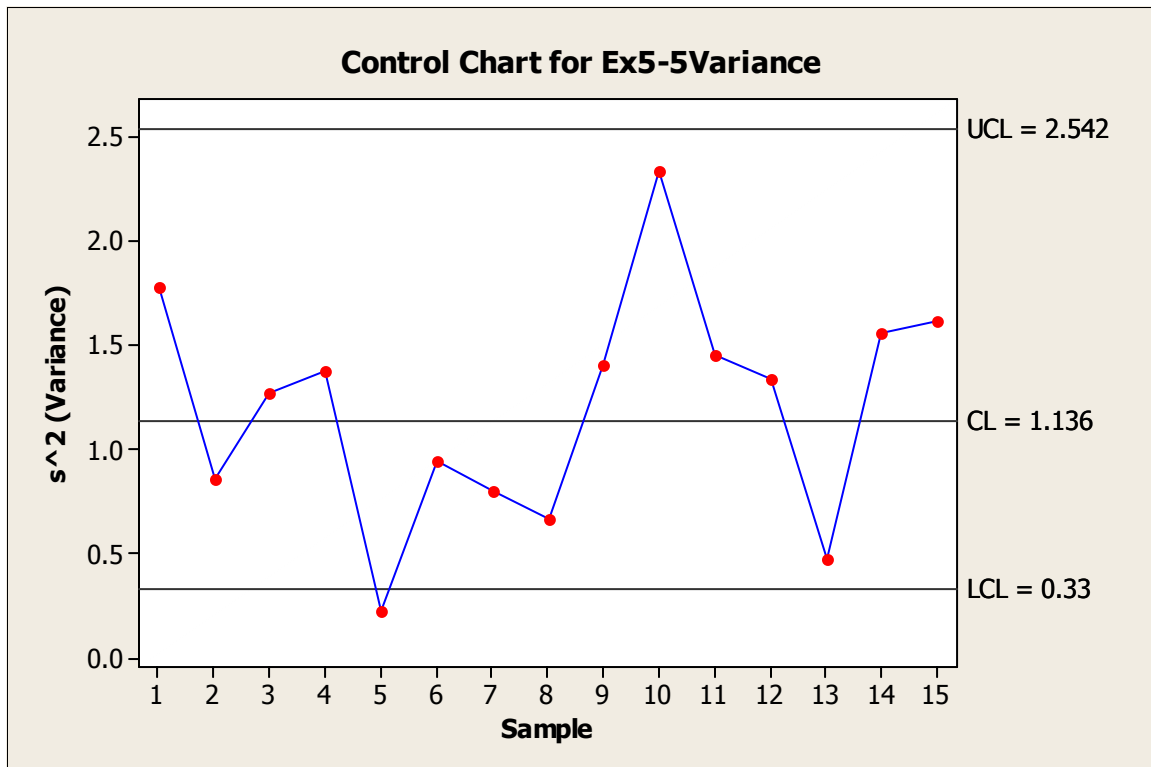
$$UCL = \bar{s}^2 / (n-1) \chi_{\alpha/2, n-1}^2 = 1.066^2 / (15-1) (\chi_{0.010/2, 15-1}^2) = 1.066^2 / (15-1) (31.32) = 2.542$$

$$LCL = \bar{s}^2 / (n-1) \chi_{1-(\alpha/2), n-1}^2 = 1.066^2 / (15-1) (\chi_{1-(0.010/2), 15-1}^2) = 1.066^2 / (15-1) (4.07) = 0.330$$

MINITAB's control chart options do not include an s^2 or variance chart. To construct an s^2 control chart, first calculate the sample standard deviations and then create a time series plot. To obtain sample standard deviations: **Stat > Basic Statistics > Store Descriptive Statistics**. "Variables" is column with sample data (Ex5-5Vol), and "By Variables" is the sample ID column (Ex5-5Sample). In "Statistics" select "Variance". Results are displayed in the session window. Copy results from the session window by holding down the keyboard "Alt" key, selecting only the variance column, and then copying & pasting to an empty worksheet column (results in Ex5-5Variance).

Graph > Time Series Plot > Simple

Control limits can be added using: **Time/Scale > Reference Lines > Y positions**



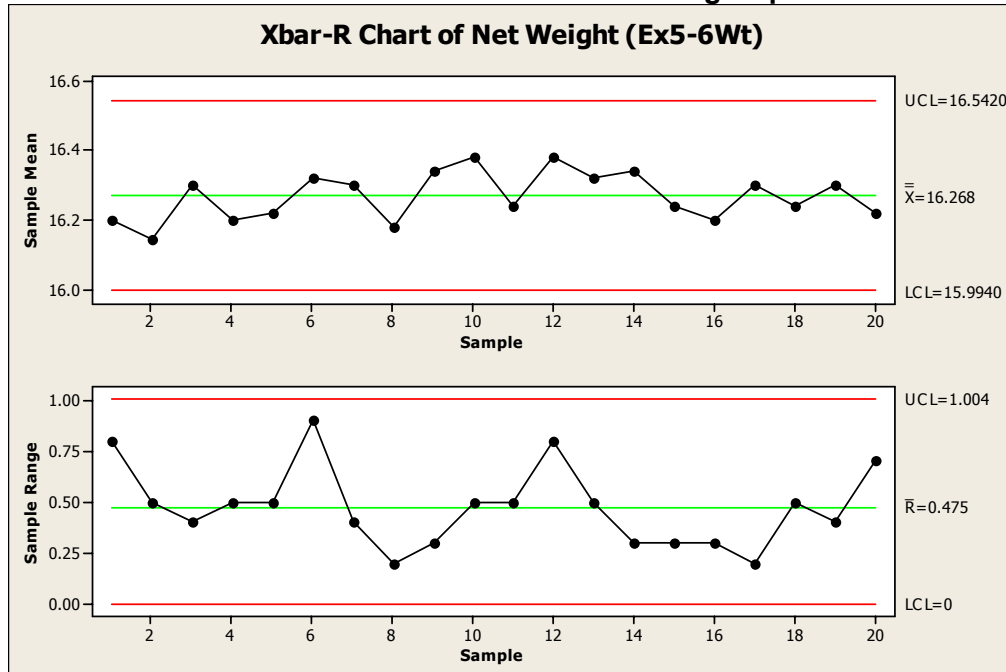
Sample 5 signals out of control below the lower control limit. Otherwise there are no runs, trends, or cycles. If the limits had been calculated using $\alpha = 0.0027$ (not tabulated in textbook), sample 5 would be within the limits, and there would be no difference in interpretation from either the $\bar{x} - s$ or the $x - R$ chart.

Chapter 5 Exercise Solutions

5-6.

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

(b)

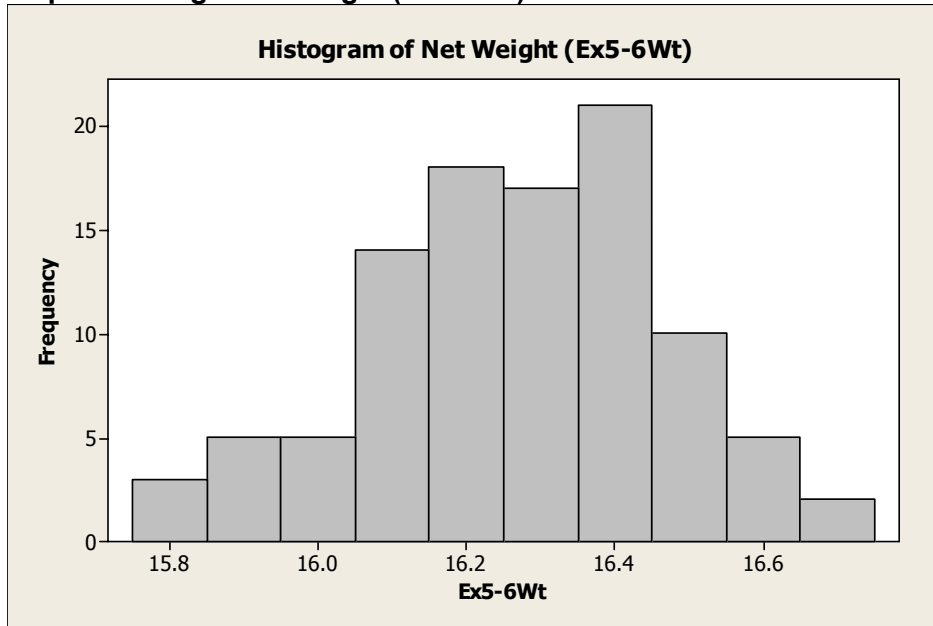
$$n = 5; \bar{\bar{x}} = 16.268; \bar{R} = 0.475; \hat{\sigma}_x = \bar{R} / d_2 = 0.475 / 2.326 = 0.204$$

Chapter 5 Exercise Solutions

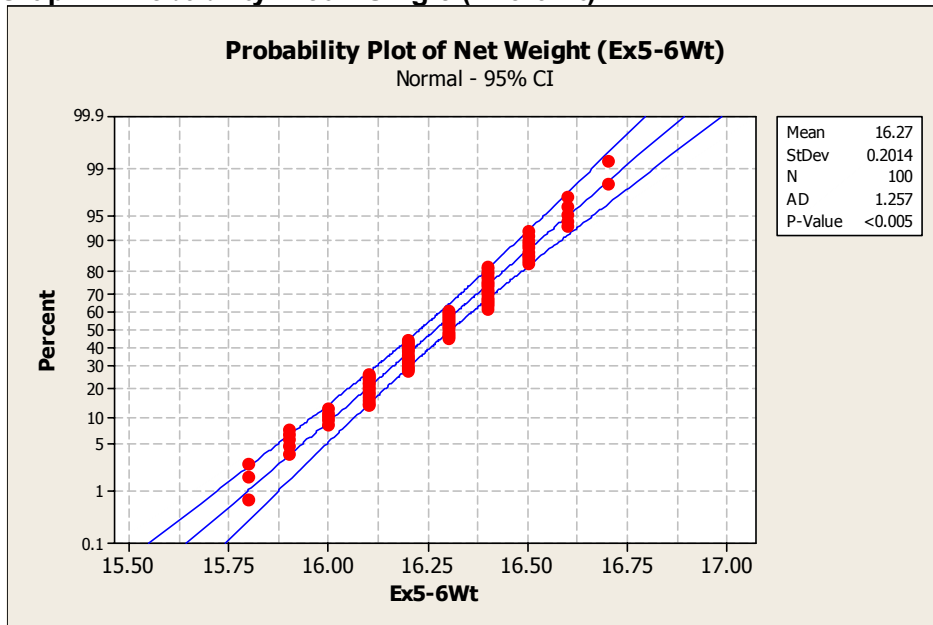
5-6 continued

(c)

MTB > Graph > Histogram > Single (Ex5-6Wt)



MTB > Graph > Probability Plot > Single (Ex5-6Wt)



Visual examination indicates that fill weights approximate a normal distribution - the histogram has one mode, and is approximately symmetrical with a bell shape. Points on the normal probability plot generally fall along a straight line.

Chapter 5 Exercise Solutions

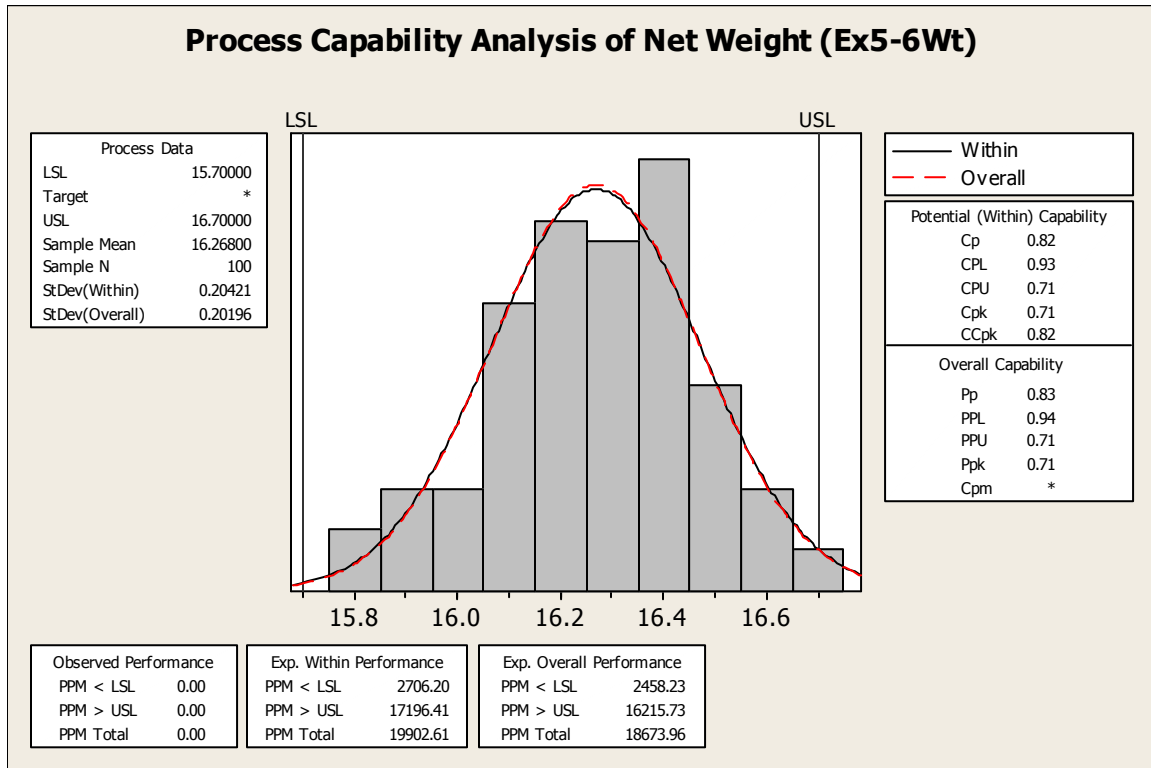
5-6 continued

(d)

$\hat{C}_p \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+0.5 - (-0.5)}{6(0.204)} = 0.82$, so the process is not capable of meeting specifications.

MTB > Stat > Quality Tools > Capability Analysis > Normal

Under "Estimate" select Rbar as method to estimate standard deviation.



(e)

$$\hat{p}_{\text{lower}} = \Pr\{x < LSL\} = \Pr\{x < 15.7\} = \Phi\left(\frac{15.7 - 16.268}{0.204}\right) = \Phi(-2.78) = 0.0027$$

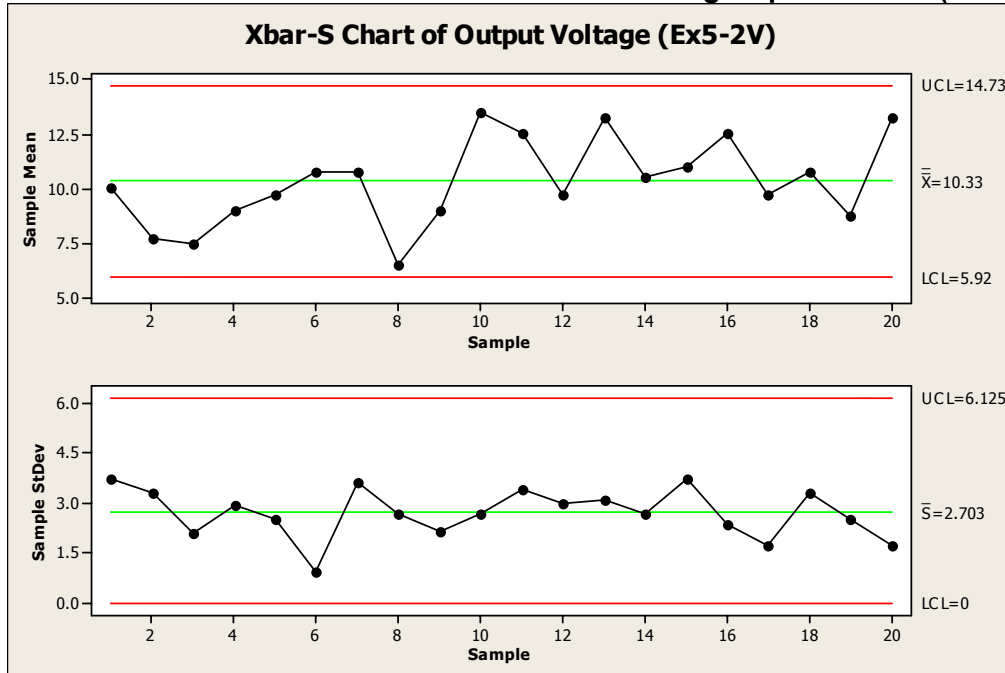
The MINITAB process capability analysis also reports

Exp. "Overall" Performance	
PPM < LSL	2458.23
PPM > USL	16215.73
PPM Total	18673.96

Chapter 5 Exercise Solutions

5-7.

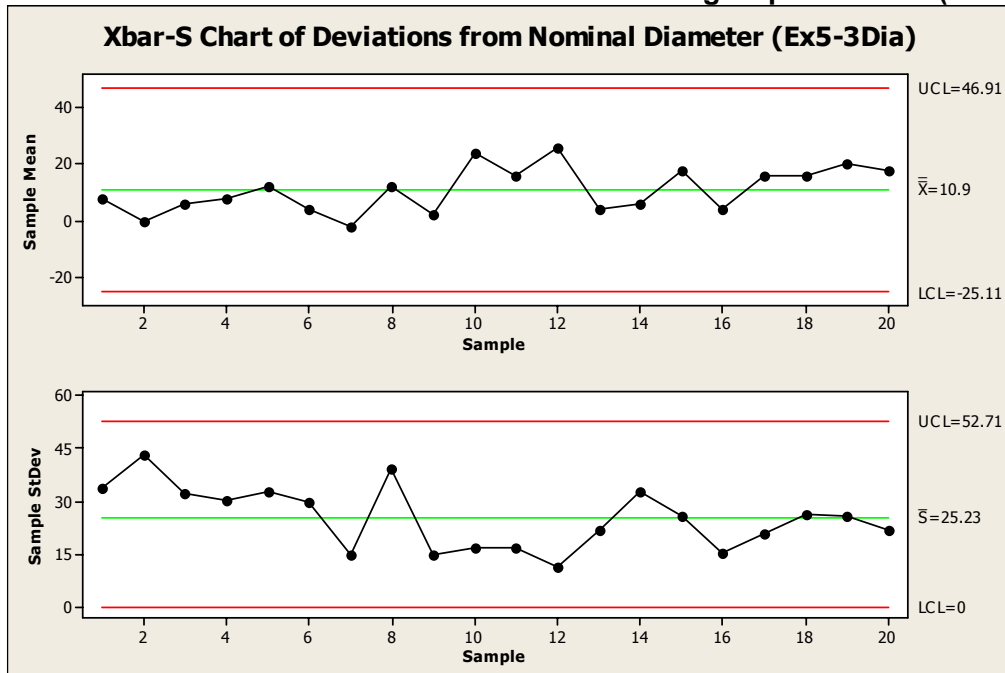
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S (Ex5-2VI)



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

5-8.

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S (Ex5-3Dia)



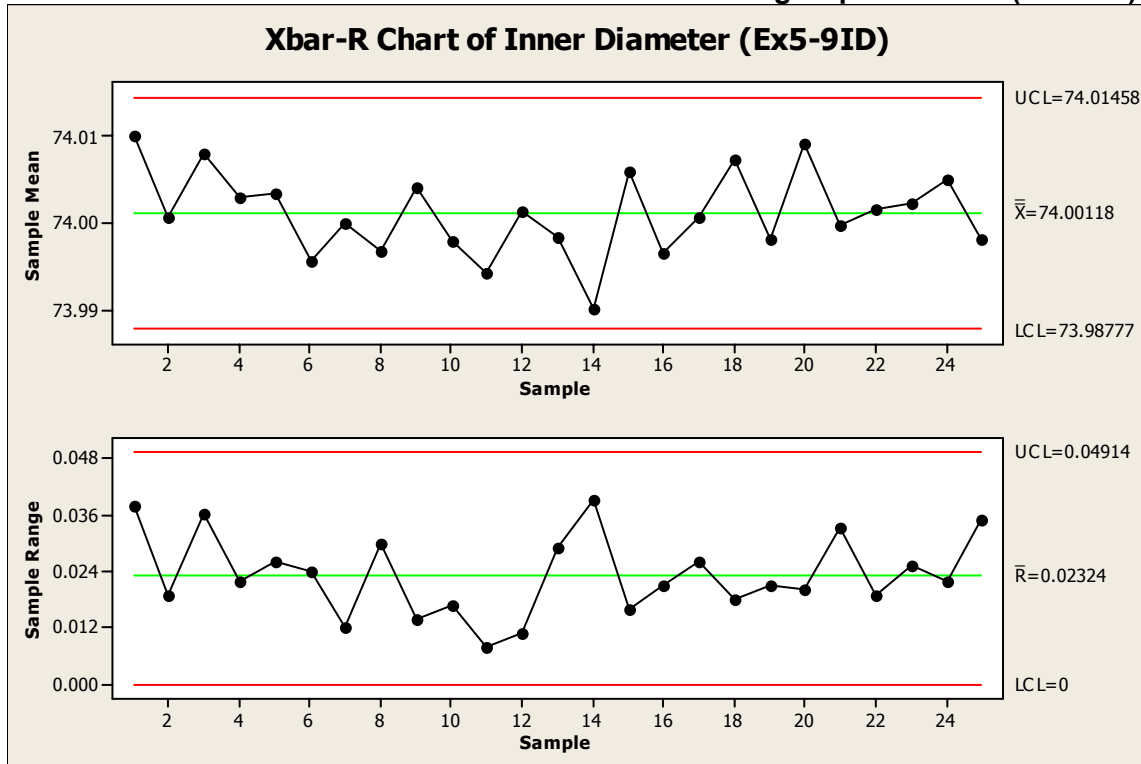
The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

Chapter 5 Exercise Solutions

5-9☺.

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R (Ex5-9ID)



The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

(b)

The control limits on the \bar{x} charts in Example 5-3 were calculated using \bar{S} to estimate σ , in this exercise \bar{R} was used to estimate σ . They will not always be the same, and in general, the \bar{x} control limits based on \bar{S} will be slightly different than limits based on \bar{R} .

Chapter 5 Exercise Solutions

5-9 continued

(c)

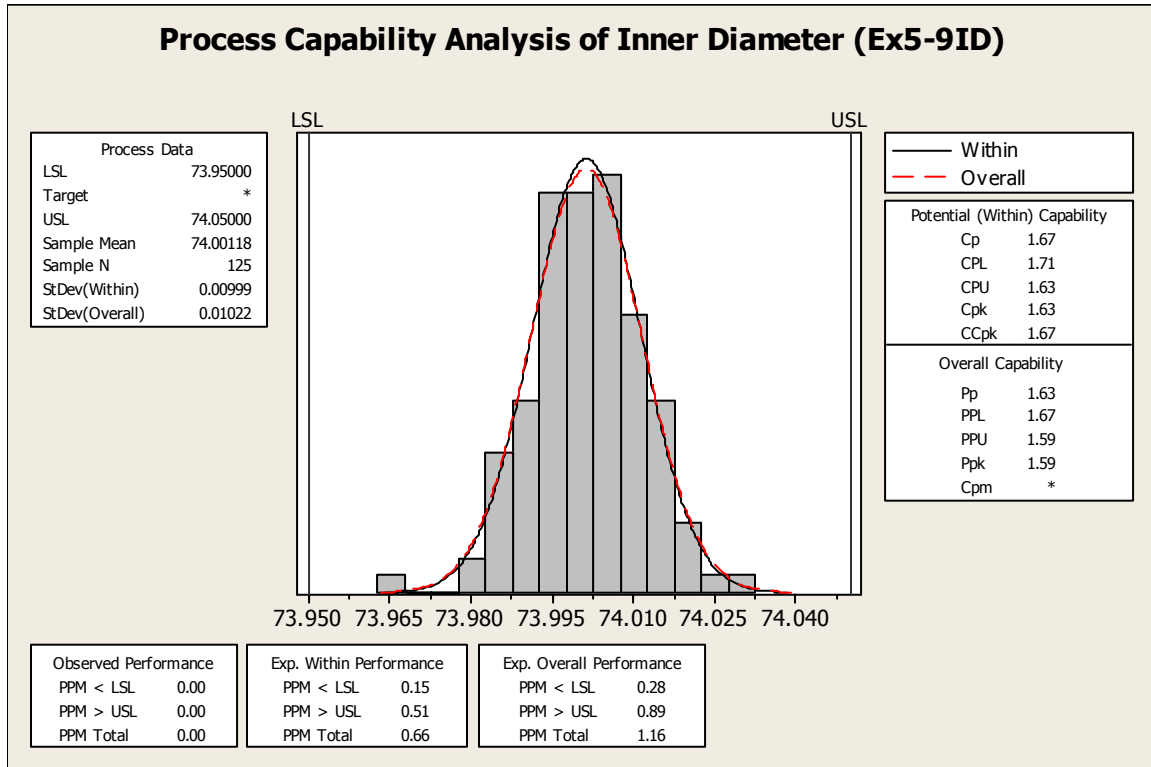
$$\hat{\sigma}_x = \bar{R} / d_2 = 0.02324 / 2.326 = 0.009991$$

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{74.05 - 73.95}{6(0.009991)} = 1.668, \text{ so the process is not capable of meeting}$$

specifications.

MTB > Stat > Quality Tools > Capability Analysis > Normal

Under “Estimate” select Rbar as method to estimate standard deviation.

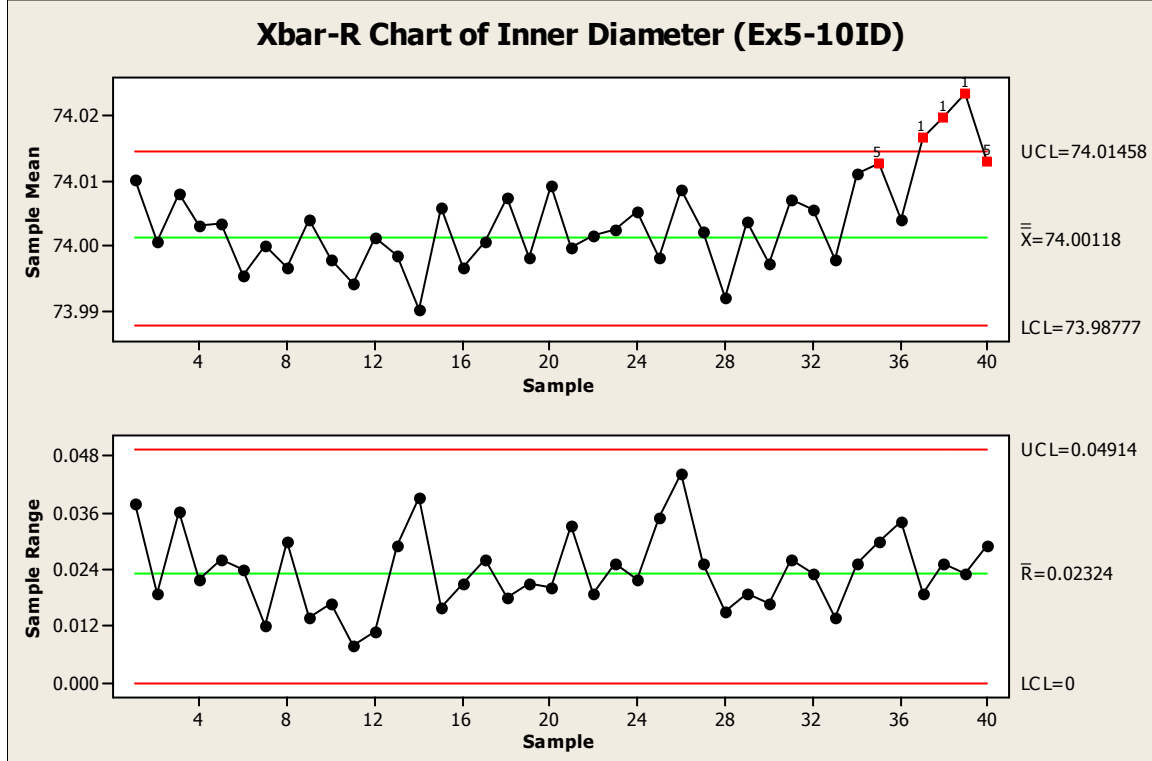


$$\begin{aligned} \hat{p} &= \Pr\{x < LSL\} + \Pr\{x > USL\} \\ &= \Pr\{x < 73.95\} + \Pr\{x > 74.05\} \\ &= \Pr\{x < 73.95\} + [1 - \Pr\{x < 74.05\}] \\ &= \Phi\left(\frac{73.95 - 74.00118}{0.009991}\right) + \left[1 - \Phi\left(\frac{74.05 - 74.00118}{0.009991}\right)\right] \\ &= \Phi(-5.123) + 1 - \Phi(4.886) \\ &= 0 + 1 - 1 \\ &= 0 \end{aligned}$$

Chapter 5 Exercise Solutions

5-10 ☺.

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R (Ex5-10ID)



Test Results for Xbar Chart of Ex5-10ID

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 37, 38, 39

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 35, 37, 38, 39, 40

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on one side of CL).

Test Failed at points: 38, 39, 40

The control charts indicate that the process is in control, until the \bar{x} -value from the 37th sample is plotted. Since this point and the three subsequent points plot above the upper control limit, an assignable cause has likely occurred, increasing the process mean.

Chapter 5 Exercise Solutions

5-11 (5-9).

$n = 10$; $\mu = 80$ in-lb; $\sigma_x = 10$ in-lb; and $A = 0.949$; $B_6 = 1.669$; $B_5 = 0.276$

centerline $_{\bar{x}} = \mu = 80$

$$UCL_{\bar{x}} = \mu + A\sigma_x = 80 + 0.949(10) = 89.49$$

$$LCL_{\bar{x}} = \mu - A\sigma_x = 80 - 0.949(10) = 70.51$$

$$\text{centerline}_S = c_4\sigma_x = 0.9727(10) = 9.727$$

$$UCL_S = B_6\sigma_x = 1.669(10) = 16.69$$

$$LCL_S = B_5\sigma_x = 0.276(10) = 2.76$$

5-12* (5-10).

$n = 6$ items/sample; $\sum_{i=1}^{50} \bar{x}_i = 2000$; $\sum_{i=1}^{50} R_i = 200$; $m = 50$ samples

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{50} \bar{x}_i}{m} = \frac{2000}{50} = 40; \quad \bar{R} = \frac{\sum_{i=1}^{50} R_i}{m} = \frac{200}{50} = 4$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 40 + 0.483(4) = 41.932$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 40 - 0.483(4) = 38.068$$

$$UCL_R = D_4\bar{R} = 2.004(4) = 8.016$$

$$LCL_R = D_3\bar{R} = 0(4) = 0$$

(b)

natural tolerance limits: $\bar{\bar{x}} \pm 3\hat{\sigma}_x = \bar{\bar{x}} \pm 3(\bar{R}/d_2) = 40 \pm 3(4/2.534) = [35.264, 44.736]$

(c)

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+5.0 - (-5.0)}{6(1.579)} = 1.056, \text{ so the process is not capable.}$$

(d)

$$\hat{p}_{\text{scrap}} = \Pr\{x < LSL\} = \Pr\{x < 36\} = \Phi\left(\frac{36 - 40}{1.579}\right) = \Phi(-2.533) = 0.0057, \text{ or } 0.57\%.$$

$$\hat{p}_{\text{rework}} = \Pr\{x > USL\} = 1 - \Pr\{x < USL\} = 1 - \Phi\left(\frac{47 - 40}{1.579}\right) = 1 - \Phi(4.433) = 1 - 0.999995 = 0.000005$$

or 0.0005%.

(e)

First, center the process at 41, not 40, to reduce scrap and rework costs. Second, reduce variability such that the natural process tolerance limits are closer to, say, $\hat{\sigma}_x \approx 1.253$.

Chapter 5 Exercise Solutions

5-13* (5-11).

$n = 4$ items/subgroup; $\sum_{i=1}^{50} \bar{x}_i = 1000$; $\sum_{i=1}^{50} S_i = 72$; $m = 50$ subgroups

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{50} \bar{x}_i}{m} = \frac{1000}{50} = 20$$

$$\bar{S} = \frac{\sum_{i=1}^{50} S_i}{m} = \frac{72}{50} = 1.44$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_3 \bar{S} = 20 + 1.628(1.44) = 22.34$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_3 \bar{S} = 20 - 1.628(1.44) = 17.66$$

$$UCL_S = B_4 \bar{S} = 2.266(1.44) = 3.26$$

$$LCL_S = B_3 \bar{S} = 0(1.44) = 0$$

(b)

$$\text{natural process tolerance limits: } \bar{\bar{x}} \pm 3\hat{\sigma}_x = \bar{\bar{x}} \pm 3\left(\frac{\bar{S}}{c_4}\right) = 20 \pm 3\left(\frac{1.44}{0.9213}\right) = [15.3, 24.7]$$

(c)

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+4.0 - (-4.0)}{6(1.44/0.9213)} = 0.85, \text{ so the process is not capable.}$$

(d)

$$\hat{p}_{\text{rework}} = \Pr\{x > USL\} = 1 - \Pr\{x \leq USL\} = 1 - \Phi\left(\frac{23 - 20}{1.44/0.9213}\right) = 1 - \Phi(1.919) = 1 - 0.9725 = 0.0275$$

or 2.75%.

$$\hat{p}_{\text{scrap}} = \Pr\{x < LSL\} = \Phi\left(\frac{15 - 20}{1.44/0.9213}\right) = \Phi(-3.199) = 0.00069, \text{ or } 0.069\%$$

$$\text{Total} = 2.88\% + 0.069\% = 2.949\%$$

(e)

$$\hat{p}_{\text{rework}} = 1 - \Phi\left(\frac{23 - 19}{1.44/0.9213}\right) = 1 - \Phi(2.56) = 1 - 0.99477 = 0.00523, \text{ or } 0.523\%$$

$$\hat{p}_{\text{scrap}} = \Phi\left(\frac{15 - 19}{1.44/0.9213}\right) = \Phi(-2.56) = 0.00523, \text{ or } 0.523\%$$

$$\text{Total} = 0.523\% + 0.523\% = 1.046\%$$

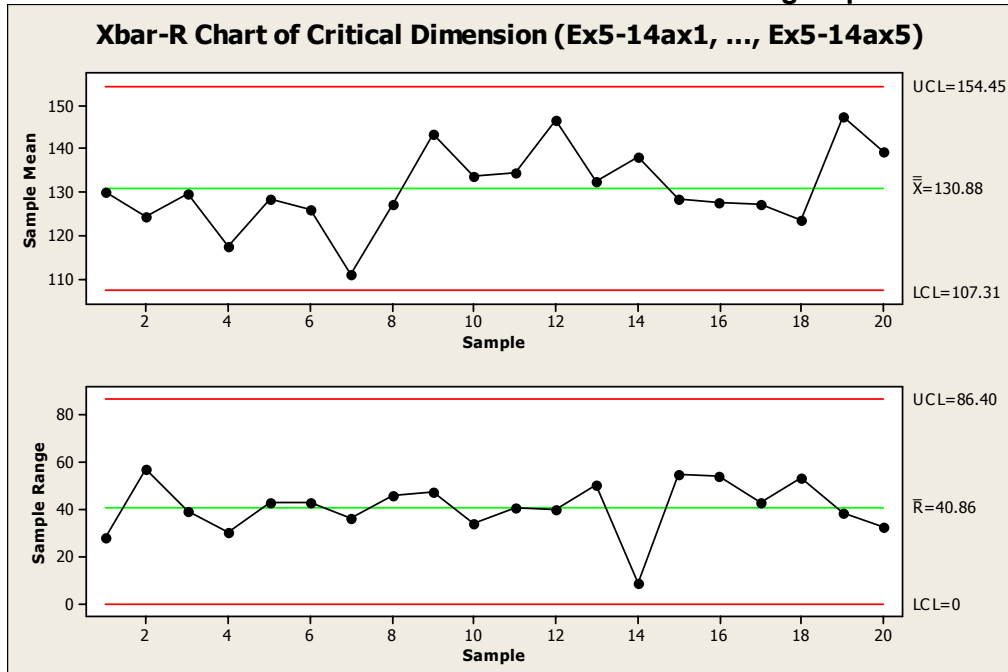
Centering the process would reduce rework, but increase scrap. A cost analysis is needed to make the final decision. An alternative would be to work to improve the process by reducing variability.

Chapter 5 Exercise Solutions

5-14 (5-12).

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R

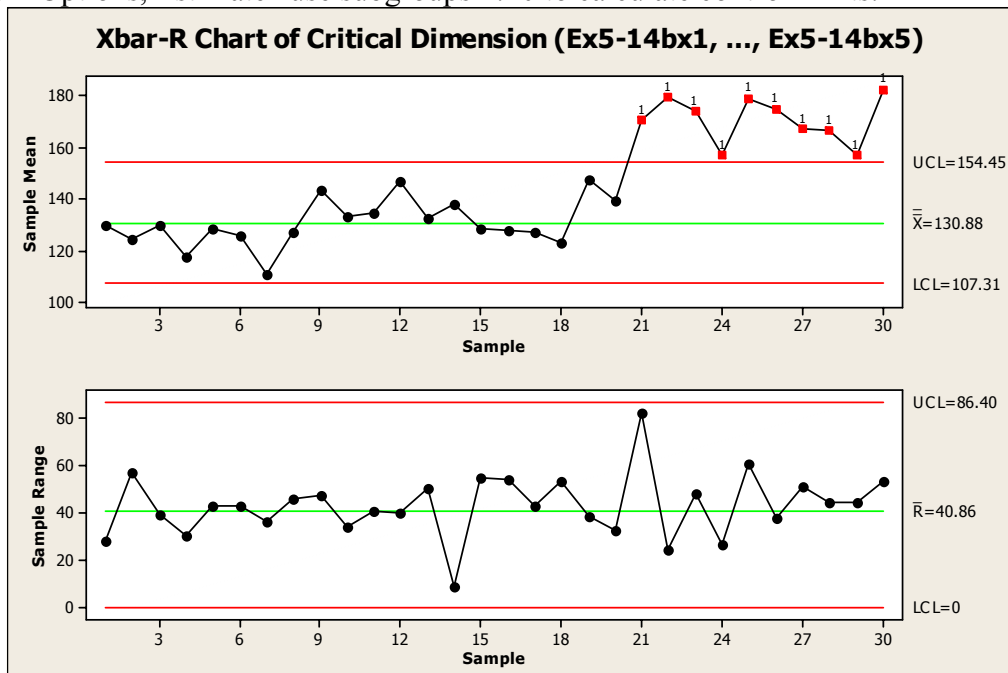


The process is in statistical control with no out-of-control signals, runs, trends, or cycles.

(b)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R

Under "Options, Estimate" use subgroups 1:20 to calculate control limits.



Starting at Sample #21, the process average has shifted to above the UCL = 154.45.

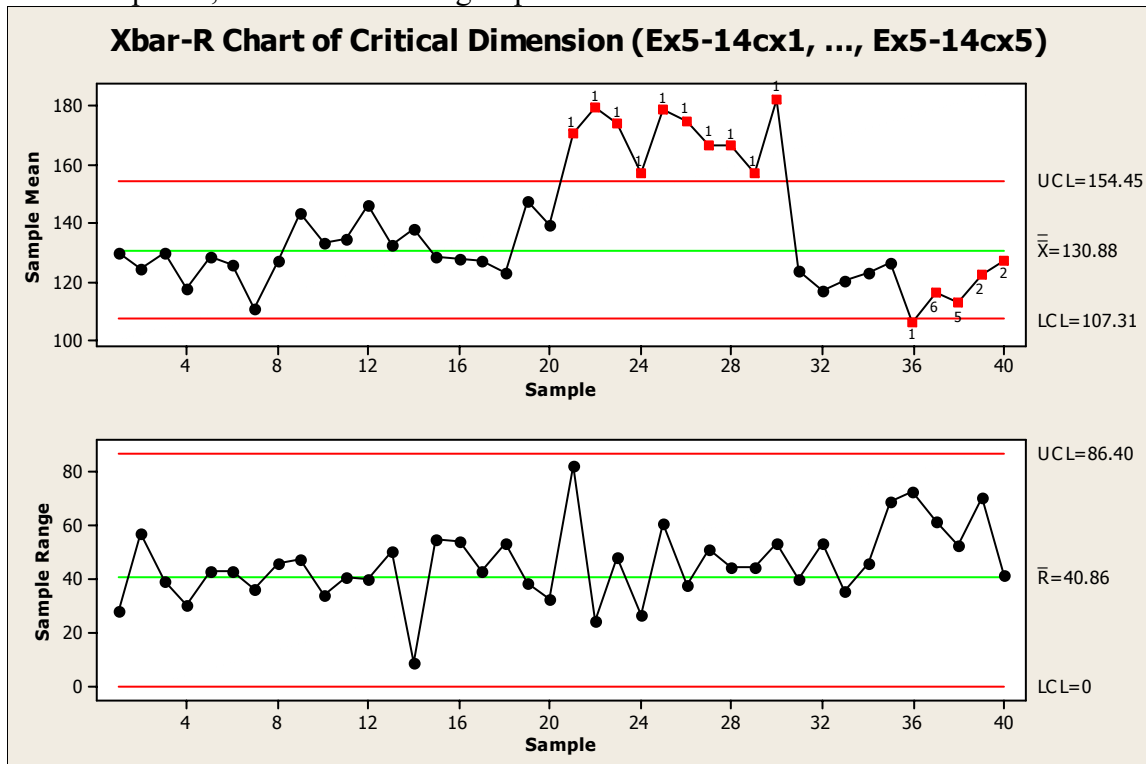
Chapter 5 Exercise Solutions

5-14 continued

(c)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R

Under "Options, Estimate" use subgroups 1:20 to calculate control limits.



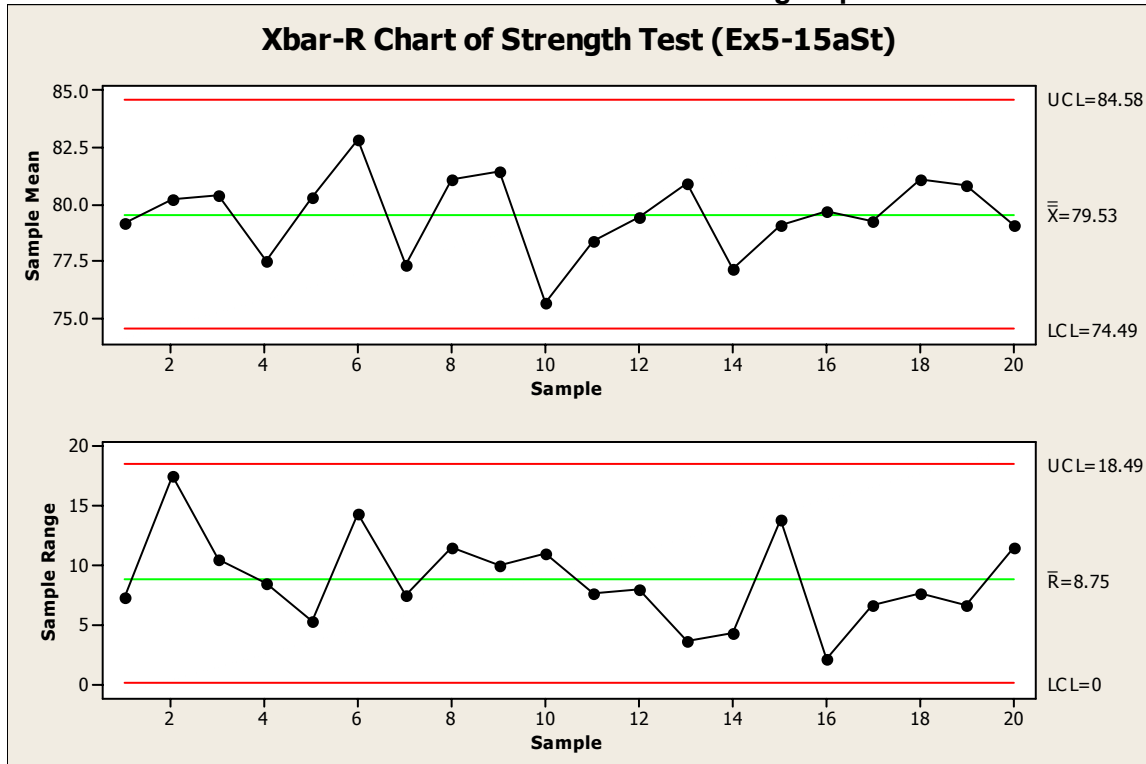
The adjustment overcompensated for the upward shift. The process average is now between $\bar{\bar{x}}$ and the LCL, with a run of ten points below the centerline, and one sample (#36) below the LCL.

Chapter 5 Exercise Solutions

5-15* (5-13).

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



Yes, the process is in control—though we should watch for a possible cyclic pattern in the averages.

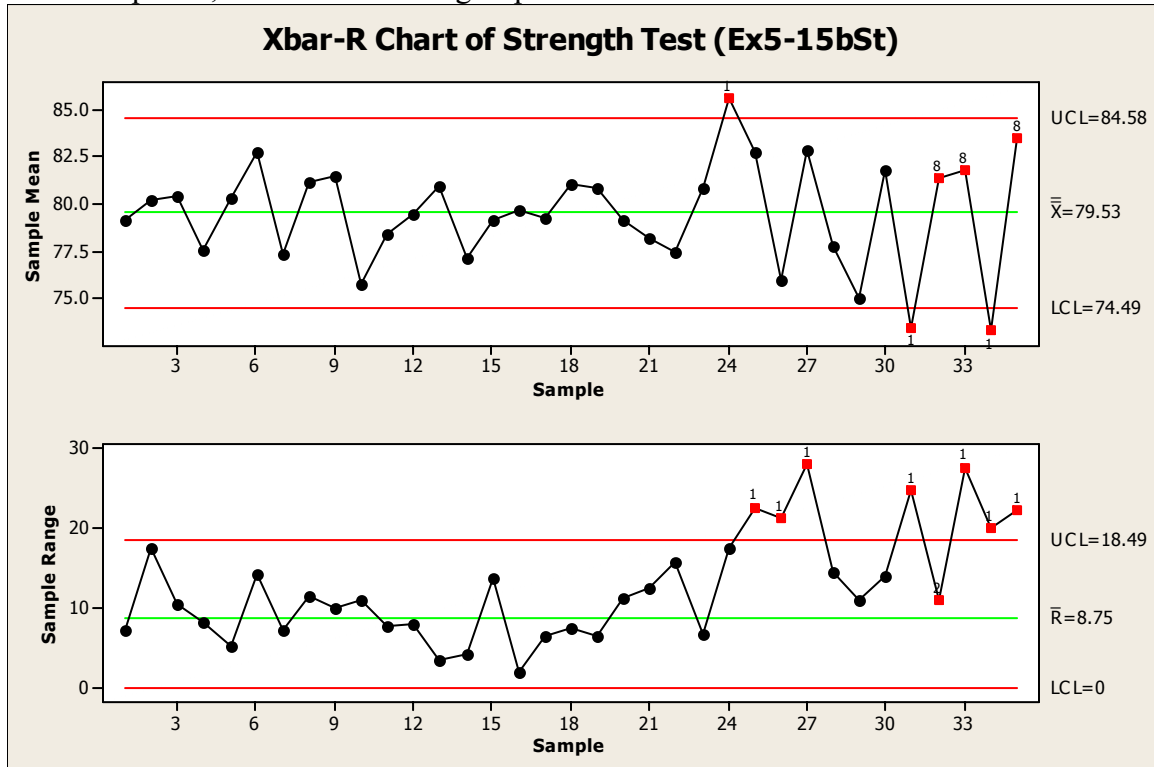
Chapter 5 Exercise Solutions

5-15 continued

(b)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R

Under "Options, Estimate" use subgroups 1:20 to calculate control limits.



Test Results for R Chart of Ex5-15bSt

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 25, 26, 27, 31, 33, 34, 35

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 32, 33, 34, 35

A strongly cyclic pattern in the averages is now evident, but more importantly, there are several out-of-control points on the range chart.

Chapter 5 Exercise Solutions

5-16 (5-14).

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S



Under “Options, Estimate” use subgroups 1:20 to calculate control limits.



Test Results for Xbar Chart of Ex5-15bSt

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 24, 31, 34

Test Results for S Chart of Ex5-15bSt

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 22, 25, 26, 27, 31, 33, 34, 35

Chapter 5 Exercise Solutions

5-16 continued

(b)

Yes, the s chart detects the change in process variability more quickly than the R chart did, at sample #22 versus sample #24.

5-17 (5-15).

$$n_{\text{old}} = 5; \bar{\bar{x}}_{\text{old}} = 34.00; \bar{R}_{\text{old}} = 4.7$$

(a)

for $n_{\text{new}} = 3$

$$UCL_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(n_{\text{new}})} \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = 34 + 1.023 \left[\frac{1.693}{2.326} \right] (4.7) = 37.50$$

$$LCL_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(n_{\text{new}})} \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = 34 - 1.023 \left[\frac{1.693}{2.326} \right] (4.7) = 30.50$$

$$UCL_R = D_{4(n_{\text{new}})} \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = 2.574 \left[\frac{1.693}{2.326} \right] (4.7) = 8.81$$

$$CL_R = \bar{R}_{\text{new}} = \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = \left[\frac{1.693}{2.326} \right] (4.7) = 3.42$$

$$LCL_R = D_{3(n_{\text{new}})} \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = 0 \left[\frac{1.693}{2.326} \right] (4.7) = 0$$

(b)

The \bar{x} control limits for $n = 5$ are “tighter” (31.29, 36.72) than those for $n = 3$ (30.50, 37.50). This means a 2σ shift in the mean would be detected more quickly with a sample size of $n = 5$.

Chapter 5 Exercise Solutions

5-17 continued

(c)

for $n = 8$

$$UCL_{\bar{x}} = \bar{\bar{x}}_{old} + A_{2(new)} \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = 34 + 0.373 \left[\frac{2.847}{2.326} \right] (4.7) = 36.15$$

$$LCL_{\bar{x}} = \bar{\bar{x}}_{old} - A_{2(new)} \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = 34 - 0.373 \left[\frac{2.847}{2.326} \right] (4.7) = 31.85$$

$$UCL_R = D_{4(new)} \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = 1.864 \left[\frac{2.847}{2.326} \right] (4.7) = 10.72$$

$$CL_R = \bar{R}_{new} = \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = \left[\frac{2.847}{2.326} \right] (4.7) = 5.75$$

$$LCL_R = D_{3(new)} \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = 0.136 \left[\frac{2.847}{2.326} \right] (4.7) = 0.78$$

(d)

The \bar{x} control limits for $n = 8$ are even "tighter" (31.85, 36.15), increasing the ability of the chart to quickly detect the 2σ shift in process mean.

5-18☺.

$n_{old} = 5$, $\bar{\bar{x}}_{old} = 74.001$, $\bar{R}_{old} = 0.023$, $n_{new} = 3$

$$UCL_{\bar{x}} = \bar{\bar{x}}_{old} + A_{2(new)} \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = 74.001 + 1.023 \left[\frac{1.693}{2.326} \right] (0.023) = 74.018$$

$$LCL_{\bar{x}} = \bar{\bar{x}}_{old} - A_{2(new)} \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = 74.001 - 1.023 \left[\frac{1.693}{2.326} \right] (0.023) = 73.984$$

$$UCL_R = D_{4(new)} \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = 2.574 \left[\frac{1.693}{2.326} \right] (0.023) = 0.043$$

$$CL_R = \bar{R}_{new} = \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = \left[\frac{1.693}{2.326} \right] (0.023) = 0.017$$

$$LCL_R = D_{3(new)} \left[\frac{d_{2(new)}}{d_{2(old)}} \right] \bar{R}_{old} = 0 \left[\frac{1.693}{2.326} \right] (0.023) = 0$$

Chapter 5 Exercise Solutions

5-19 (5-16).

$$n = 7; \quad \sum_{i=1}^{35} \bar{x}_i = 7805; \quad \sum_{i=1}^{35} R_i = 1200; \quad m = 35 \text{ samples}$$

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{35} \bar{x}_i}{m} = \frac{7805}{35} = 223$$

$$\bar{R} = \frac{\sum_{i=1}^{35} R_i}{m} = \frac{1200}{35} = 34.29$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R} = 223 + 0.419(34.29) = 237.37$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R} = 223 - 0.419(34.29) = 208.63$$

$$UCL_R = D_4 \bar{R} = 1.924(34.29) = 65.97$$

$$LCL_R = D_3 \bar{R} = 0.076(34.29) = 2.61$$

(b)

$$\hat{\mu} = \bar{\bar{x}} = 223; \quad \hat{\sigma}_x = \bar{R} / d_2 = 34.29 / 2.704 = 12.68$$

(c)

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+35 - (-35)}{6(12.68)} = 0.92, \text{ the process is not capable of meeting}$$

specifications.

$$\begin{aligned} \hat{p} &= \Pr\{x > USL\} + \Pr\{x < LSL\} = 1 - \Pr\{x < USL\} + \Pr\{x < LSL\} = 1 - \Pr\{x \leq 255\} + \Pr\{x \leq 185\} \\ &= 1 - \Phi\left(\frac{255 - 223}{12.68}\right) + \Phi\left(\frac{185 - 223}{12.68}\right) = 1 - \Phi(2.52) + \Phi(-3.00) = 1 - 0.99413 + 0.00135 = 0.0072 \end{aligned}$$

(d)

The process mean should be located at the nominal dimension, 220, to minimize non-conforming units.

$$\hat{p} = 1 - \Phi\left(\frac{255 - 220}{12.68}\right) + \Phi\left(\frac{185 - 220}{12.68}\right) = 1 - \Phi(2.76) + \Phi(-2.76) = 1 - 0.99711 + 0.00289 = 0.00578$$

Chapter 5 Exercise Solutions

5-20 (5-17).

$$n = 5; \quad \sum_{i=1}^{25} \bar{x}_i = 662.50; \quad \sum_{i=1}^{25} R_i = 9.00; \quad m = 25 \text{ samples}$$

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{25} \bar{x}_i}{m} = \frac{662.50}{25} = 26.50$$

$$\bar{R} = \frac{\sum_{i=1}^{25} R_i}{m} = \frac{9.00}{25} = 0.36$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R} = 26.50 + 0.577(0.36) = 26.71$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R} = 26.50 - 0.577(0.36) = 26.29$$

$$UCL_R = D_4 \bar{R} = 2.114(0.36) = 0.76$$

$$LCL_R = D_3 \bar{R} = 0(0.36) = 0$$

(b)

$$\hat{\sigma}_x = \bar{R} / d_2 = 0.36 / 2.326 = 0.155$$

$$\hat{p} = \Pr\{x > USL\} + \Pr\{x < LSL\} = 1 - \Pr\{x \leq USL\} + \Pr\{x < LSL\}$$

$$\begin{aligned} &= 1 - \Phi\left(\frac{26.90 - 26.50}{0.155}\right) + \Phi\left(\frac{25.90 - 26.50}{0.155}\right) = 1 - \Phi(2.58) + \Phi(-3.87) = 1 - 0.99506 + 0.00005 \\ &= 0.00499 \end{aligned}$$

(c)

$$\begin{aligned} \hat{p} &= 1 - \Phi\left(\frac{26.90 - 26.40}{0.155}\right) + \Phi\left(\frac{25.90 - 26.40}{0.155}\right) = 1 - \Phi(3.23) + \Phi(-3.23) \\ &= 1 - 0.99938 + 0.00062 = 0.00124 \end{aligned}$$

Chapter 5 Exercise Solutions

5-21 (5-18).

$n = 5$; $\bar{\bar{x}} = 20.0$; $\bar{S} = 1.5$; $m = 50$ samples

(a)

$$\hat{\sigma}_x = \bar{S} / c_4 = 1.5 / 0.9400 = 1.60$$

(b)

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_3\bar{S} = 20.0 + 1.427(1.5) = 22.14$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_3\bar{S} = 20.0 - 1.427(1.5) = 17.86$$

$$UCL_S = B_4\bar{S} = 2.089(1.5) = 3.13$$

$$LCL_S = B_3\bar{S} = 0(1.5) = 0$$

(c)

$$\Pr\{\text{in control}\} = \Pr\{LCL \leq \bar{x} \leq UCL\} = \Pr\{\bar{x} \leq UCL\} - \Pr\{\bar{x} \leq LCL\}$$

$$= \Phi\left(\frac{22.14 - 22}{1.6/\sqrt{5}}\right) - \Phi\left(\frac{17.86 - 22}{1.6/\sqrt{5}}\right) = \Phi(0.20) - \Phi(-5.79)$$

$$= 0.57926 - 0 = 0.57926$$

5-22 (5-19).

$$\Pr\{\text{detect}\} = 1 - \Pr\{\text{not detect}\} = 1 - [\Pr\{LCL \leq \bar{x} \leq UCL\}] = 1 - [\Pr\{\bar{x} \leq UCL\} - \Pr\{\bar{x} \leq LCL\}]$$

$$= 1 - \left[\Phi\left(\frac{UCL_{\bar{x}} - \mu_{\text{new}}}{\sigma_x/\sqrt{n}}\right) - \Phi\left(\frac{LCL_{\bar{x}} - \mu_{\text{new}}}{\sigma_x/\sqrt{n}}\right) \right] = 1 - \left[\Phi\left(\frac{209 - 188}{6/\sqrt{4}}\right) - \Phi\left(\frac{191 - 188}{6/\sqrt{4}}\right) \right]$$

$$= 1 - \Phi(7) + \Phi(1) = 1 - 1 + 0.84134 = 0.84134$$

5-23 (5-20).

$X \sim N$; $n = 5$; $\bar{\bar{x}} = 104$; $\bar{R} = 9.30$; $USL = 110$; $LSL = 90$

$\hat{\sigma}_x = \bar{R} / d_2 = 9.30 / 2.326 = 3.998$ and $6\hat{\sigma}_x = 6(3.998) = 23.99$ is larger than the width of the tolerance band, $2(10) = 20$. So, even if the mean is located at the nominal dimension, 100, not all of the output will meet specification.

$$\hat{C}_P = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+10 - (-10)}{6(3.998)} = 0.8338$$

Chapter 5 Exercise Solutions

5-24* (5-21).

$n = 2$; $\mu = 10$; $\sigma_x = 2.5$. These are standard values.

(a)

$$\text{centerline}_{\bar{x}} = \mu = 10$$

$$\text{UCL}_{\bar{x}} = \mu + A\sigma_x = 10 + 2.121(2.5) = 15.30$$

$$\text{LCL}_{\bar{x}} = \mu - A\sigma_x = 10 - 2.121(2.5) = 4.70$$

(b)

$$\text{centerline}_R = d_2\sigma_x = 1.128(2.5) = 2.82$$

$$\text{UCL}_R = D_2\sigma = 3.686(2.5) = 9.22$$

$$\text{LCL}_R = D_1\sigma = 0(2.5) = 0$$

(c)

$$\text{centerline}_S = c_4\sigma_x = 0.7979(2.5) = 1.99$$

$$\text{UCL}_S = B_6\sigma = 2.606(2.5) = 6.52$$

$$\text{LCL}_S = B_5\sigma = 0(2.5) = 0$$

Chapter 5 Exercise Solutions

5-25 (5-22).

$n = 5$; $\bar{\bar{x}} = 20$; $\bar{R} = 4.56$; $m = 25$ samples

(a)

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 20 + 0.577(4.56) = 22.63$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 20 - 0.577(4.56) = 17.37$$

$$UCL_R = D_4\bar{R} = 2.114(4.56) = 9.64$$

$$LCL_R = D_3\bar{R} = 0(4.56) = 0$$

(b)

$$\hat{\sigma}_x = \bar{R} / d_2 = 4.56 / 2.326 = 1.96$$

(c)

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+5 - (-5)}{6(1.96)} = 0.85, \text{ so the process is not capable of meeting}$$

specifications.

(d)

$$\Pr\{\text{not detect}\} = \Pr\{LCL \leq \bar{x} \leq UCL\} = \Pr\{\bar{x} \leq UCL\} - \Pr\{\bar{x} \leq LCL\}$$

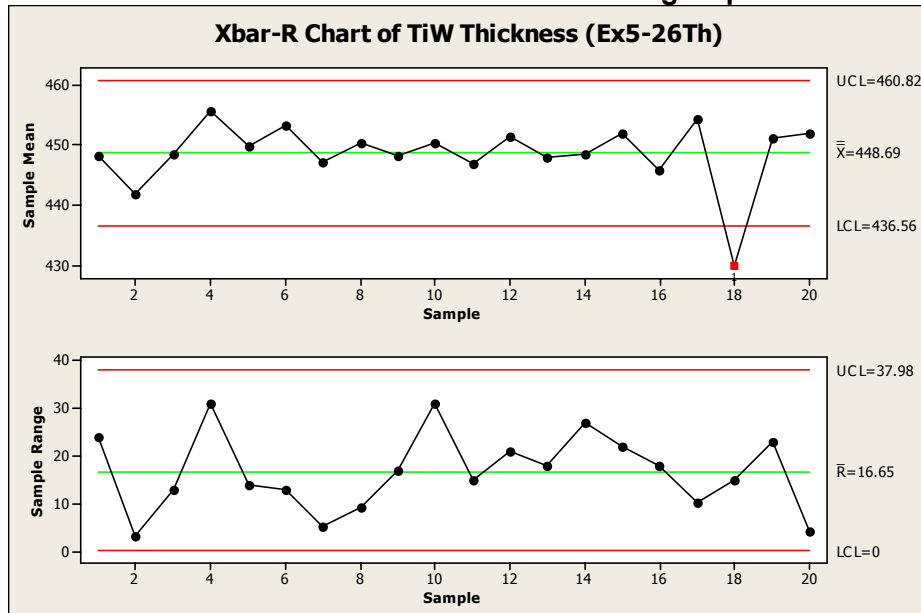
$$= \Phi\left(\frac{UCL_{\bar{x}} - \mu_{\text{new}}}{\hat{\sigma}_x / \sqrt{n}}\right) - \Phi\left(\frac{LCL_{\bar{x}} - \mu_{\text{new}}}{\hat{\sigma}_x / \sqrt{n}}\right) = \Phi\left(\frac{22.63 - 24}{1.96 / \sqrt{5}}\right) - \Phi\left(\frac{17.37 - 24}{1.96 / \sqrt{5}}\right)$$

$$= \Phi(-1.56) + \Phi(-7.56) = 0.05938 - 0 = 0.05938$$

Chapter 5 Exercise Solutions

5-26 ☺.

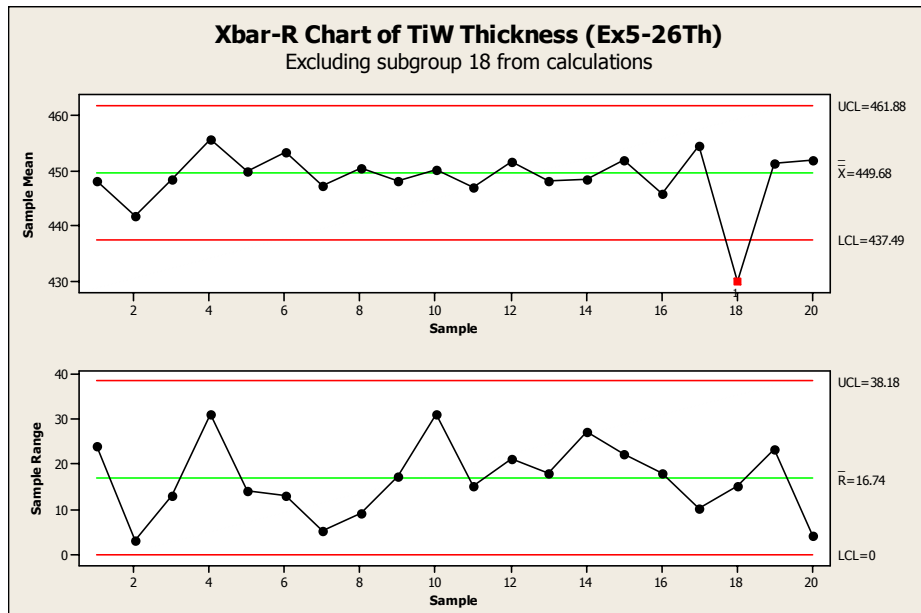
MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



Test Results for Xbar Chart of Ex5-26Th

TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points: 18

The process is out of control on the \bar{x} chart at subgroup 18. Excluding subgroup 18 from control limits calculations:



Test Results for Xbar Chart of Ex5-26Th

TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points: 18

No additional subgroups are beyond the control limits, so these limits can be used for future production.

Chapter 5 Exercise Solutions

5-26 continued

(b)

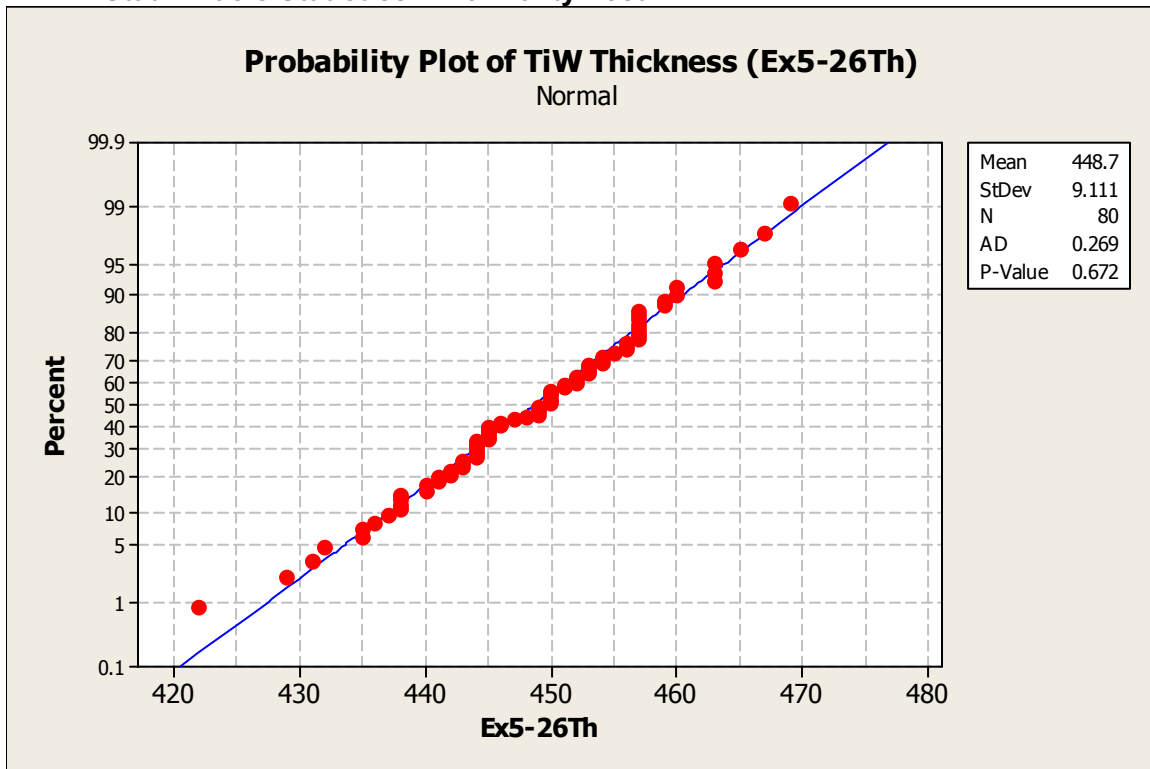
Excluding subgroup 18:

$$\bar{x} = 449.68$$

$$\hat{\sigma}_x = \bar{R} / d_2 = 16.74 / 2.059 = 8.13$$

(c)

MTB > Stat > Basic Statistics > Normality Test



A normal probability plot of the TiW thickness measurements shows the distribution is close to normal.

Chapter 5 Exercise Solutions

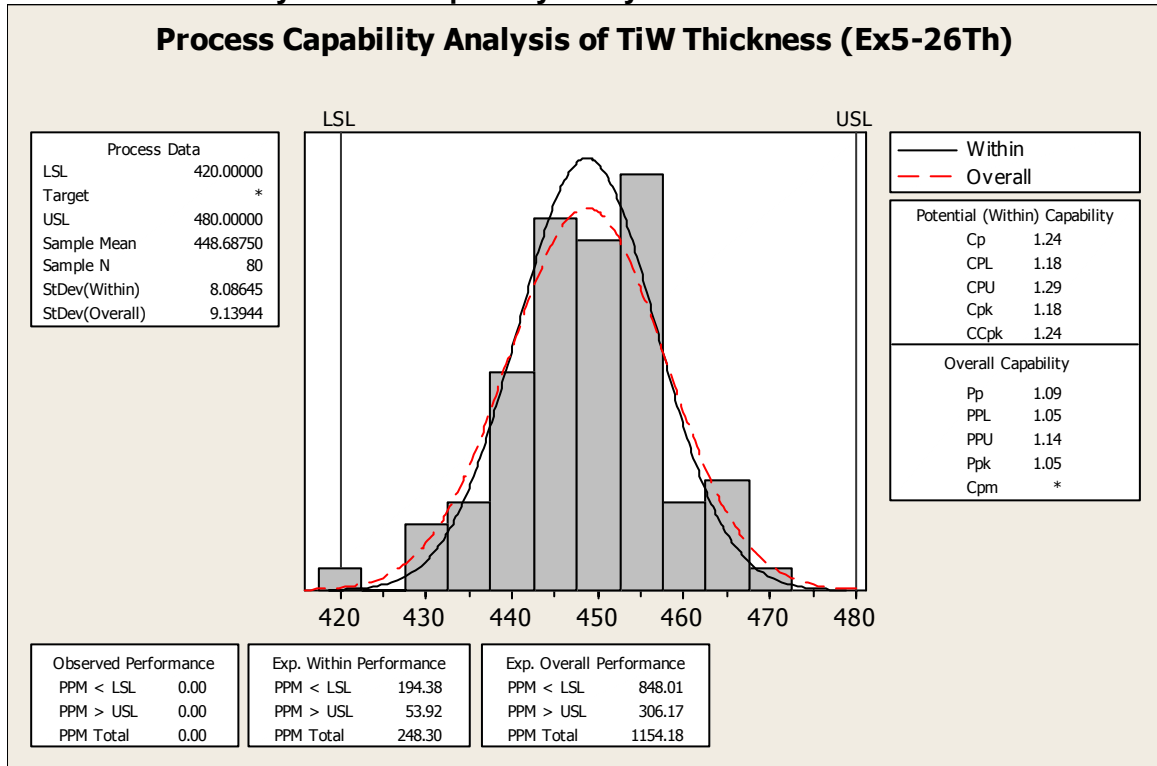
5-26 continued

(d)

USL = +30, LSL = -30

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{+30 - (-30)}{6(8.13)} = 1.23, \text{ so the process is capable.}$$

MTB > Stat > Quality Tools > Capability Analysis > Normal

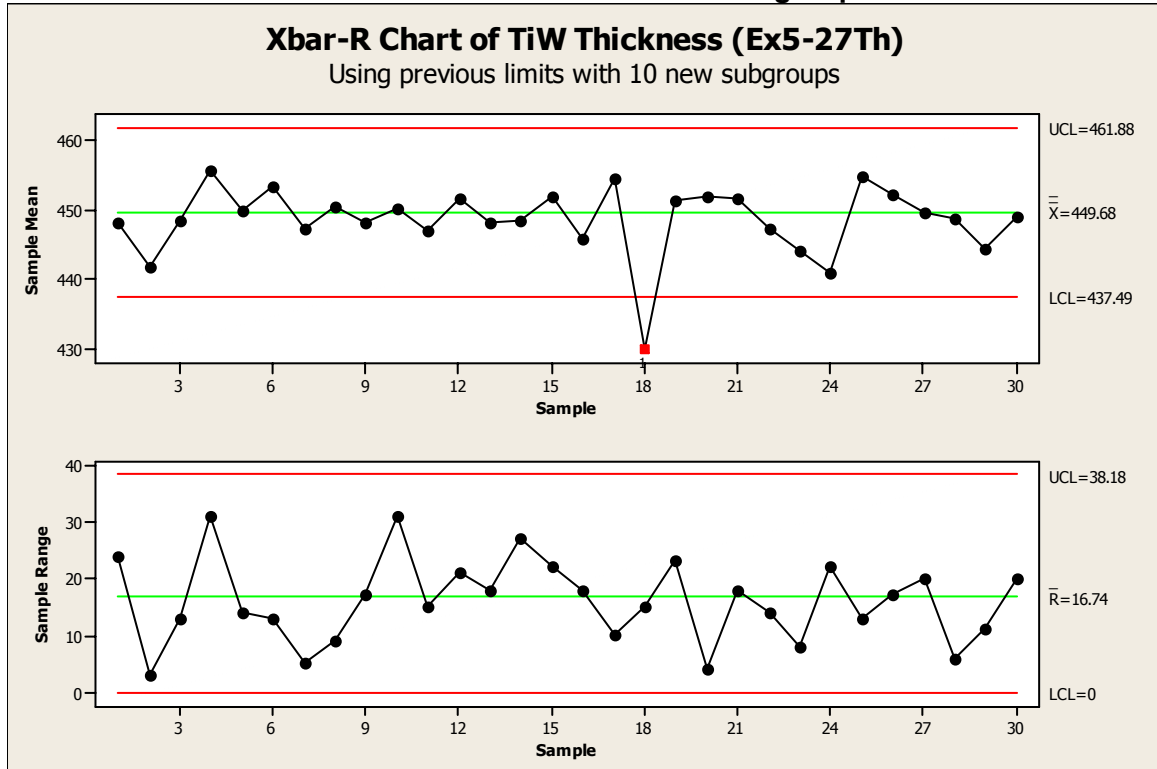


The Potential (Within) Capability, $C_p = 1.24$, is estimated from the within-subgroup variation, or in other words, σ_x is estimated using \bar{R} . This is the same result as the manual calculation.

Chapter 5 Exercise Solutions

5-27☺.

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



Test Results for Xbar Chart of Ex5-27Th

TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points: 18

The process continues to be in a state of statistical control.

Chapter 5 Exercise Solutions

5-28☺.

$$n_{\text{old}} = 4; \quad \bar{\bar{x}}_{\text{old}} = 449.68; \quad \bar{R}_{\text{old}} = 16.74; \quad n_{\text{new}} = 2$$

$$UCL_{\bar{x}} = \bar{\bar{x}}_{\text{old}} + A_{2(n_{\text{new}})} \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = 449.68 + 1.880 \left[\frac{1.128}{2.059} \right] (16.74) = 466.92$$

$$LCL_{\bar{x}} = \bar{\bar{x}}_{\text{old}} - A_{2(n_{\text{new}})} \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = 449.68 - 1.880 \left[\frac{1.128}{2.059} \right] (16.74) = 432.44$$

$$UCL_R = D_{4(n_{\text{new}})} \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = 3.267 \left[\frac{1.128}{2.059} \right] (16.74) = 29.96$$

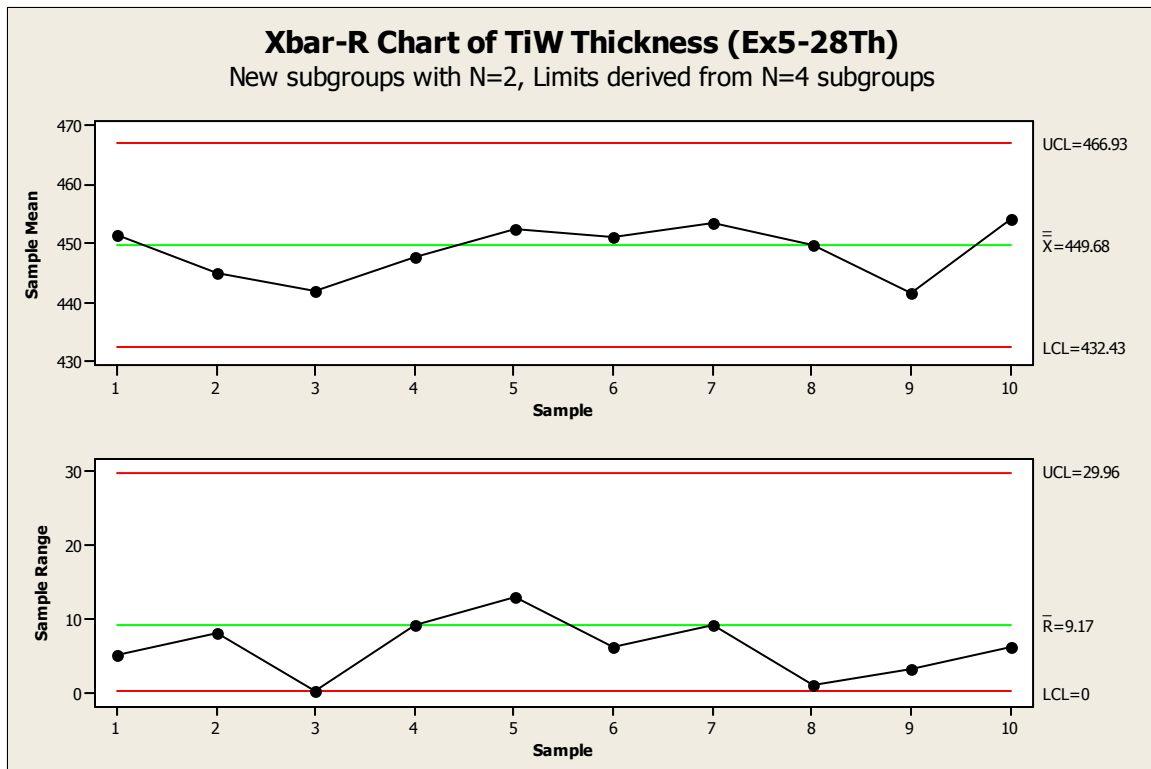
$$CL_R = \bar{R}_{\text{new}} = \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = \left[\frac{1.128}{2.059} \right] (16.74) = 9.17$$

$$LCL_R = D_{3(n_{\text{new}})} \left[\frac{d_{2(n_{\text{new}})}}{d_{2(n_{\text{old}})}} \right] \bar{R}_{\text{old}} = 0 \left[\frac{1.128}{2.059} \right] (16.74) = 0$$

$$\hat{\sigma}_{\text{new}} = \bar{R}_{\text{new}} / d_{2(n_{\text{new}})} = 9.17 / 1.128 = 8.13$$

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R

Select Xbar-R options, Parameters, and enter new parameter values.



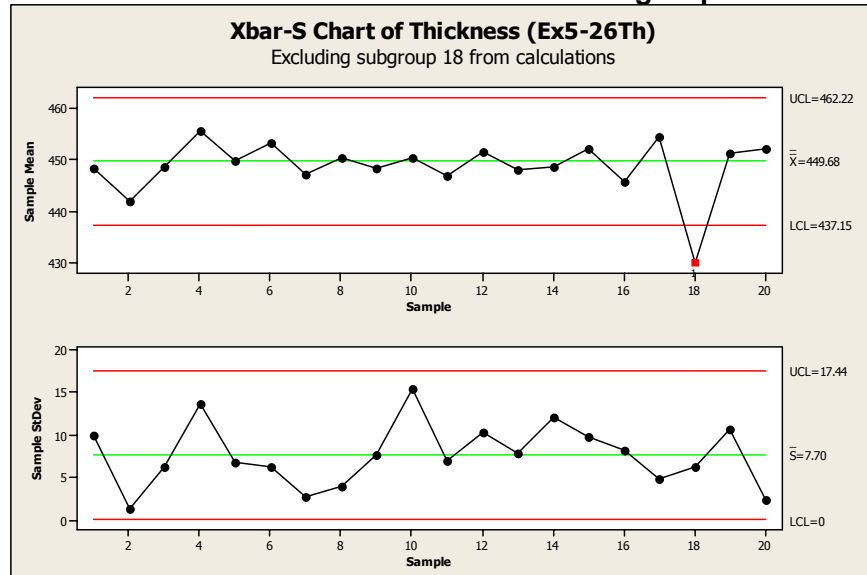
The process remains in statistical control.

Chapter 5 Exercise Solutions

5-29☺.

The process is out of control on the \bar{x} chart at subgroup 18. After finding assignable cause, exclude subgroup 18 from control limits calculations:

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-S

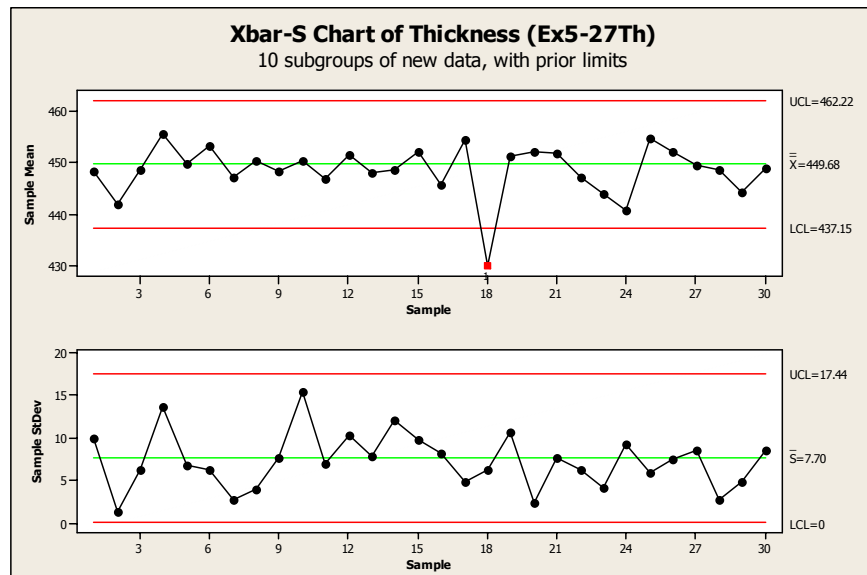


Xbar-S Chart of Ex5-26Th

Test Results for Xbar Chart of Ex5-26Th

TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points: 18

No additional subgroups are beyond the control limits, so these limits can be used for future production.



The process remains in statistical control.

Chapter 5 Exercise Solutions

5-30 (5-23).

$$n = 6; \quad \sum_{i=1}^{30} \bar{x}_i = 6000; \quad \sum_{i=1}^{30} R_i = 150; \quad m = 30 \text{ samples}$$

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{30} \bar{x}_i}{m} = \frac{6000}{30} = 200$$

$$\bar{\bar{R}} = \frac{\sum_{i=1}^{30} R_i}{m} = \frac{150}{30} = 5$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{\bar{R}} = 200 + 0.483(5) = 202.42$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{\bar{R}} = 200 - 0.483(5) = 197.59$$

$$\text{UCL}_R = D_4 \bar{\bar{R}} = 2.004(5) = 10.02$$

$$\text{LCL}_R = D_3 \bar{\bar{R}} = 0(5) = 0$$

(b)

$$\hat{\sigma}_x = \bar{R} / d_2 = 5 / 2.534 = 1.97$$

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+5 - (-5)}{6(1.97)} = 0.85$$

The process is not capable of meeting specification. Even though the process is centered at nominal, the variation is large relative to the tolerance.

(c)

$$\begin{aligned} \beta - \text{risk} &= \Pr\{\text{not detect}\} = \Phi\left(\frac{202.42 - 199}{1.97/\sqrt{6}}\right) - \Phi\left(\frac{197.59 - 199}{1.97/\sqrt{6}}\right) \\ &= \Phi(4.25) - \Phi(-1.75) = 1 - 0.04006 = 0.95994 \end{aligned}$$

Chapter 5 Exercise Solutions

5-31 (5-24).

$$\mu_0 = 100; \quad L = 3; \quad n = 4; \quad \sigma = 6; \quad \mu_1 = 92$$

$$k = (\mu_1 - \mu_0) / \sigma = (92 - 100) / 6 = -1.33$$

$$\begin{aligned} \Pr\{\text{detecting shift on 1st sample}\} &= 1 - \Pr\{\text{not detecting shift on 1st sample}\} \\ &= 1 - \beta \\ &= 1 - \left[\Phi(L - k\sqrt{n}) - \Phi(-L - k\sqrt{n}) \right] \\ &= 1 - \left[\Phi(3 - (-1.33)\sqrt{4}) - \Phi(-3 - (-1.33)\sqrt{4}) \right] \\ &= 1 - [\Phi(5.66) - \Phi(-0.34)] \\ &= 1 - [1 - 0.37] \\ &= 0.37 \end{aligned}$$

5-32 (5-25).

(a)

$$\bar{\bar{x}} = 104.05; \quad \bar{R} = 3.95$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 104.05 + 0.577(3.95) = 106.329$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 104.05 - 0.577(3.95) = 101.771$$

$$UCL_R = D_4\bar{R} = 2.114(3.95) = 8.350$$

$$LCL_R = D_3\bar{R} = 0(3.95) = 0$$

Sample #4 is out of control on the Range chart. So, excluding #4 and recalculating:

$$\bar{\bar{x}} = 104; \quad \bar{R} = 3.579$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 104 + 0.577(3.579) = 106.065$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 104 - 0.577(3.579) = 101.935$$

$$UCL_R = D_4\bar{R} = 2.114(3.579) = 7.566$$

$$LCL_R = D_3\bar{R} = 0(3.579) = 0$$

(b)

$$\text{Without sample \#4, } \hat{\sigma}_x = \bar{R} / d_2 = 3.579 / 2.326 = 1.539$$

(c)

$$UNTL = \bar{\bar{x}} + 3\hat{\sigma}_x = 104 + 3(1.539) = 108.62$$

$$LNNTL = \bar{\bar{x}} - 3\hat{\sigma}_x = 104 - 3(1.539) = 99.38$$

Chapter 5 Exercise Solutions

5-32 continued

(d)

$$\hat{p} = 1 - \Phi\left(\frac{107-104}{1.539}\right) + \Phi\left(\frac{99-104}{1.539}\right) = 1 - \Phi(1.95) + \Phi(-3.25) = 1 - 0.9744 + 0.0006 = 0.0262$$

(e)

To reduce the fraction nonconforming, first center the process at nominal.

$$\hat{p} = 1 - \Phi\left(\frac{107-103}{1.539}\right) + \Phi\left(\frac{99-103}{1.539}\right) = 1 - \Phi(2.60) + \Phi(-2.60) = 1 - 0.9953 + 0.0047 = 0.0094$$

Next work on reducing the variability; if $\hat{\sigma}_x = 0.667$, then almost 100% of parts will be within specification.

$$\hat{p} = 1 - \Phi\left(\frac{107-103}{0.667}\right) + \Phi\left(\frac{99-103}{0.667}\right) = 1 - \Phi(5.997) + \Phi(-5.997) = 1 - 1.0000 + 0.0000 = 0.0000$$

5-33 (5-26).

$$n = 5; \quad \sum_{i=1}^{30} \bar{x}_i = 607.8; \quad \sum_{i=1}^{30} R_i = 144; \quad m = 30$$

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^m \bar{x}_i}{m} = \frac{607.8}{30} = 20.26$$

$$\bar{R} = \frac{\sum_{i=1}^m R_i}{m} = \frac{144}{30} = 4.8$$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R} = 20.26 + 0.577(4.8) = 23.03$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R} = 20.26 - 0.577(4.8) = 17.49$$

$$UCL_R = D_4 \bar{R} = 2.114(4.8) = 10.147$$

$$LCL_R = D_3 \bar{R} = 0(4.8) = 0$$

(b)

$$\hat{\sigma}_x = \bar{R} / d_2 = 4.8 / 2.326 = 2.064$$

$$\hat{p} = \Pr\{x < LSL\} = \Phi\left(\frac{16 - 20.26}{2.064}\right) = \Phi(-2.064) = 0.0195$$

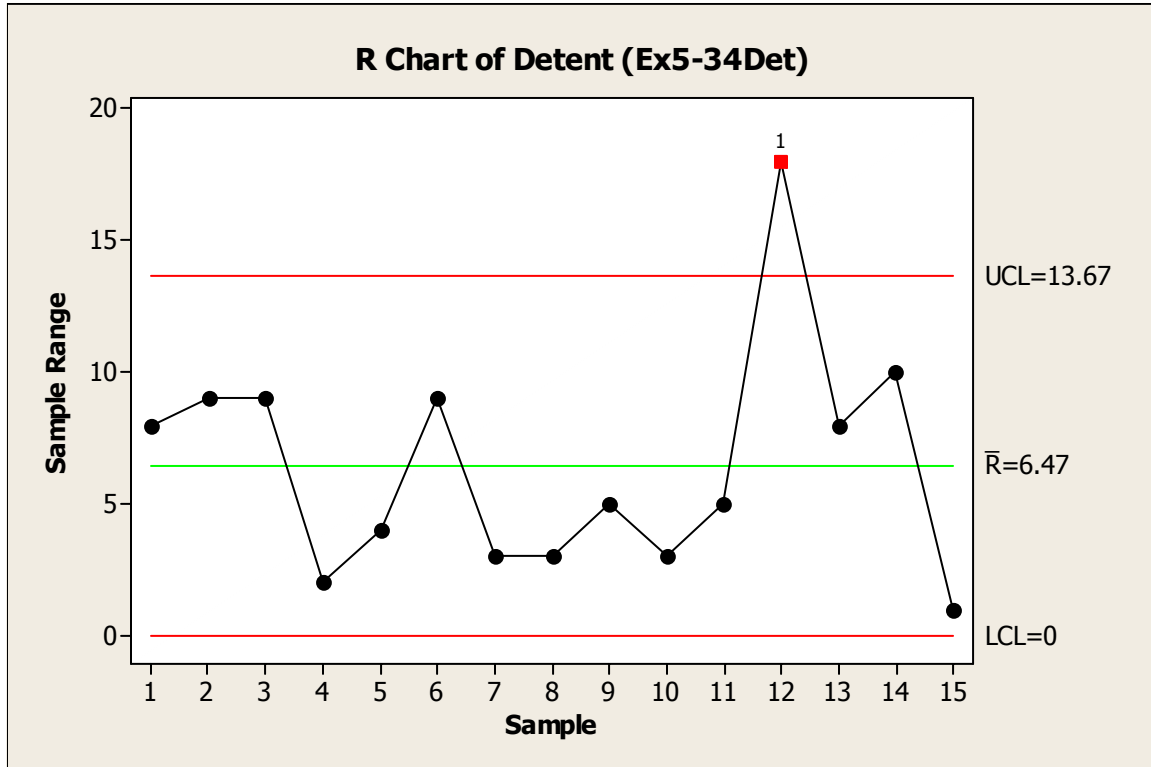
Chapter 5 Exercise Solutions

5-34 (5-27).

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > R

Under "Options, Estimate" select Rbar as method to estimate standard deviation.



Test Results for R Chart of Ex5-34Det

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 12

Process is not in statistical control -- sample #12 exceeds the upper control limit on the Range chart.

Chapter 5 Exercise Solutions

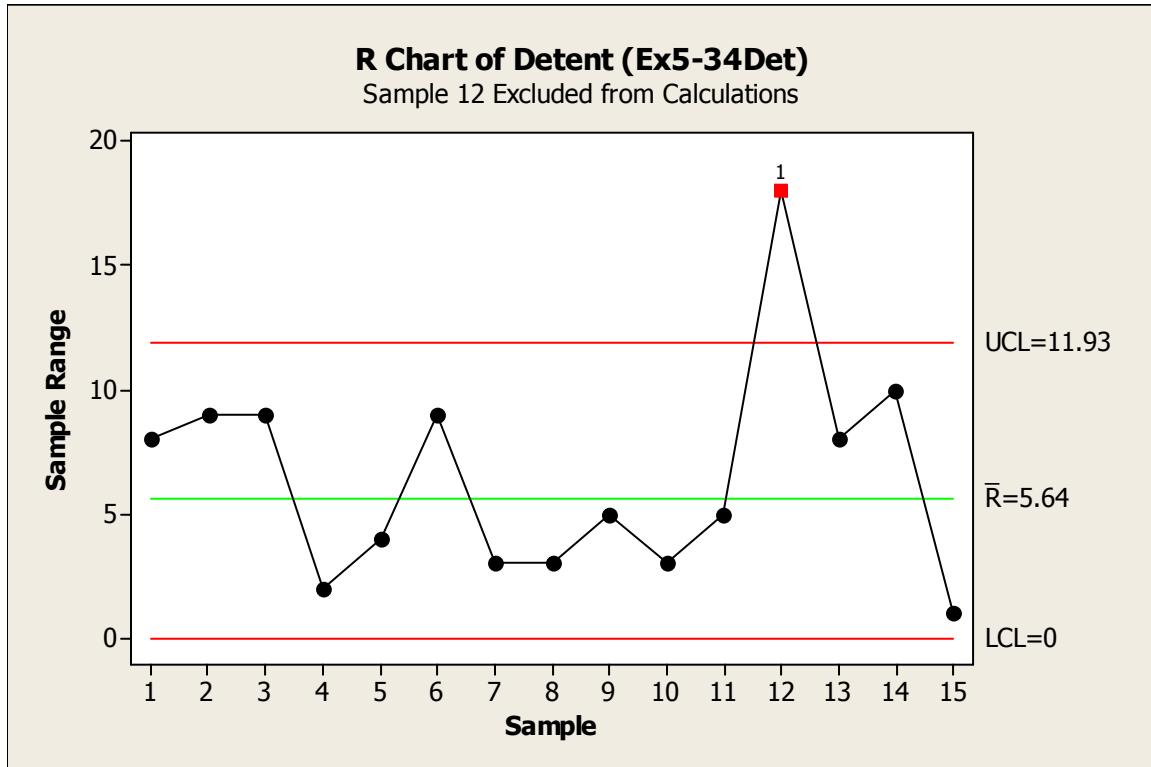
5-34 continued

(b)

Excluding Sample Number 12:

MTB > Stat > Control Charts > Variables Charts for Subgroups > R

Under “Options, Estimate” omit subgroup 12 and select Rbar.



Test Results for R Chart of Ex5-34Det

TEST 1. One point more than 3.00 standard deviations from center line.
Test Failed at points: 12

(c)

Without sample #12: $\hat{\sigma}_x = \bar{R} / d_2 = 5.64 / 2.326 = 2.42$

(d)

Assume the cigar lighter detent is normally distributed. Without sample #12:

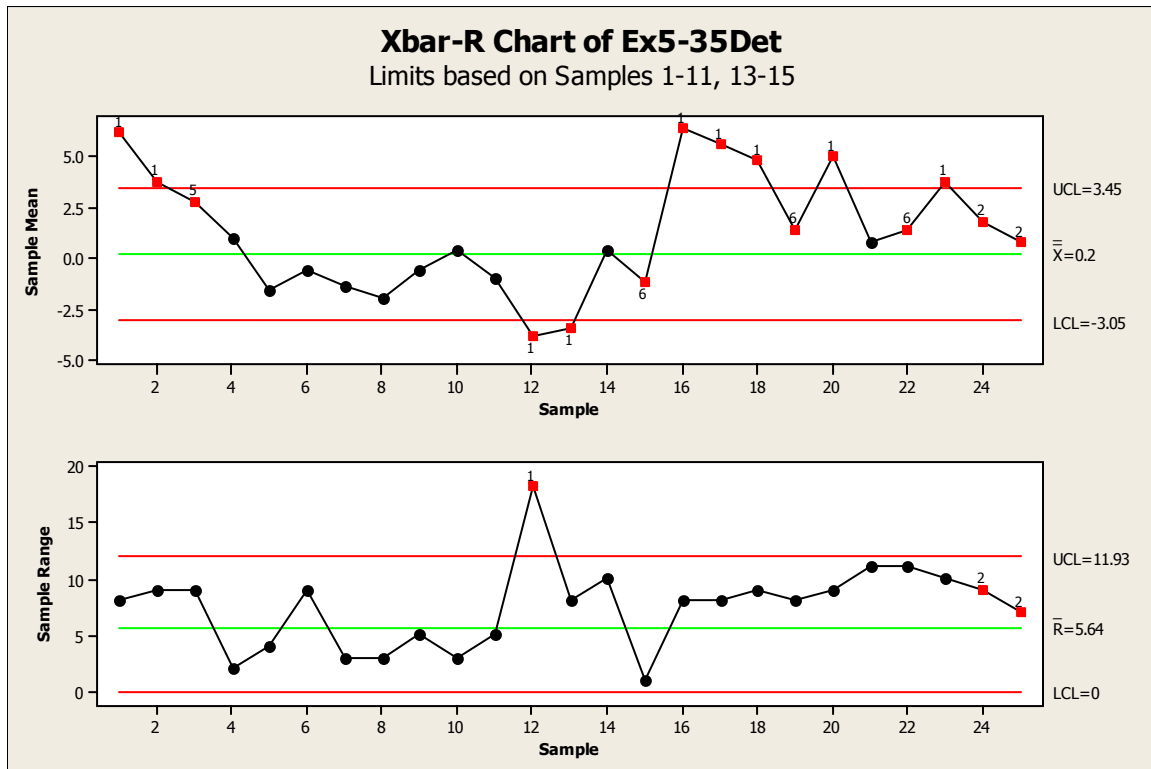
$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{0.3220 - 0.3200}{6(2.42 \times 0.0001)} = 1.38$$

Chapter 5 Exercise Solutions

5-35 (5-28).

MTB > Stat > Control Charts > Variables Charts for Subgroups > R

Under "Options, Estimate" use subgroups 1:11 and 13:15, and select Rbar.



Test Results for Xbar Chart of Ex5-35Det

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 1, 2, 12, 13, 16, 17, 18, 20, 23

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 24, 25

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 2, 3, 13, 17, 18, 20

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on one side of CL).

Test Failed at points: 15, 19, 20, 22, 23, 24

Test Results for R Chart of Ex5-35Det

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 12

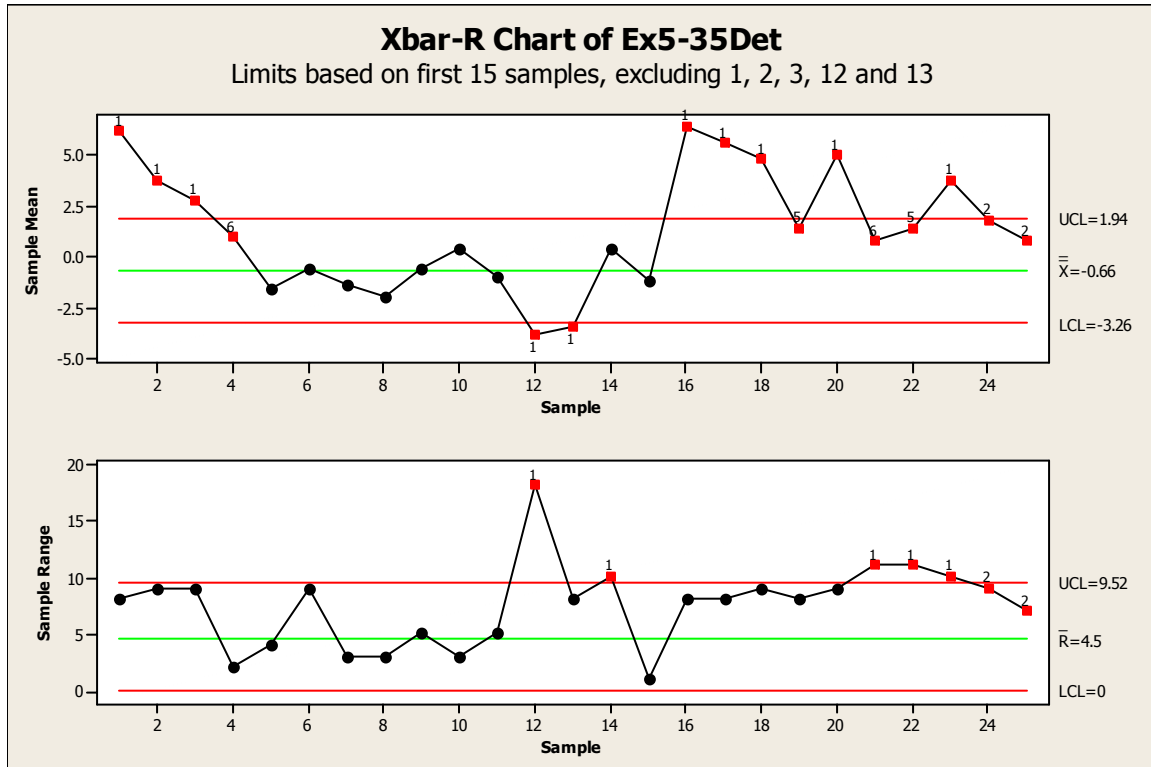
TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 24, 25

Chapter 5 Exercise Solutions

5-35 continued

We are trying to establish trial control limits from the first 15 samples to monitor future production. Note that samples 1, 2, 12, and 13 are out of control on the \bar{x} chart. If these samples are removed and the limits recalculated, sample 3 is also out of control on the \bar{x} chart. Removing sample 3 gives



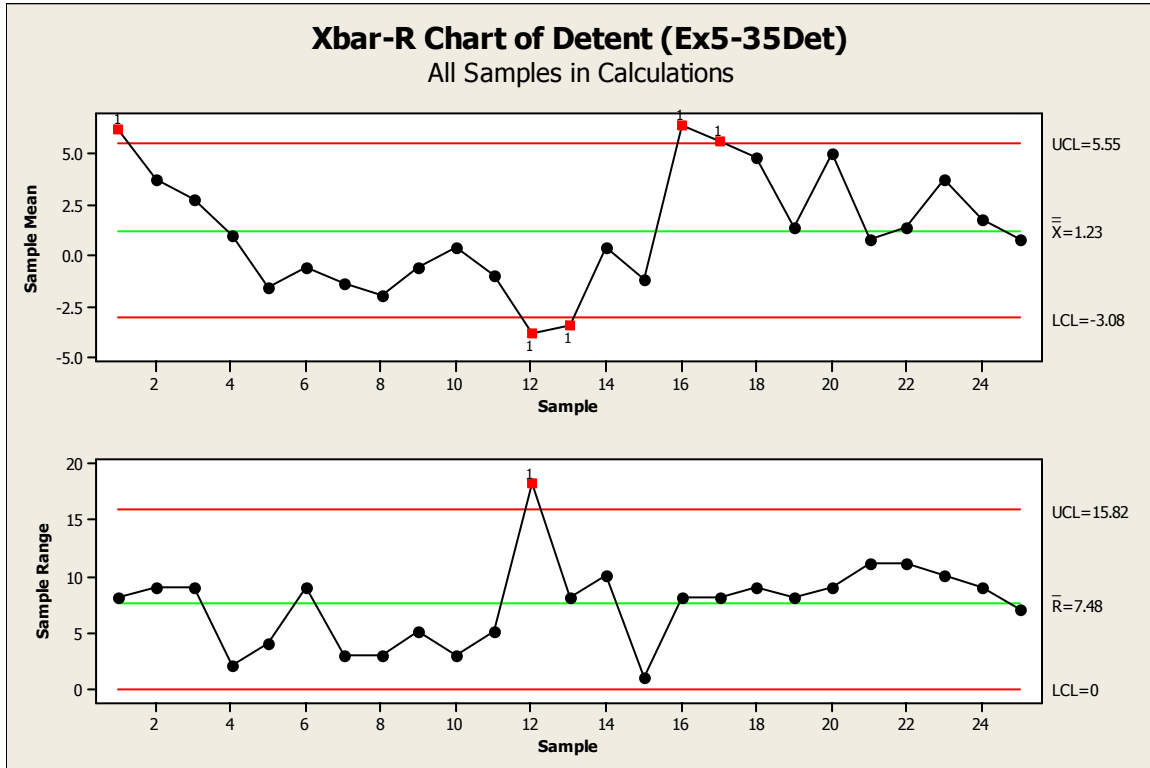
Sample 14 is now out of control on the R chart. No additional samples are out of control on the \bar{x} chart. While the limits on the above charts may be used to monitor future production, the fact that 6 of 15 samples were out of control and eliminated from calculations is an early indication of process instability.

(a) Given the large number of points after sample 15 beyond both the \bar{x} and R control limits on the charts above, the process appears to be unstable.

Chapter 5 Exercise Solutions

5-35 continued

(b)



With Test 1 only:

Test Results for Xbar Chart of Ex5-35Det

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 1, 12, 13, 16, 17

Test Results for R Chart of Ex5-35Det

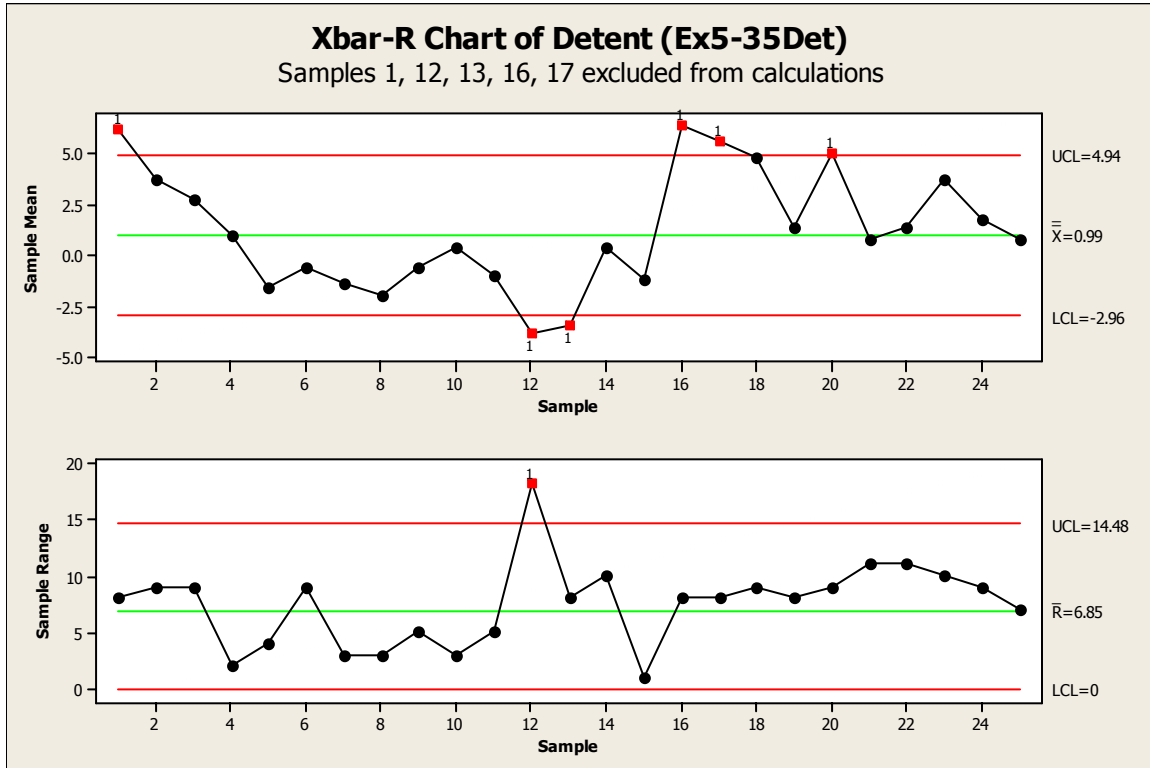
TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 12

Chapter 5 Exercise Solutions

5-35 (b) continued

Removing samples 1, 12, 13, 16, and 17 from calculations:



With Test 1 only:

Test Results for Xbar Chart of Ex5-35Det

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 1, 12, 13, 16, 17, 20

Test Results for R Chart of Ex5-35Det

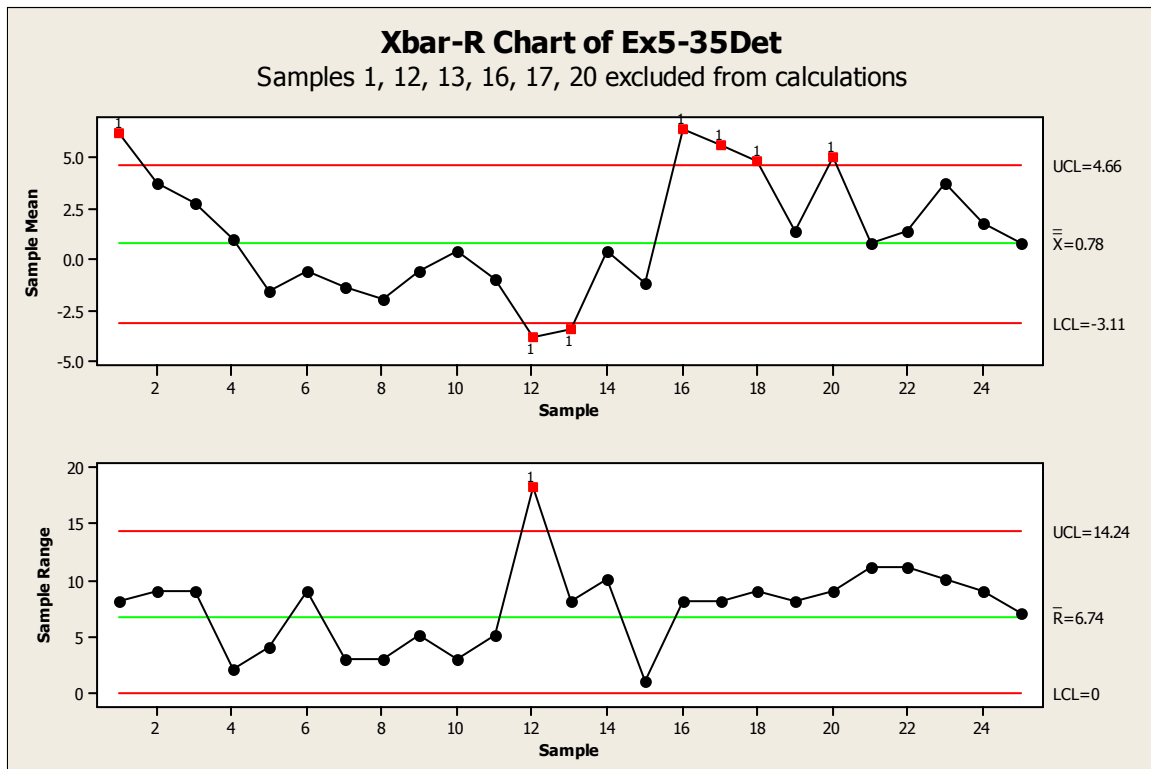
TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 12

Chapter 5 Exercise Solutions

5-35 continued

Sample 20 is now also out of control. Removing sample 20 from calculations,



With Test 1 only:

Test Results for Xbar Chart of Ex5-35Det

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 1, 12, 13, 16, 17, 18, 20

Test Results for R Chart of Ex5-35Det

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 12

Sample 18 is now out-of-control, for a total 7 of the 25 samples, with runs of points both above and below the centerline. This suggests that the process is inherently unstable, and that the sources of variation need to be identified and removed.

Chapter 5 Exercise Solutions

5-36 (5-29).

(a)

$$n = 5; \quad m_x = 20; \quad m_y = 10; \quad \sum_{i=1}^{20} R_{x,i} = 18.608; \quad \sum_{i=1}^{10} R_{y,i} = 6.978$$

$$\hat{\sigma}_x = \bar{R}_x / d_2 = \left(\sum_{i=1}^{20} R_{x,i} / m_x \right) / d_2 = (18.608 / 20) / 2.326 = 0.400$$

$$\hat{\sigma}_y = \bar{R}_y / d_2 = \left(\sum_{i=1}^{10} R_{y,i} / m_y \right) / d_2 = (6.978 / 10) / 2.326 = 0.300$$

(b)

Want $\Pr\{(x - y) < 0.09\} = 0.006$. Let $z = x - y$. Then

$$\hat{\sigma}_z = \sqrt{\hat{\sigma}_x^2 + \hat{\sigma}_y^2} = \sqrt{0.4^2 + 0.3^2} = 0.500$$

$$\Phi\left(\frac{0.09 - z}{\hat{\sigma}_z}\right) = 0.006$$

$$\Phi^{-1}\left(\frac{0.09 - z}{0.500}\right) = \Phi(0.006)$$

$$\left(\frac{0.09 - z}{0.500}\right) = -2.5121$$

$$z = +2.5121(0.500) + 0.09 = 1.346$$

5-37 (5-30).

$$n = 6; \quad \sum_{i=1}^{30} \bar{x}_i = 12,870; \quad \sum_{i=1}^{30} R_i = 1350; \quad m = 30$$

(a)

$$\bar{R} = \frac{\sum_{i=1}^m R_i}{m} = \frac{1350}{30} = 45.0$$

$$UCL_R = D_4 \bar{R} = 2.004(45.0) = 90.18$$

$$LCL_R = D_3 \bar{R} = 0(45.0) = 0$$

(b)

$$\hat{\mu} = \bar{\bar{x}} = \frac{\sum_{i=1}^m \bar{x}_i}{m} = \frac{12,870}{30} = 429.0$$

$$\hat{\sigma}_x = \bar{R} / d_2 = 45.0 / 2.534 = 17.758$$

Chapter 5 Exercise Solutions

5-37 continued

(c)

$$USL = 440 + 40 = 480; \quad LSL = 440 - 40 = 400$$

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{480 - 400}{6(17.758)} = 0.751$$

$$\hat{p} = 1 - \Phi\left(\frac{480 - 429}{17.758}\right) + \Phi\left(\frac{400 - 429}{17.758}\right) = 1 - \Phi(2.87) + \Phi(-1.63) = 1 - 0.9979 + 0.0516 = 0.0537$$

(d)

To minimize fraction nonconforming the mean should be located at the nominal dimension (440) for a constant variance.

5-38 (5-31).

$$n = 4; \quad \sum_{i=1}^{30} \bar{x}_i = 12,870; \quad \sum_{i=1}^{30} S_i = 410; \quad m = 30$$

(a)

$$\bar{S} = \frac{\sum_{i=1}^m S_i}{m} = \frac{410}{30} = 13.667$$

$$UCL_S = B_4 \bar{S} = 2.266(13.667) = 30.969$$

$$LCL_S = B_3 \bar{S} = 0(13.667) = 0$$

(b)

$$\hat{\mu} = \bar{\bar{x}} = \frac{\sum_{i=1}^m \bar{x}_i}{m} = \frac{12,870}{30} = 429.0$$

$$\hat{\sigma}_x = \bar{S} / c_4 = 13.667 / 0.9213 = 14.834$$

Chapter 5 Exercise Solutions

5-39 (5-32).

(a)

$$n = 4; \mu = 100; \sigma_x = 8$$

$$UCL_{\bar{x}} = \mu + 2\sigma_{\bar{x}} = \mu + 2\left(\sigma_x/\sqrt{n}\right) = 100 + 2\left(8/\sqrt{4}\right) = 108$$

$$LCL_{\bar{x}} = \mu - 2\sigma_{\bar{x}} = \mu - 2\left(\sigma_x/\sqrt{n}\right) = 100 - 2\left(8/\sqrt{4}\right) = 92$$

(b)

$$k = Z_{\alpha/2} = Z_{0.005/2} = Z_{0.0025} = 2.807$$

$$UCL_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\sigma_x/\sqrt{n}\right) = 100 + 2.807\left(8/\sqrt{4}\right) = 111.228$$

$$LCL_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\sigma_x/\sqrt{n}\right) = 100 - 2.807\left(8/\sqrt{4}\right) = 88.772$$

5-40 (5-33).

$$n = 5; UCL_{\bar{x}} = 104; \text{centerline}_{\bar{x}} = 100; LCL_{\bar{x}} = 96; k = 3; \mu = 98; \sigma_x = 8$$

Pr{out-of-control signal by at least 3rd plot point}

$$= 1 - \text{Pr}\{\text{not detected by 3rd sample}\} = 1 - [\text{Pr}\{\text{not detected}\}]^3$$

$$\text{Pr}\{\text{not detected}\} = \text{Pr}\{LCL_{\bar{x}} \leq \bar{x} \leq UCL_{\bar{x}}\} = \text{Pr}\{\bar{x} \leq UCL_{\bar{x}}\} - \text{Pr}\{\bar{x} \leq LCL_{\bar{x}}\}$$

$$= \Phi\left(\frac{UCL_{\bar{x}} - \mu}{\sigma_{\bar{x}}}\right) - \Phi\left(\frac{LCL_{\bar{x}} - \mu}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{104 - 98}{8/\sqrt{5}}\right) - \Phi\left(\frac{96 - 98}{8/\sqrt{5}}\right) = \Phi(1.68) - \Phi(-0.56)$$

$$= 0.9535 - 0.2877 = 0.6658$$

$$1 - [\text{Pr}\{\text{not detected}\}]^3 = 1 - (0.6658)^3 = 0.7049$$

5-41 (5-34).

$$ARL_1 = \frac{1}{1 - \beta} = \frac{1}{1 - \text{Pr}\{\text{not detect}\}} = \frac{1}{1 - 0.6658} = 2.992$$

5-42 (5-35).

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x} = \frac{USL - LSL}{6(\bar{S}/c_4)} = \frac{202.50 - 197.50}{6(1.000/0.9213)} = 0.7678$$

The process is not capable of meeting specifications.

Chapter 5 Exercise Solutions

5-43 (5-36).

$$n = 4; \mu = 200; \sigma_x = 10$$

(a)

$$\text{centerline}_S = c_4\sigma = 0.9213(10) = 9.213$$

$$\text{UCL}_S = B_6\sigma_x = 2.088(10) = 20.88$$

$$\text{LCL}_S = B_5\sigma_x = 0(10) = 0$$

(b)

$$k = Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$$

$$\text{UCL}_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\sigma_x/\sqrt{n}\right) = 200 + 1.96\left(10/\sqrt{4}\right) = 209.8$$

$$\text{LCL}_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\sigma_x/\sqrt{n}\right) = 200 - 1.96\left(10/\sqrt{4}\right) = 190.2$$

5-44 (5-37).

$$n = 9; \text{USL} = 600 + 20 = 620; \text{LSL} = 600 - 20 = 580$$

(a)

$$\hat{C}_P = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{\text{USL} - \text{LSL}}{6(\bar{R}/d_2)} = \frac{620 - 580}{6(17.82/2.970)} = 1.111$$

Process is capable of meeting specifications.

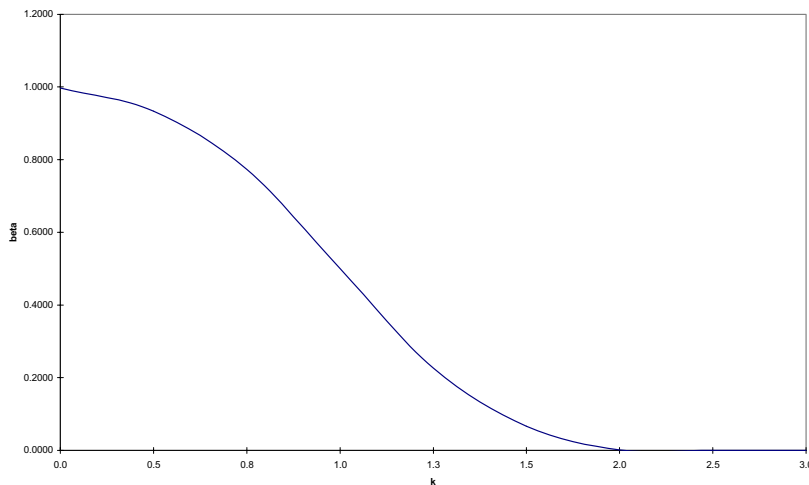
(b)

$$n = 9; L = 3; \beta = \Phi(L - k\sqrt{n}) - \Phi(-L - k\sqrt{n})$$

for $k = \{0, 0.5, 0.75, 1.0, 1.25, 1.5, 2.0, 2.5, 3.0\}$,

$$\beta = \{0.9974, 0.9332, 0.7734, 0.5, 0.2266, 0.0668, 0.0013, 0.0000, 0.0000\}$$

Operating Characteristic Curve
for $n = 9, L = 3$



Chapter 5 Exercise Solutions

5-45 (5-38).

$$n = 7; \sum_{i=1}^{30} \bar{x}_i = 2700; \sum_{i=1}^{30} R_i = 120; m = 30$$

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^m \bar{x}_i}{m} = \frac{2700}{30} = 90; \bar{R} = \frac{\sum_{i=1}^m R_i}{m} = \frac{120}{30} = 4$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R} = 90 + 0.419(4) = 91.676$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R} = 90 - 0.419(4) = 88.324$$

$$\text{UCL}_R = D_4 \bar{R} = 1.924(4) = 7.696$$

$$\text{LCL}_R = D_3 \bar{R} = 0.076(4) = 0.304$$

(b)

$$\hat{\sigma}_x = \bar{R} / d_2 = 4 / 2.704 = 1.479$$

(c)

$$\bar{S} = c_4 \hat{\sigma}_x = 0.9594(1.479) = 1.419$$

$$\text{UCL}_S = 1.882(1.419) = 2.671$$

$$\text{LCL}_S = 0.118(1.419) = 0.167$$

5-46 (5-39).

$$n = 9; \mu = 600; \sigma_x = 12; \alpha = 0.01$$

$$k = Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005} = 2.576$$

$$\text{UCL}_{\bar{x}} = \mu + k\sigma_{\bar{x}} = \mu + k\left(\sigma_x / \sqrt{n}\right) = 600 + 2.576(12/\sqrt{9}) = 610.3$$

$$\text{LCL}_{\bar{x}} = \mu - k\sigma_{\bar{x}} = \mu - k\left(\sigma_x / \sqrt{n}\right) = 600 - 2.576(12/\sqrt{9}) = 589.7$$

5-47 (5-40).

$$\hat{\sigma}_x = \bar{R} / d_2 = 20.59 / 2.059 = 10$$

$$\Pr\{\text{detect shift on 1st sample}\} = \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Pr\{\bar{x} < \text{LCL}\} + 1 - \Pr\{\bar{x} \leq \text{UCL}\}$$

$$= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) = \Phi\left(\frac{785 - 790}{10/\sqrt{4}}\right) + 1 - \Phi\left(\frac{815 - 790}{10/\sqrt{4}}\right)$$

$$= \Phi(-1) + 1 - \Phi(5) = 0.1587 + 1 - 1.0000 = 0.1587$$

Chapter 5 Exercise Solutions

5-48 (5-41).

$$ARL_1 = \frac{1}{1 - \beta} = \frac{1}{1 - \Pr\{\text{not detect}\}} = \frac{1}{\Pr\{\text{detect}\}} = \frac{1}{0.1587} = 6.30$$

5-49 (5-42).

(a)

$$\hat{\sigma}_x = \bar{R} / d_2 = 8.91 / 2.970 = 3.000$$

$$\begin{aligned} \alpha &= \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} = \Phi\left(\frac{\text{LCL} - \bar{x}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \bar{x}}{\sigma_{\bar{x}}}\right) \\ &= \Phi\left(\frac{357 - 360}{3/\sqrt{9}}\right) + 1 - \Phi\left(\frac{363 - 360}{3/\sqrt{9}}\right) = \Phi(-3) + 1 - \Phi(3) = 0.0013 + 1 - 0.9987 = 0.0026 \end{aligned}$$

(b)

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}_x} = \frac{+6 - (-6)}{6(3)} = 0.667$$

The process is not capable of producing all items within specification.

(c)

$$\mu_{\text{new}} = 357$$

$$\begin{aligned} \Pr\{\text{not detect on 1st sample}\} &= \Pr\{\text{LCL} \leq \bar{x} \leq \text{UCL}\} = \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\hat{\sigma}_x/\sqrt{n}}\right) - \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\hat{\sigma}_x/\sqrt{n}}\right) \\ &= \Phi\left(\frac{363 - 357}{3/\sqrt{9}}\right) - \Phi\left(\frac{357 - 357}{3/\sqrt{9}}\right) = \Phi(6) - \Phi(0) = 1.0000 - 0.5000 = 0.5000 \end{aligned}$$

(d)

$$\alpha = 0.01; \quad k = Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005} = 2.576$$

$$\text{UCL}_{\bar{x}} = \bar{x} + k\sigma_{\bar{x}} = \bar{x} + k\left(\hat{\sigma}_x/\sqrt{n}\right) = 360 + 2.576\left(3/\sqrt{9}\right) = 362.576$$

$$\text{LCL}_{\bar{x}} = 360 - 2.576\left(3/\sqrt{9}\right) = 357.424$$

Chapter 5 Exercise Solutions

5-50 (5-43).

(a)

$$\hat{\sigma}_x = \bar{R} / d_2 = 8.236 / 2.059 = 4.000$$

(b)

$$\bar{S} = c_4 \hat{\sigma}_x = 0.9213(4) = 3.865$$

$$UCL_S = B_4 \bar{S} = 2.266(3.865) = 8.351$$

$$LCL_S = B_3 \bar{S} = 0(3.865) = 0$$

(c)

$$\begin{aligned}\hat{p} &= \Pr\{x < LSL\} + \Pr\{x > USL\} = \Phi\left(\frac{LSL - \bar{x}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{USL - \bar{x}}{\hat{\sigma}_x}\right) \\ &= \Phi\left(\frac{595 - 620}{4}\right) + 1 - \Phi\left(\frac{625 - 620}{4}\right) \\ &= \Phi(-6.25) + 1 - \Phi(1.25) = 0.0000 + 1 - 0.8944 = 0.1056\end{aligned}$$

(d)

To reduce the fraction nonconforming, try moving the center of the process from its current mean of 620 closer to the nominal dimension of 610. Also consider reducing the process variability.

(e)

$$\begin{aligned}\Pr\{\text{detect on 1st sample}\} &= \Pr\{\bar{x} < LCL\} + \Pr\{\bar{x} > UCL\} \\ &= \Phi\left(\frac{LCL - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{UCL - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) \\ &= \Phi\left(\frac{614 - 610}{4/\sqrt{4}}\right) + 1 - \Phi\left(\frac{626 - 610}{4/\sqrt{4}}\right) \\ &= \Phi(2) + 1 - \Phi(8) = 0.9772 + 1 - 1.0000 = 0.9772\end{aligned}$$

(f)

$$\begin{aligned}\Pr\{\text{detect by 3rd sample}\} &= 1 - \Pr\{\text{not detect by 3rd sample}\} \\ &= 1 - (\Pr\{\text{not detect}\})^3 = 1 - (1 - 0.9772)^3 = 1.0000\end{aligned}$$

Chapter 5 Exercise Solutions

5-51 (5-44).

(a)

$$\hat{\mu} = \bar{\bar{x}} = 706.00; \quad \hat{\sigma}_x = \bar{S} / c_4 = 1.738 / 0.9515 = 1.827$$

(b)

$$\text{UNTL} = \bar{\bar{x}} + 3\hat{\sigma}_x = 706 + 3(1.827) = 711.48$$

$$\text{LNTL} = 706 - 3(1.827) = 700.52$$

(c)

$$\begin{aligned} \hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\ &= \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) \\ &= \Phi\left(\frac{703 - 706}{1.827}\right) + 1 - \Phi\left(\frac{709 - 706}{1.827}\right) \\ &= \Phi(-1.642) + 1 - \Phi(1.642) = 0.0503 + 1 - 0.9497 = 0.1006 \end{aligned}$$

(d)

$$\begin{aligned} \Pr\{\text{detect on 1st sample}\} &= \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} \\ &= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x}}}\right) \\ &= \Phi\left(\frac{703.8 - 702}{1.827/\sqrt{6}}\right) + 1 - \Phi\left(\frac{708.2 - 702}{1.827/\sqrt{6}}\right) \\ &= \Phi(2.41) + 1 - \Phi(8.31) = 0.9920 + 1 - 1.0000 = 0.9920 \end{aligned}$$

(e)

$$\begin{aligned} \Pr\{\text{detect by 3rd sample}\} &= 1 - \Pr\{\text{not detect by 3rd sample}\} \\ &= 1 - (\Pr\{\text{not detect}\})^3 = 1 - (1 - 0.9920)^3 = 1.0000 \end{aligned}$$

Chapter 5 Exercise Solutions

5-52 (5-45).

(a)

$$\hat{\mu} = \bar{\bar{x}} = 700; \quad \hat{\sigma}_x = \bar{S} / c_4 = 7.979 / 0.9213 = 8.661$$

(b)

$$\begin{aligned}\hat{p} &= \Pr\{x < \text{LSL}\} + \Pr\{x > \text{USL}\} \\ &= \Phi\left(\frac{\text{LSL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) + 1 - \Phi\left(\frac{\text{USL} - \bar{\bar{x}}}{\hat{\sigma}_x}\right) \\ &= \Phi\left(\frac{690 - 700}{8.661}\right) + 1 - \Phi\left(\frac{720 - 700}{8.661}\right) \\ &= \Phi(-1.15) + 1 - \Phi(2.31) = 0.1251 + 1 - 0.9896 = 0.1355\end{aligned}$$

(c)

$$\begin{aligned}\alpha &= \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} \\ &= \Phi\left(\frac{\text{LCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \bar{\bar{x}}}{\sigma_{\bar{x}}}\right) \\ &= \Phi\left(\frac{690 - 700}{8.661/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 700}{8.661/\sqrt{4}}\right) \\ &= \Phi(-2.31) + 1 - \Phi(2.31) = 0.0104 + 1 - 0.9896 = 0.0208\end{aligned}$$

(d)

$$\begin{aligned}\Pr\{\text{detect on 1st sample}\} &= \Pr\{\bar{x} < \text{LCL}\} + \Pr\{\bar{x} > \text{UCL}\} \\ &= \Phi\left(\frac{\text{LCL} - \mu_{\text{new}}}{\sigma_{\bar{x}, \text{new}}}\right) + 1 - \Phi\left(\frac{\text{UCL} - \mu_{\text{new}}}{\sigma_{\bar{x}, \text{new}}}\right) \\ &= \Phi\left(\frac{690 - 693}{12/\sqrt{4}}\right) + 1 - \Phi\left(\frac{710 - 693}{12/\sqrt{4}}\right) \\ &= \Phi(-0.5) + 1 - \Phi(2.83) = 0.3085 + 1 - 0.9977 = 0.3108\end{aligned}$$

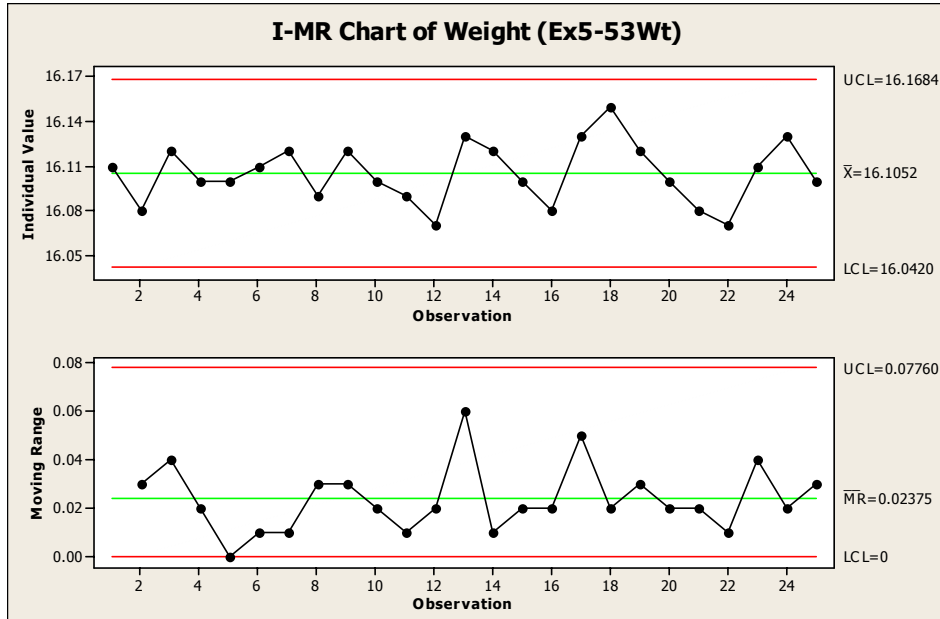
(e)

$$\text{ARL}_1 = \frac{1}{1 - \beta} = \frac{1}{1 - \Pr\{\text{not detect}\}} = \frac{1}{\Pr\{\text{detect}\}} = \frac{1}{0.3108} = 3.22$$

Chapter 5 Exercise Solutions

5-53 (5-46).

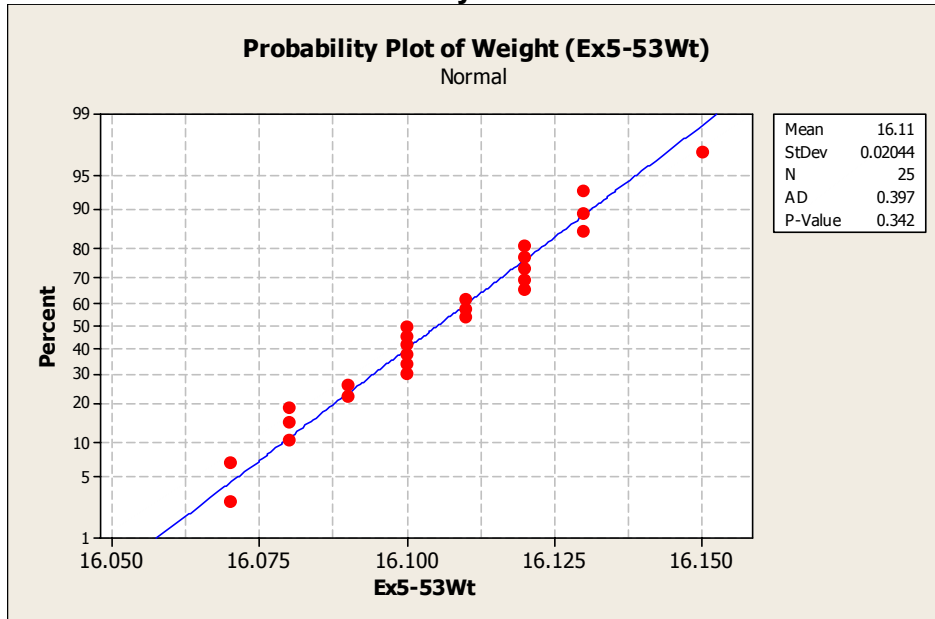
MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



There may be a “sawtooth” pattern developing on the Individuals chart.

$$\bar{\bar{x}} = 16.1052; \hat{\sigma}_x = 0.021055; \overline{MR2} = 0.02375$$

MTB > Stat > Basic Statistics > Normality Test



Visual examination of the normal probability indicates that the assumption of normally distributed coffee can weights is valid.

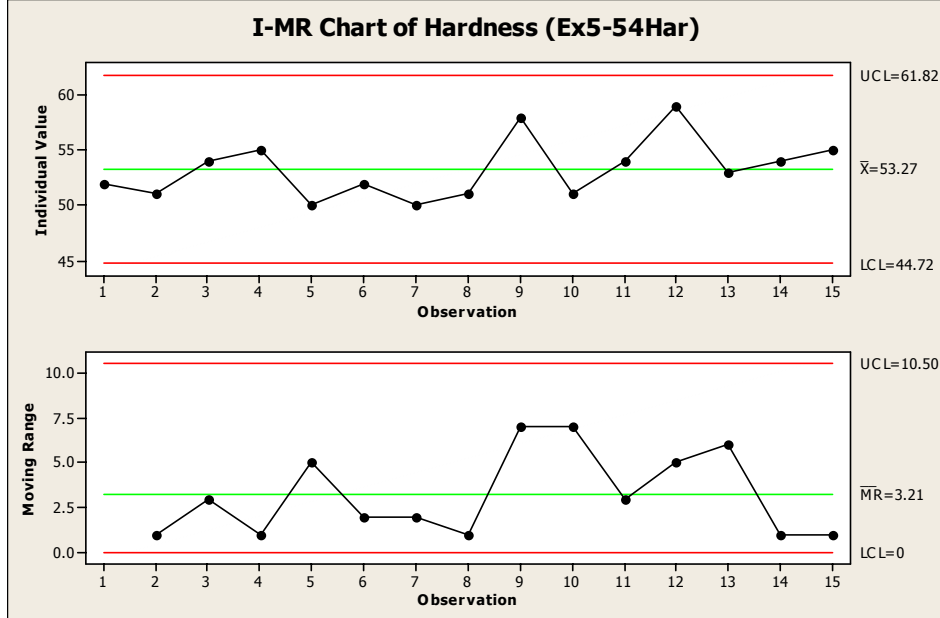
$$\% \text{underfilled} = 100\% \times \Pr\{x < 16\}$$

$$= 100\% \times \Phi\left(\frac{16 - 16.1052}{0.021055}\right) = 100\% \times \Phi(-4.9964) = 0.00003\%$$

Chapter 5 Exercise Solutions

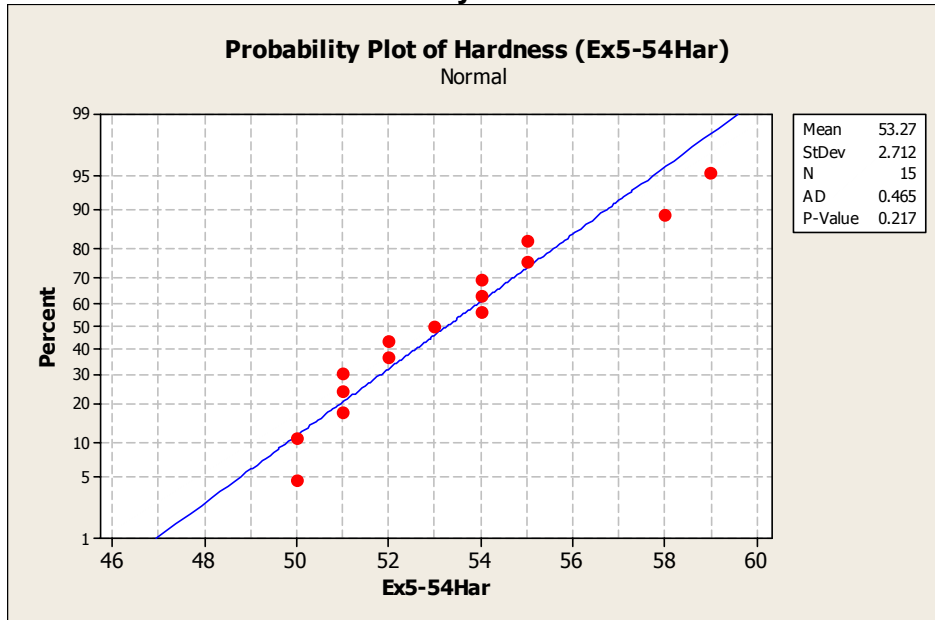
5-54(5-47).

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



$$\bar{\bar{x}} = 53.2667; \hat{\sigma}_x = 2.84954; \overline{MR2} = 3.21429$$

MTB > Stat > Basic Statistics > Normality Test



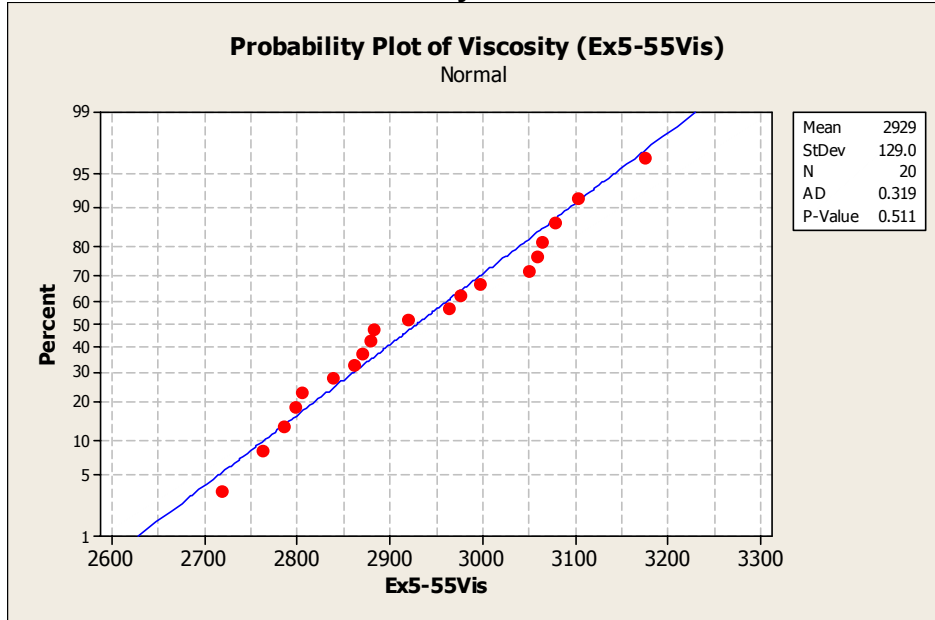
Although the observations at the tails are not very close to the straight line, the p -value is greater than 0.05, indicating that it may be reasonable to assume that hardness is normally distributed.

Chapter 5 Exercise Solutions

5-55 (5-48).

(a)

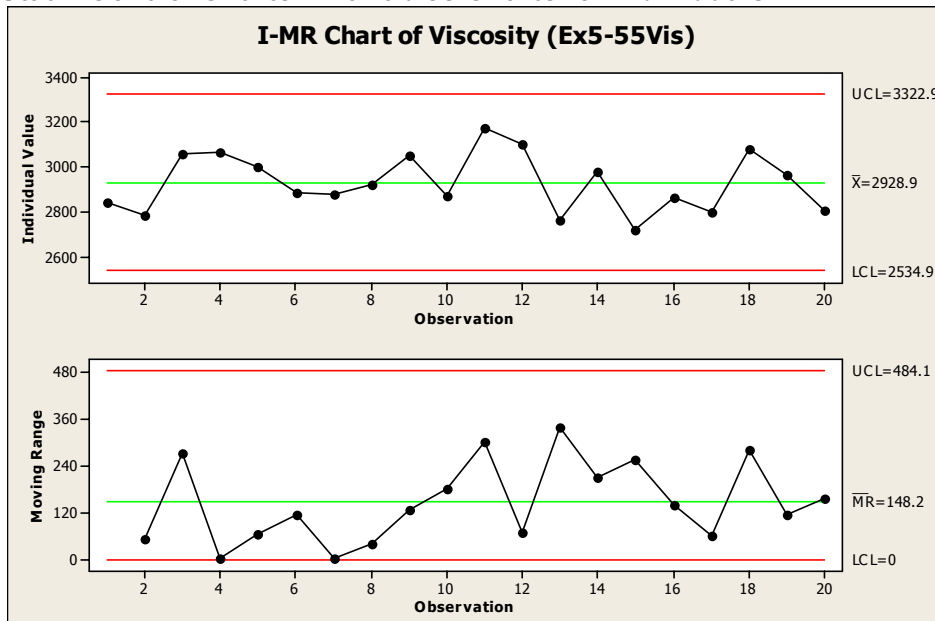
MTB > Stat > Basic Statistics > Normality Test



Viscosity measurements do appear to follow a normal distribution.

(b)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



The process appears to be in statistical control, with no out-of-control points, runs, trends, or other patterns.

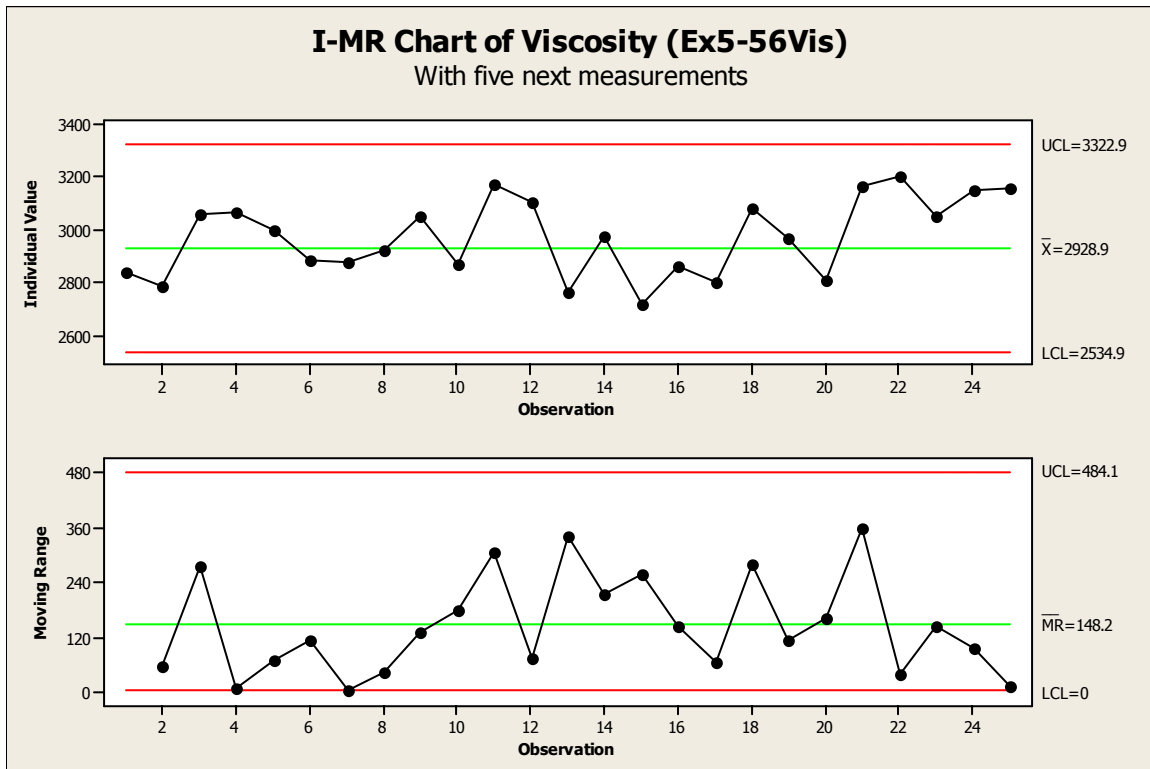
(c)

$$\hat{\mu} = \bar{\bar{x}} = 2928.9; \quad \hat{\sigma}_x = 131.346; \quad \overline{MR2} = 148.158$$

Chapter 5 Exercise Solutions

5-56 (5-49).

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



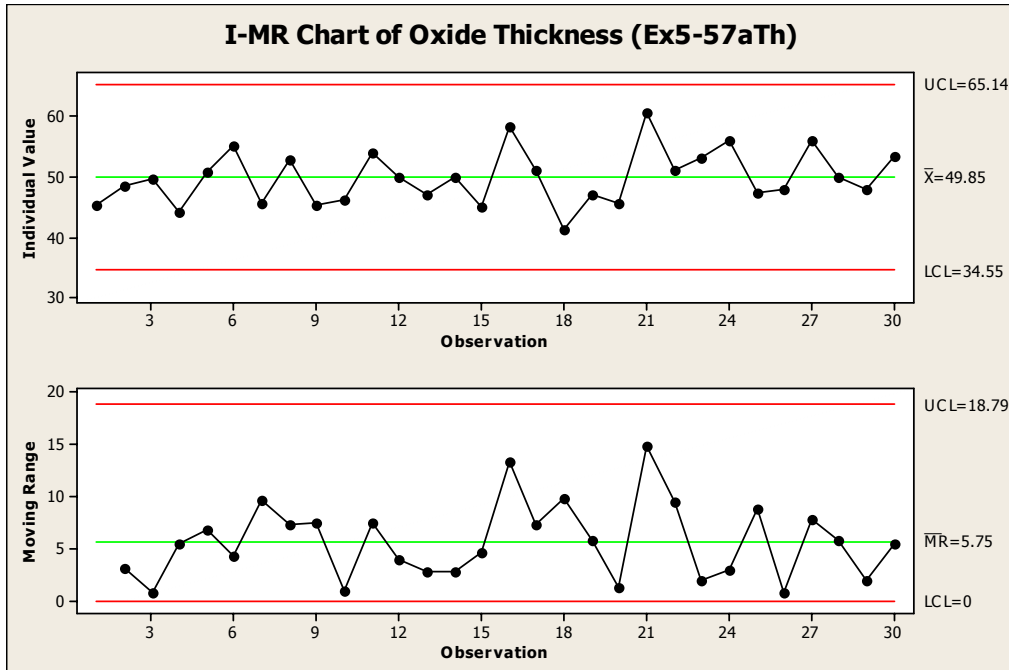
All points are inside the control limits. However all of the new points on the *I* chart are above the center line, indicating that a shift in the mean may have occurred.

Chapter 5 Exercise Solutions

5-57 (5-50).

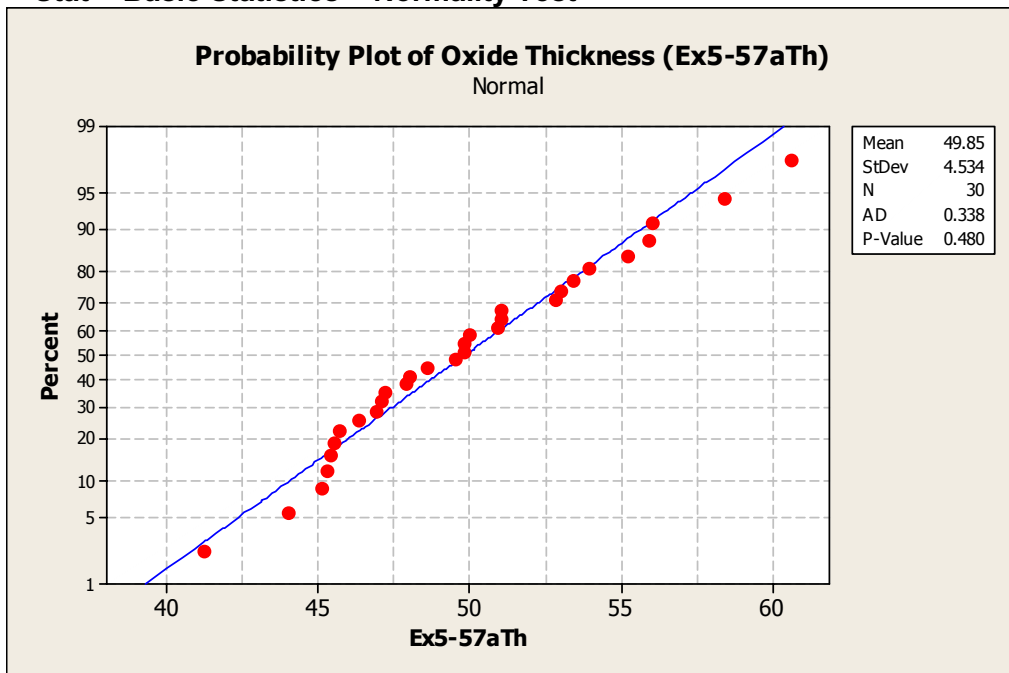
(a)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



The process is in statistical control.

MTB > Stat > Basic Statistics > Normality Test



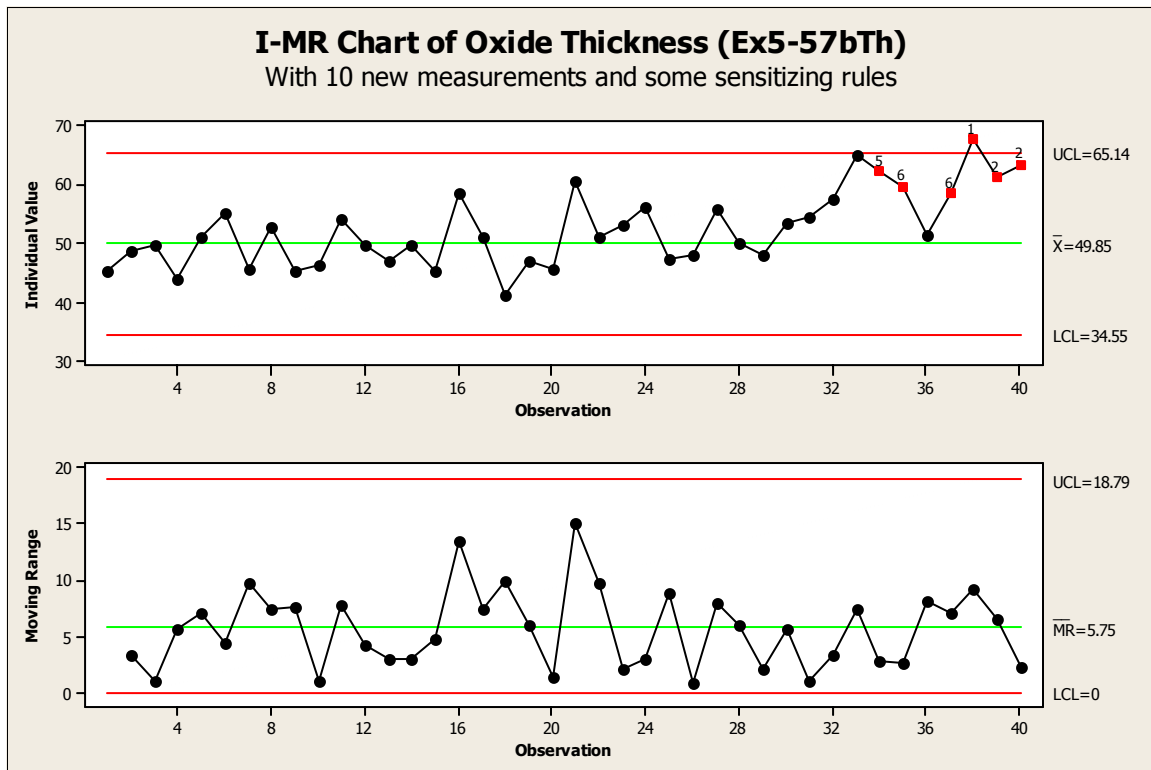
The normality assumption is reasonable.

Chapter 5 Exercise Solutions

5-57 continued

(b)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



Test Results for I Chart of Ex5-57bTh

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 38

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 38, 39, 40

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 34, 39, 40

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on one side of CL).

Test Failed at points: 35, 37, 38, 39, 40

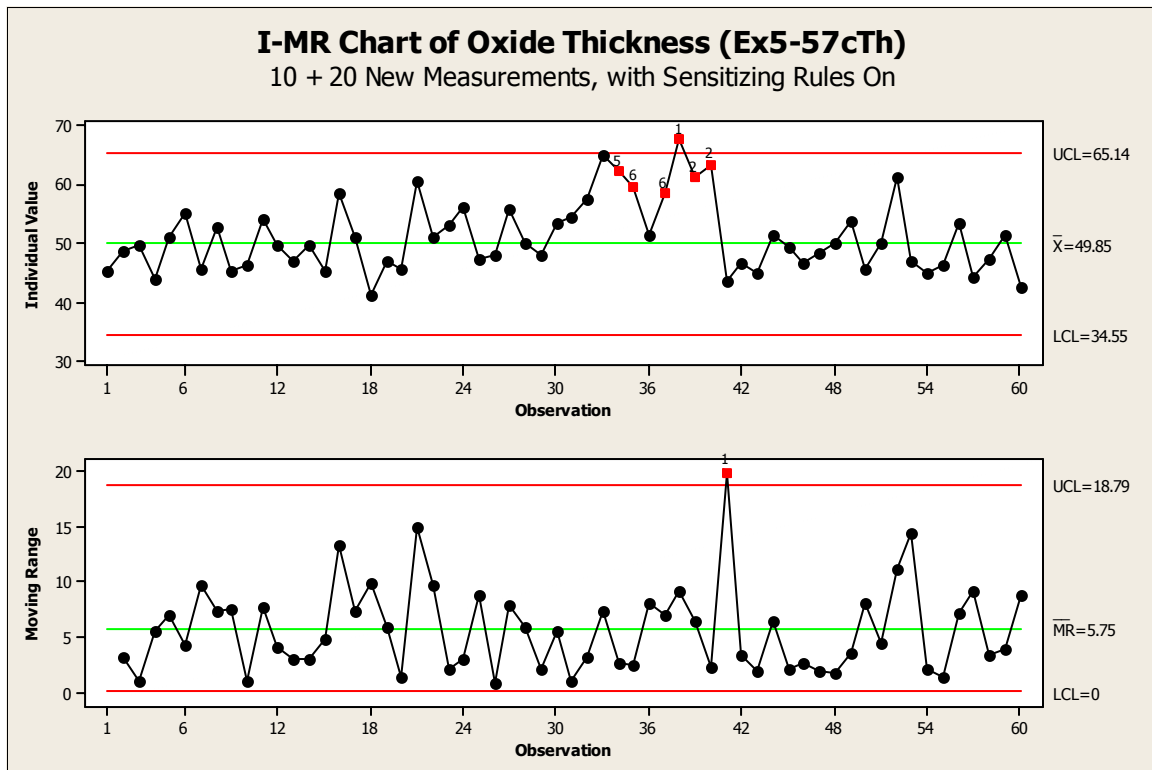
We have turned on some of the sensitizing rules in MINITAB to illustrate their use. There is a run above the centerline, several 4 of 5 beyond 1 sigma, and several 2 of 3 beyond 2 sigma on the x chart. However, even without use of the sensitizing rules, it is clear that the process is out of control during this period of operation.

Chapter 5 Exercise Solutions

5-57 continued

(c)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



The process has been returned to a state of statistical control.

Chapter 5 Exercise Solutions

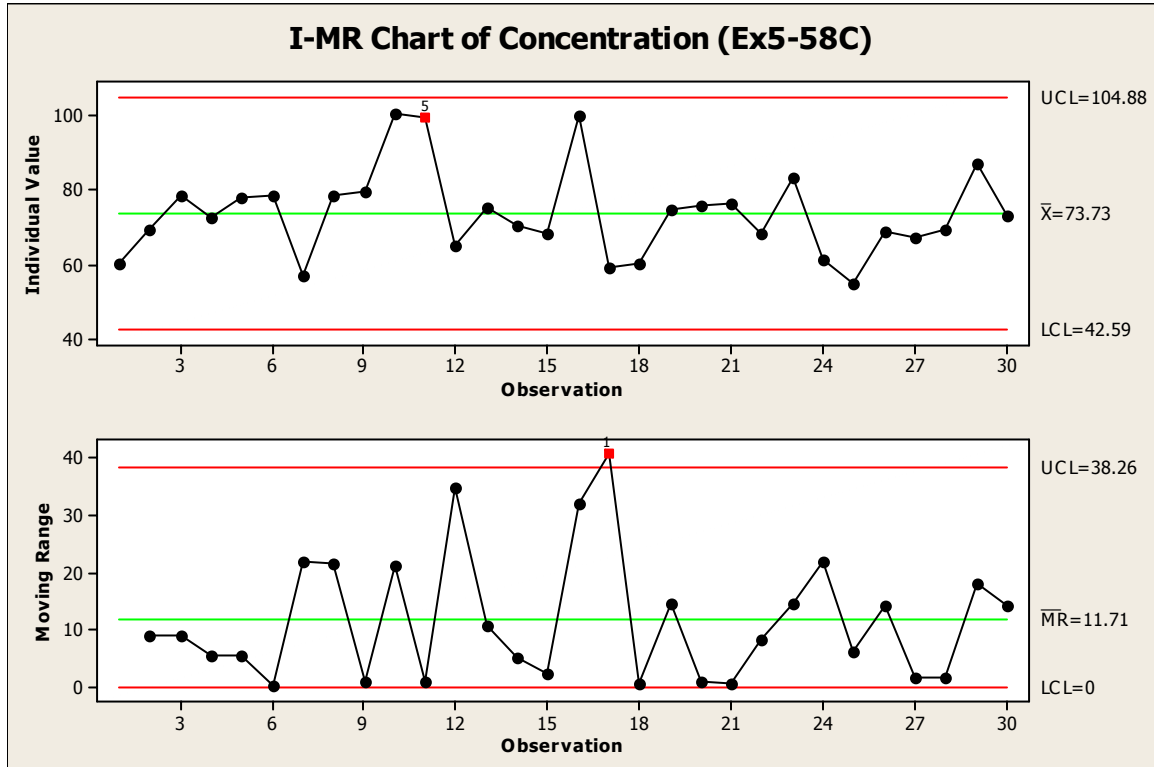
5-58 (5-51).

(a)

The normality assumption is a little bothersome for the concentration data, in particular due to the curve of the larger values and three distant values.

(b)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



Test Results for I Chart of Ex5-58C

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 11

Test Results for MR Chart of Ex5-58C

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 17

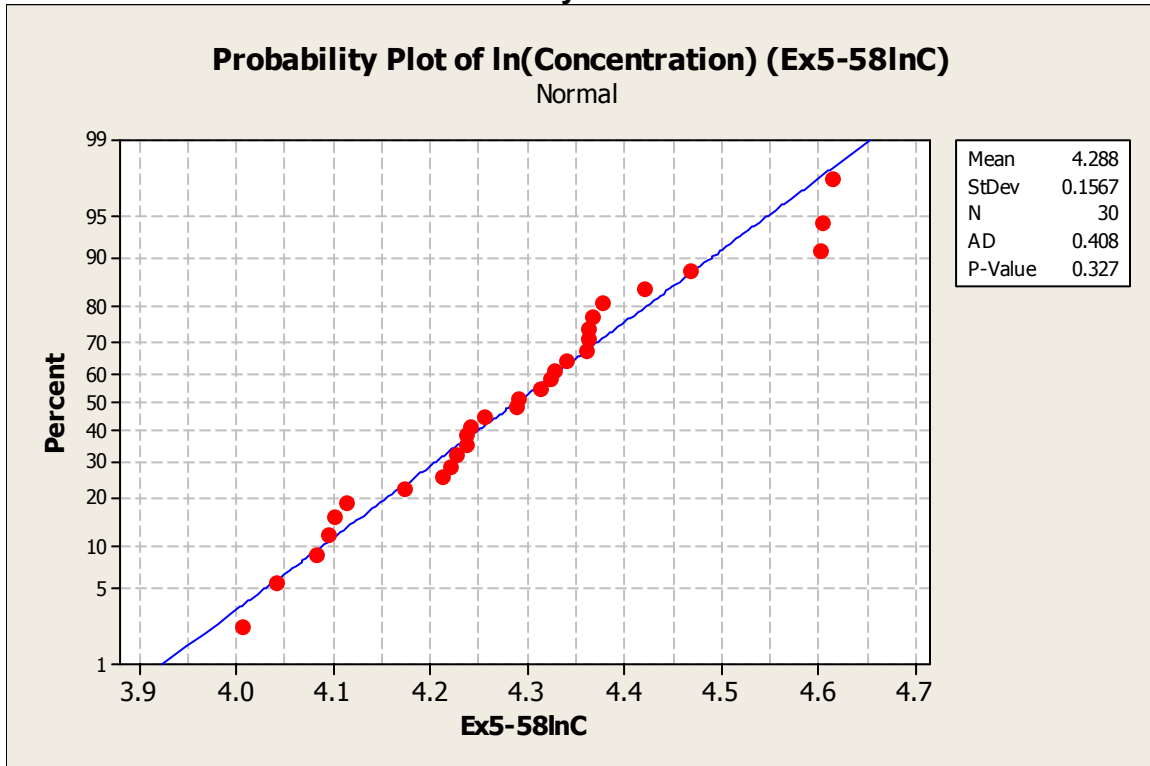
The process is not in control, with two Western Electric rule violations.

Chapter 5 Exercise Solutions

5-58 continued

(c)

MTB > Stat > Basic Statistics > Normality Test



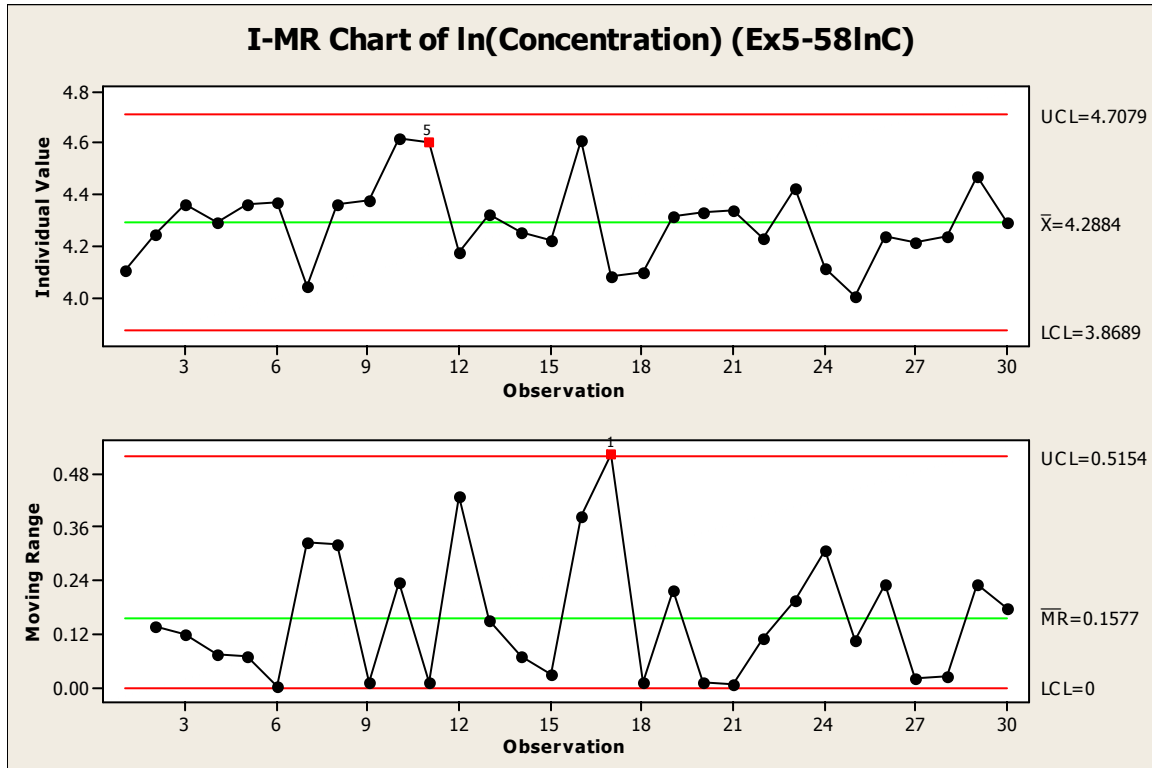
The normality assumption is still troubling for the natural log of concentration, again due to the curve of the larger values and three distant values.

Chapter 5 Exercise Solutions

5-58 continued

(d)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



Test Results for I Chart of Ex5-58lnC

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 11

Test Results for MR Chart of Ex5-58lnC

TEST 1. One point more than 3.00 standard deviations from center line.

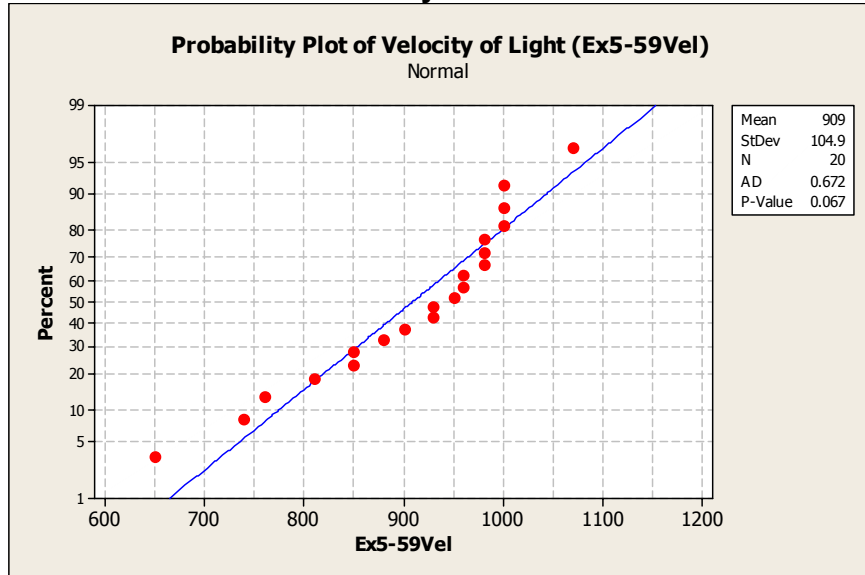
Test Failed at points: 17

The process is still not in control, with the same to Western Electric Rules violations. There does not appear to be much difference between the two control charts (actual and natural log).

Chapter 5 Exercise Solutions

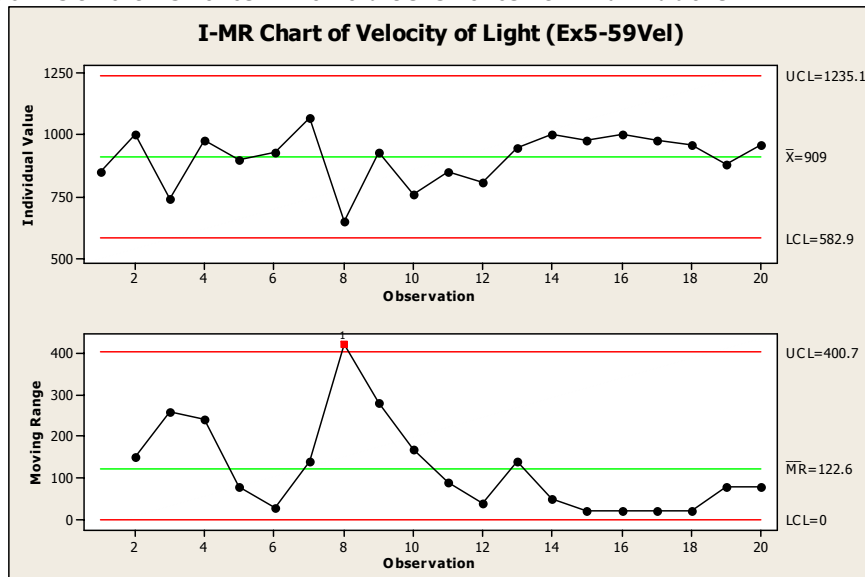
5-59 ☺.

MTB > Stat > Basic Statistics > Normality Test



Velocity of light measurements are approximately normally distributed.

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



I-MR Chart of Ex5-59Vel

Test Results for MR Chart of Ex5-59Vel

TEST 1. One point more than 3.00 standard deviations from center line.
 Test Failed at points: 8

The out-of-control signal on the moving range chart indicates a significantly large difference between successive measurements (7 and 8). Since neither of these measurements seems unusual, use all data for control limits calculations.

There may also be an early indication of less variability in the later measurements. For now, consider the process to be in a state of statistical process control.

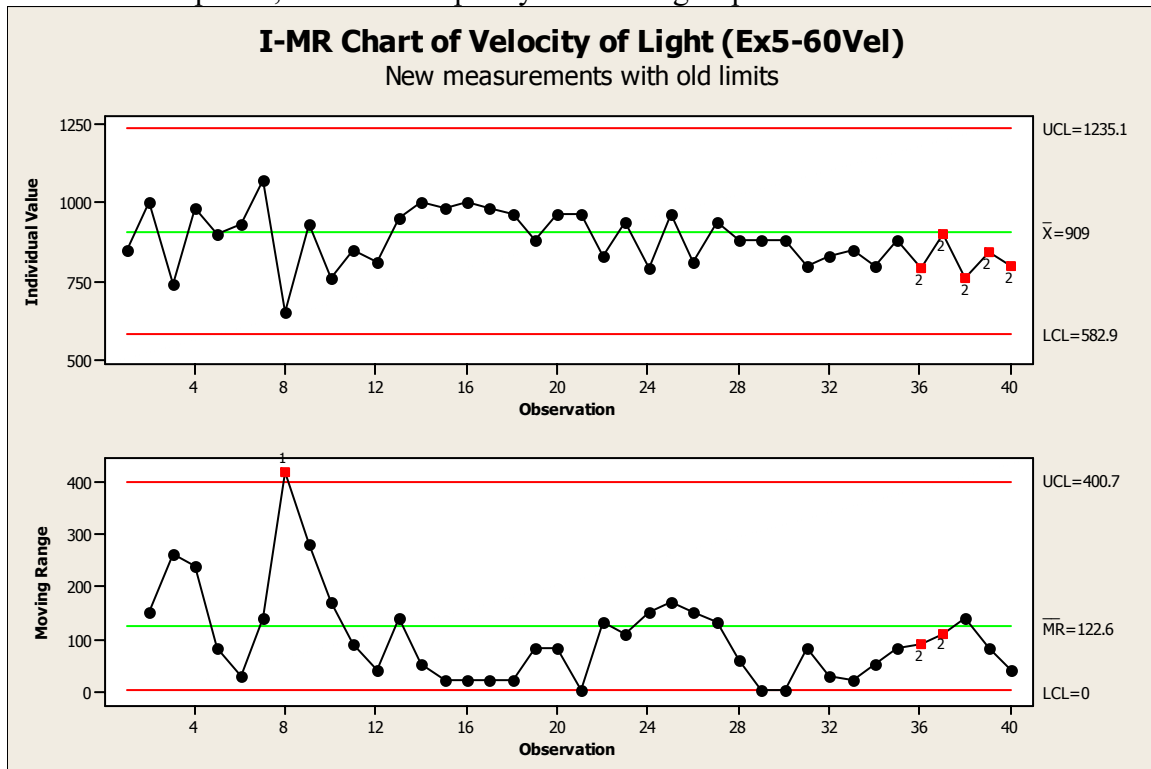
Chapter 5 Exercise Solutions

5-60☺.

(a)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR

Select I-MR Options, Estimate to specify which subgroups to use in calculations



I-MR Chart of Ex5-60Vel

Test Results for I Chart of Ex5-60Vel

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 36, 37, 38, 39, 40

Test Results for MR Chart of Ex5-60Vel

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 8

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 36, 37

The velocity of light in air is not changing, however the method of measuring is producing varying results—this is a chart of the measurement process. There is a distinct downward trend in measurements, meaning the method is producing gradually smaller measurements.

(b)

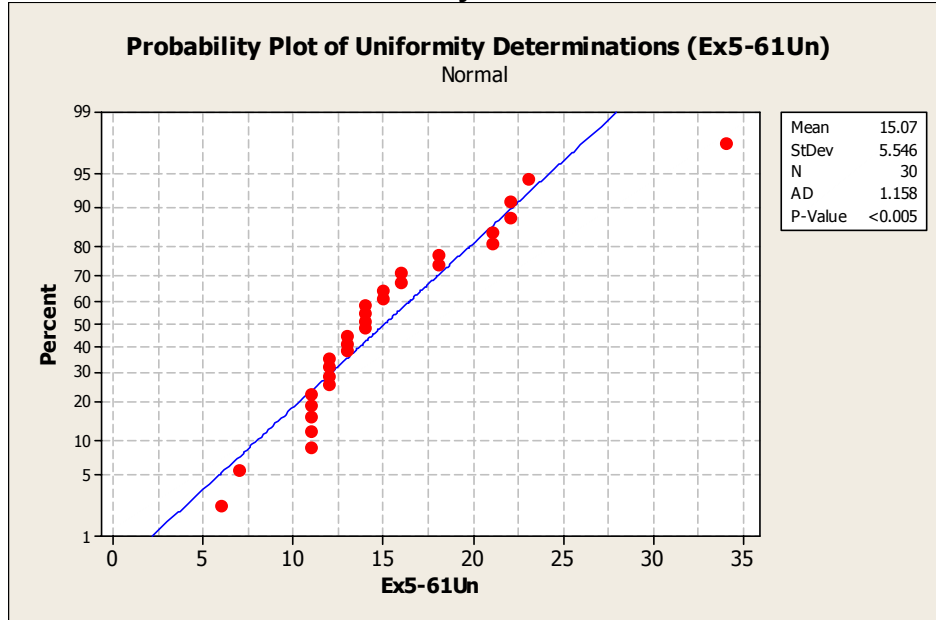
Early measurements exhibit more variability than the later measurements, which is reflected in the number of observations below the centerline of the moving range chart.

Chapter 5 Exercise Solutions

5-61 ☺.

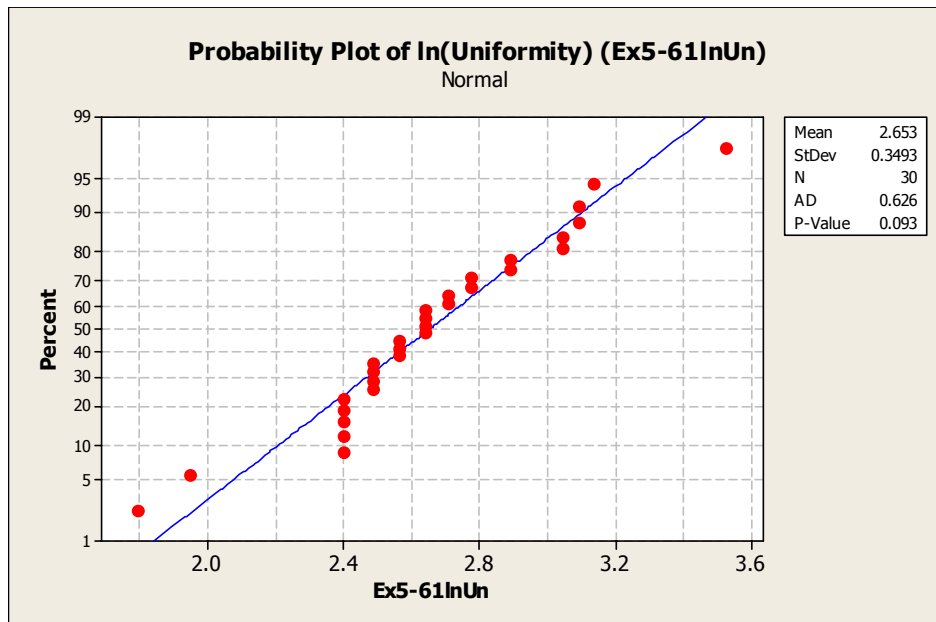
(a)

MTB > Stat > Basic Statistics > Normality Test



The data are not normally distributed, as evidenced by the “S”- shaped curve to the plot points on a normal probability plot, as well as the Anderson-Darling test p-value.

The data are skewed right, so a compressive transform such as natural log or square-root may be appropriate.



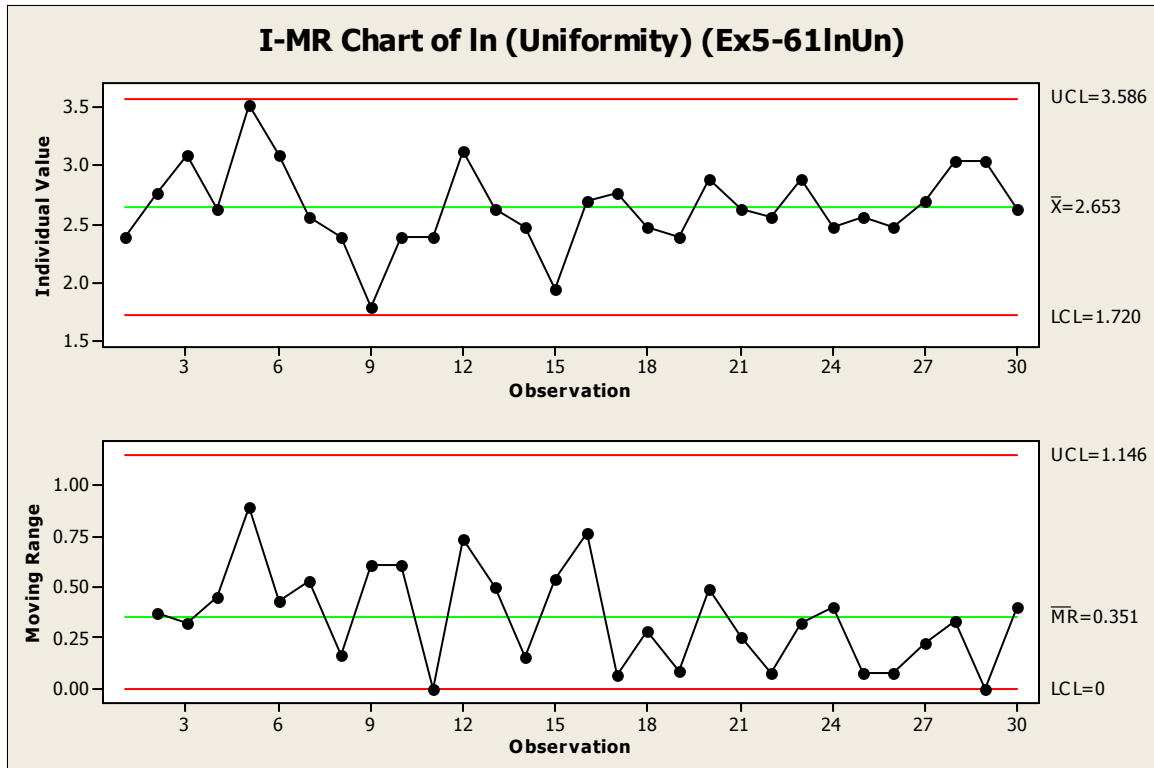
The distribution of the natural-log transformed uniformity measurements is approximately normally distributed.

Chapter 5 Exercise Solutions

5-61 continued

(b)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



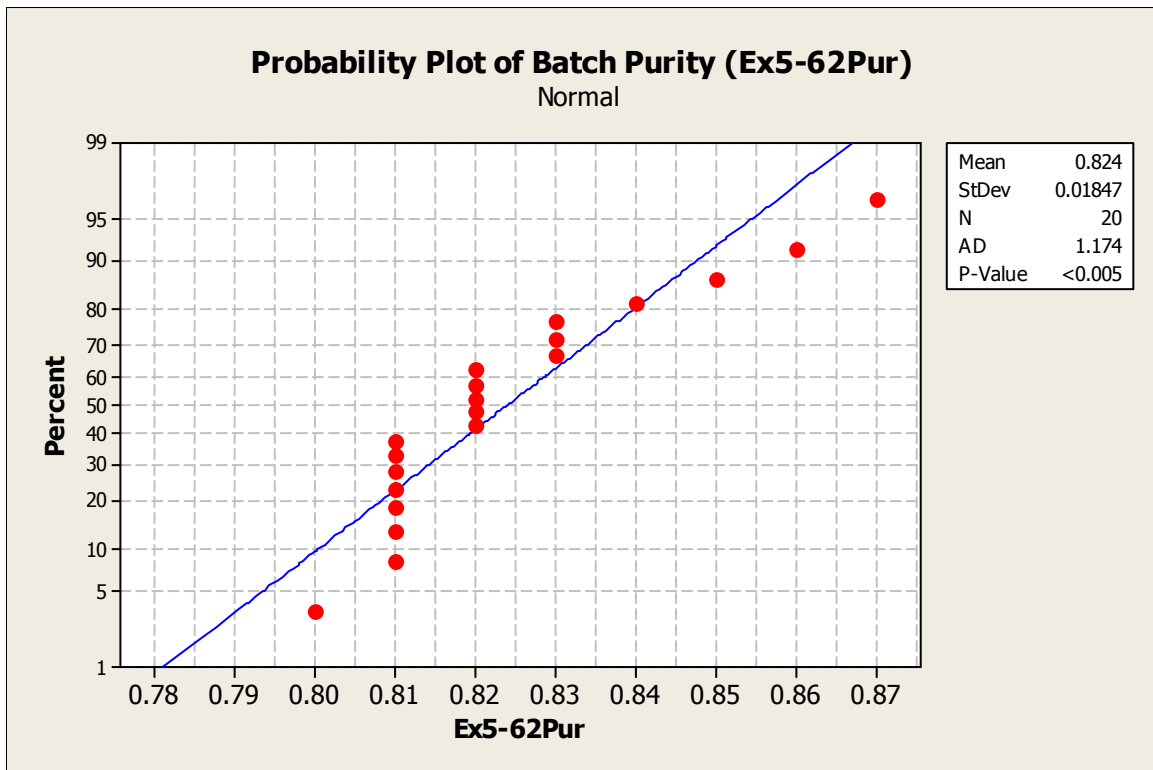
The etching process appears to be in statistical control.

Chapter 5 Exercise Solutions

5-62 (5-52).

(a)

MTB > Stat > Basic Statistics > Normality Test



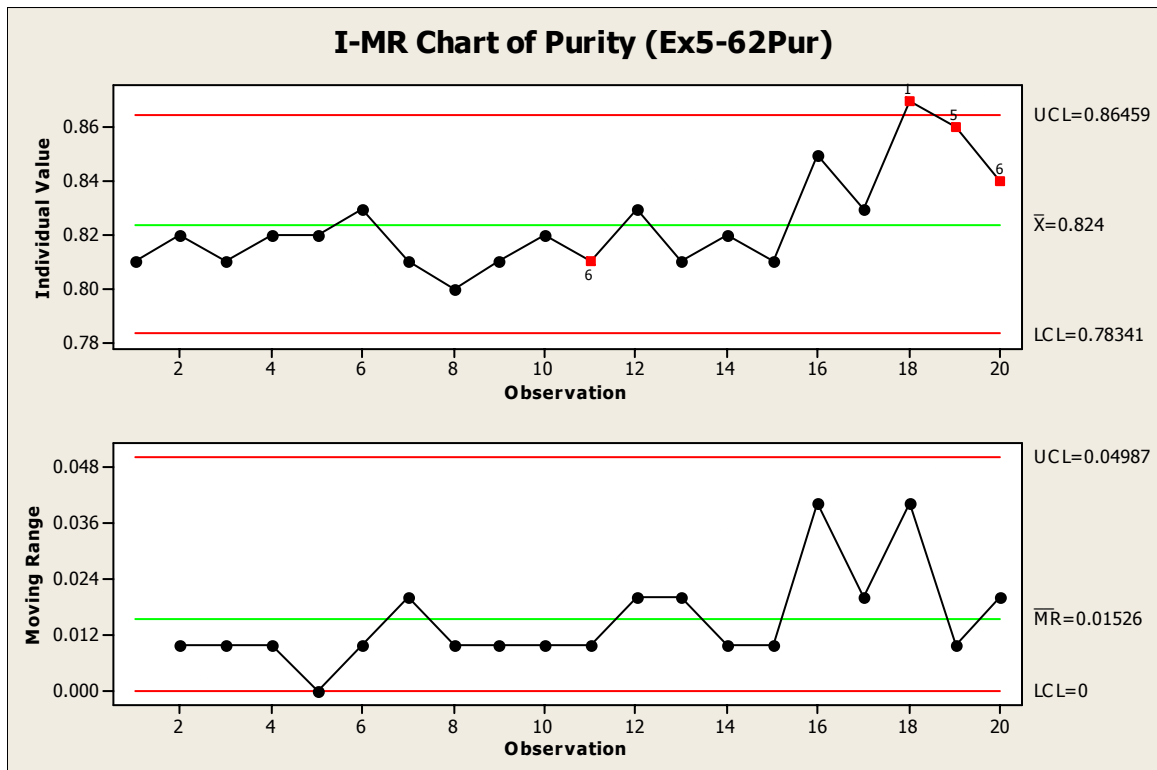
Purity is not normally distributed.

Chapter 5 Exercise Solutions

5-62 continued

(b)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



Test Results for I Chart of Ex5-62Pur

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 18

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 19

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on one side of CL).

Test Failed at points: 11, 20

The process is not in statistical control.

(c)

all data: $\hat{\mu} = 0.824$, $\hat{\sigma}_x = 0.0135$

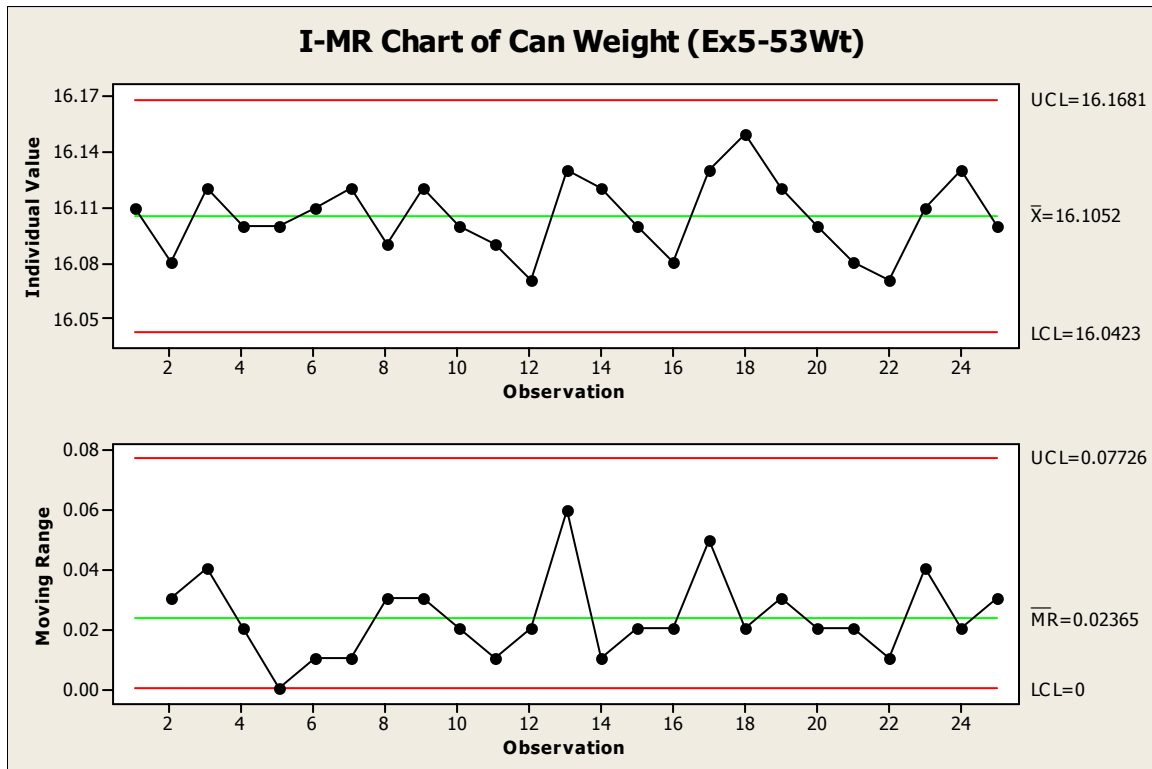
without sample 18: $\hat{\mu} = 0.8216$, $\hat{\sigma}_x = 0.0133$

Chapter 5 Exercise Solutions

5-63 (5-53).

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR

Select "Estimate" to change the method of estimating sigma



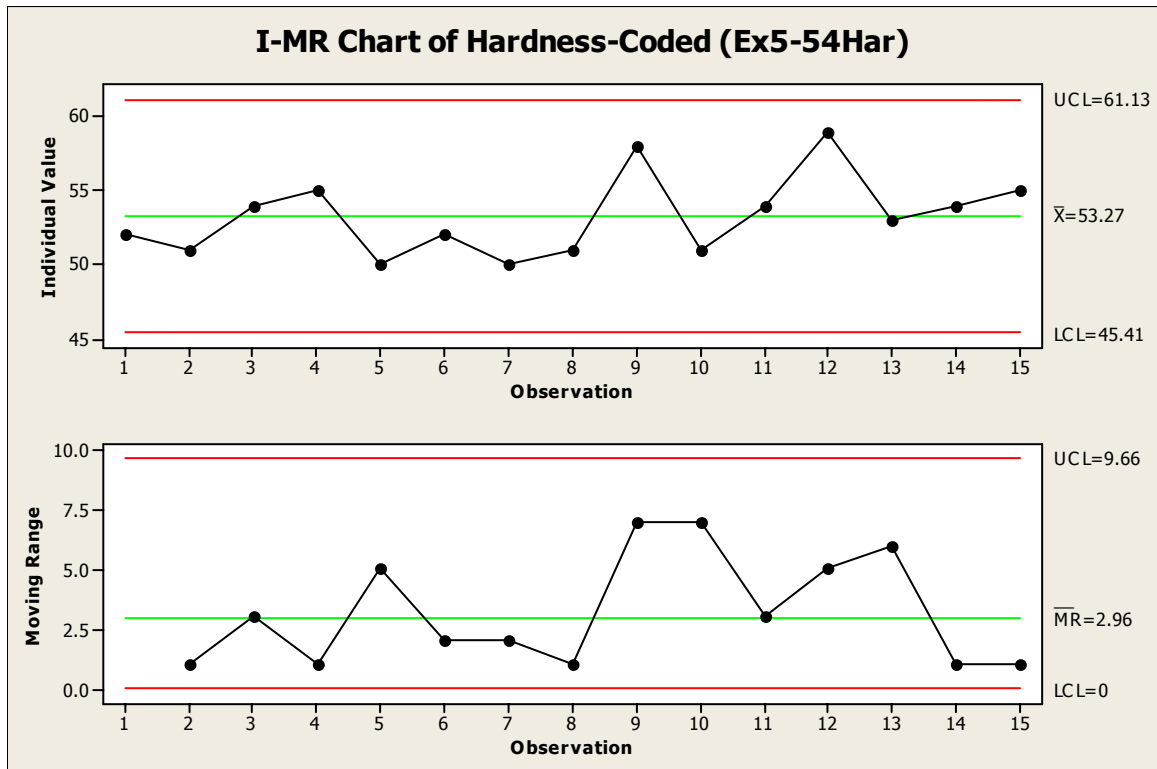
There is no difference between this chart and the one in Exercise 5-53; control limits for both are essentially the same.

Chapter 5 Exercise Solutions

5-64 (5-54).

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR

Select "Estimate" to change the method of estimating sigma



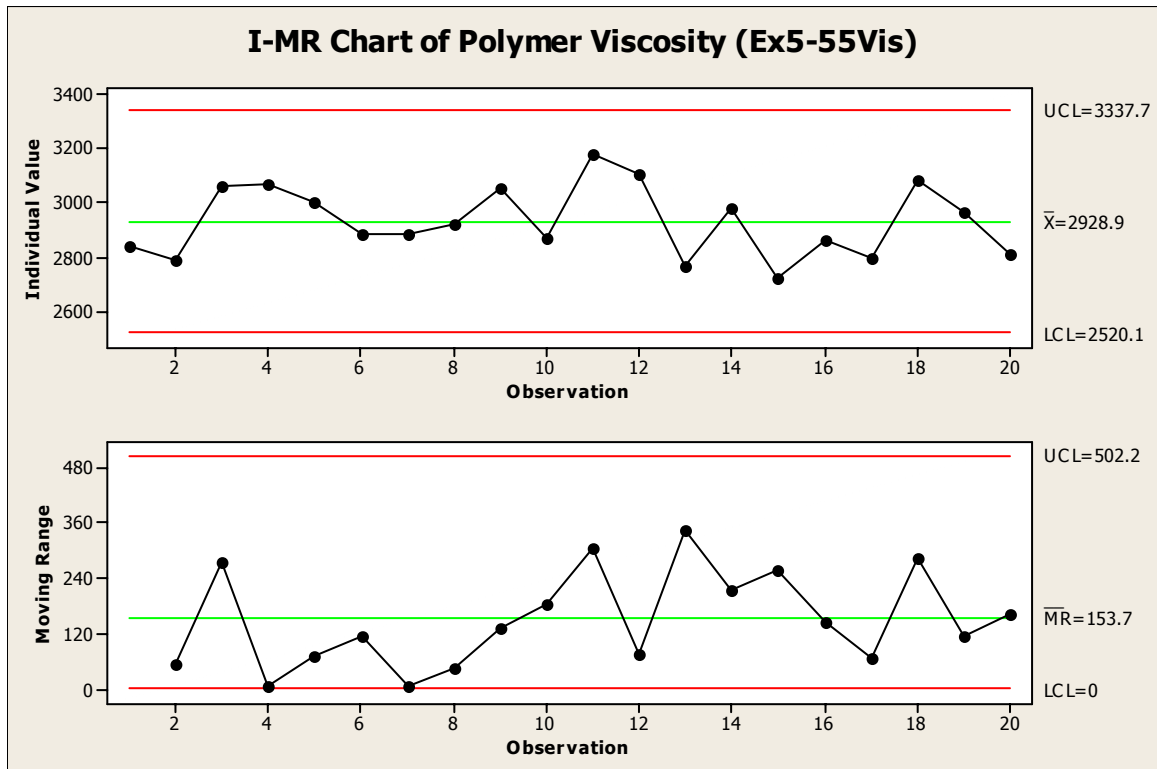
The median moving range method gives slightly tighter control limits for both the Individual and Moving Range charts, with no practical difference for this set of observations.

Chapter 5 Exercise Solutions

5-65 (5-55).

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR

Select "Estimate" to change the method of estimating sigma



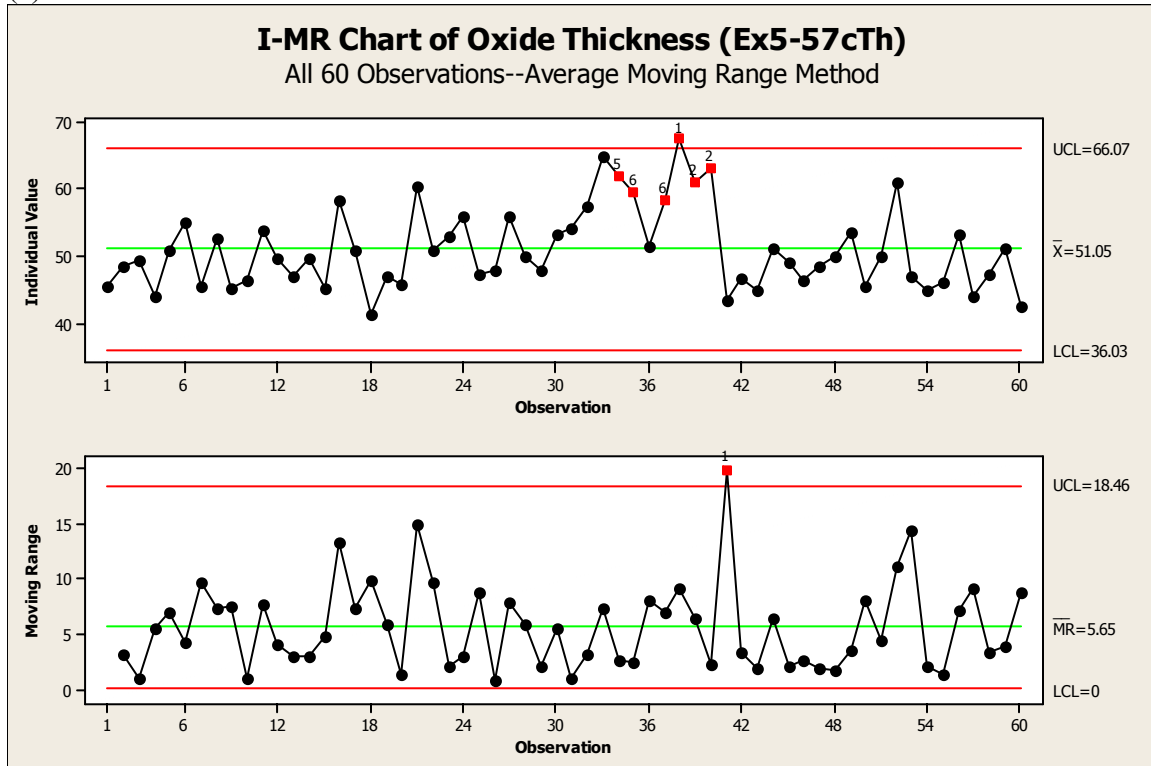
The median moving range method gives slightly wider control limits for both the Individual and Moving Range charts, with no practical meaning for this set of observations.

Chapter 5 Exercise Solutions

5-66 (5-56).

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR

(a)



Test Results for I Chart of Ex5-57cTh

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 38

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 38, 39, 40

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 34, 39, 40

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on one side of CL).

Test Failed at points: 35, 37, 38, 39, 40

Test Results for MR Chart of Ex5-57cTh

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 41

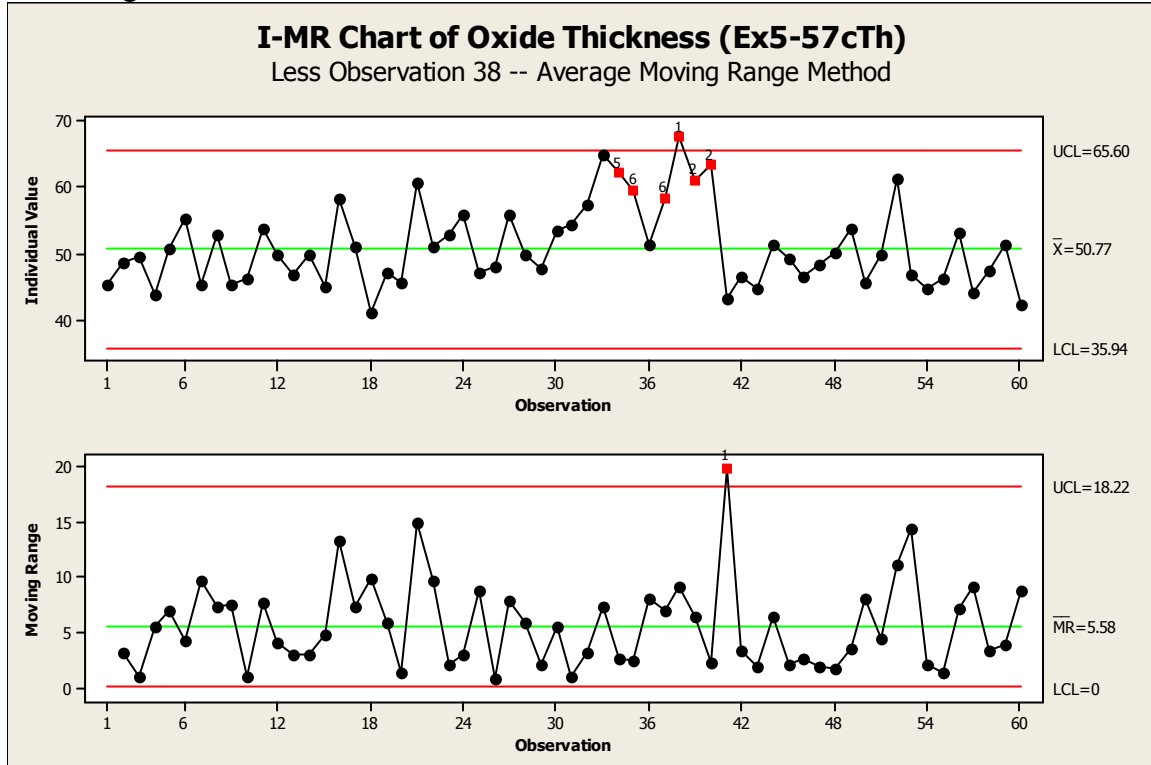
Recall that observations on the Moving Range chart are correlated with those on the Individuals chart—that is, the out-of-control signal on the MR chart for observation 41 is reflected by the shift between observations 40 and 41 on the Individuals chart.

Remove observation 38 and recalculate control limits.

Chapter 5 Exercise Solutions

5-66 (a) continued

Excluding observation 38 from calculations:



Test Results for I Chart of Ex5-57cTh

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 38

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 38, 39, 40

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 34, 39, 40

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on one side of CL).

Test Failed at points: 35, 37, 38, 39, 40

Test Results for MR Chart of Ex5-57cTh

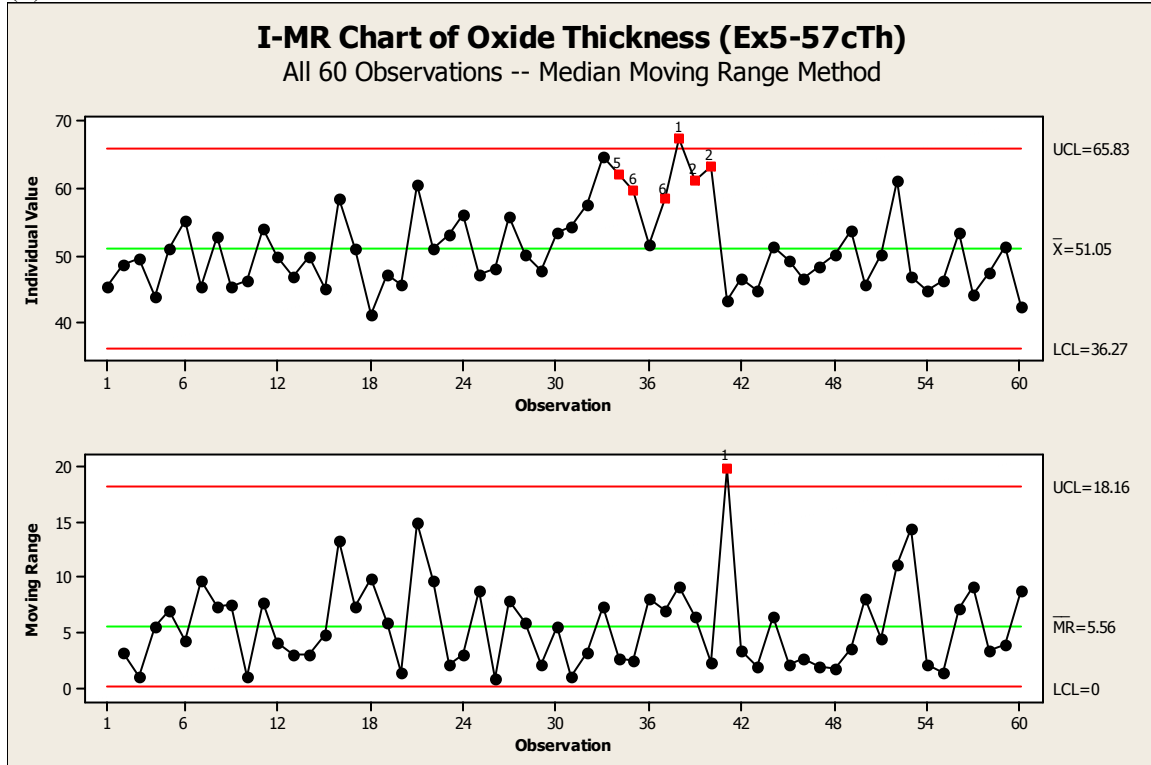
TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 41

Chapter 5 Exercise Solutions

5-66 continued

(b)



Test Results for I Chart of Ex5-57cTh

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 38

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 38, 39, 40

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 34, 39, 40

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on one side of CL).

Test Failed at points: 35, 37, 38, 39, 40

Test Results for MR Chart of Ex5-57cTh

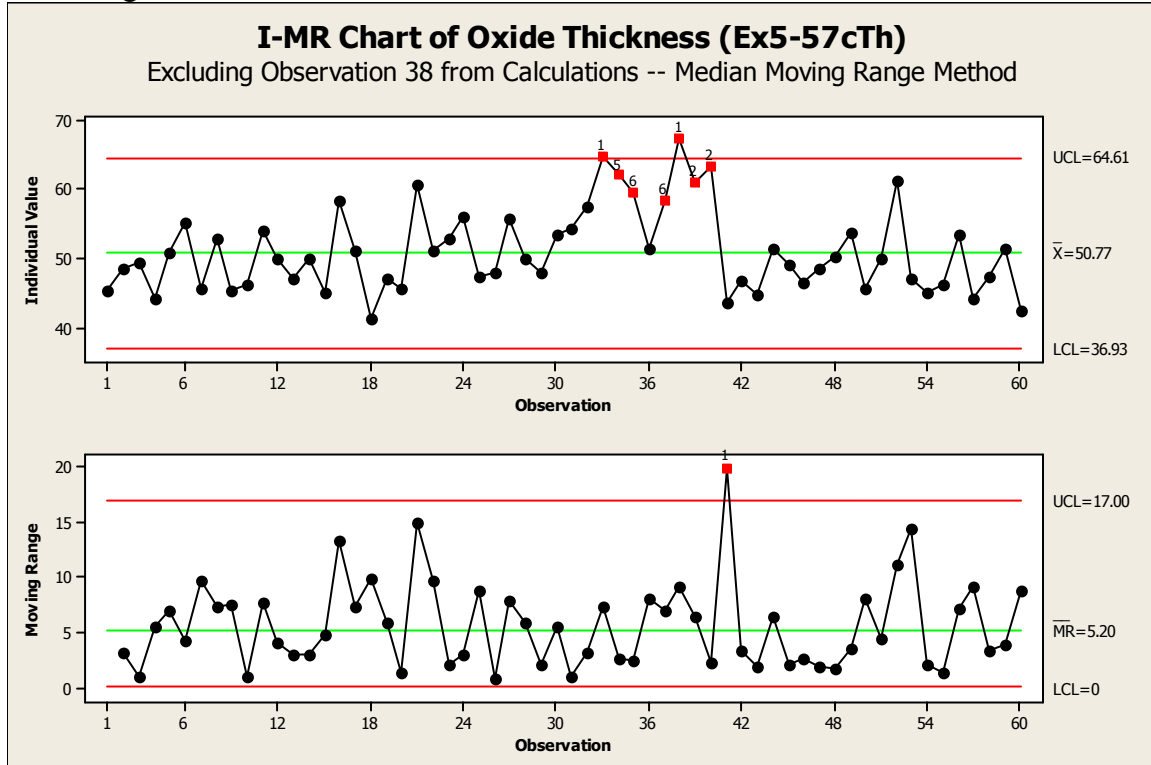
TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 41

Chapter 5 Exercise Solutions

5-66 (b) continued

Excluding observation 38 from calculations:



Test Results for I Chart of Ex5-57cTh

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 33, 38

TEST 2. 9 points in a row on same side of center line.

Test Failed at points: 38, 39, 40

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 34, 39, 40

TEST 6. 4 out of 5 points more than 1 standard deviation from center line (on one side of CL).

Test Failed at points: 35, 37, 38, 39, 40

Test Results for MR Chart of Ex5-57cTh

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 41

(c)

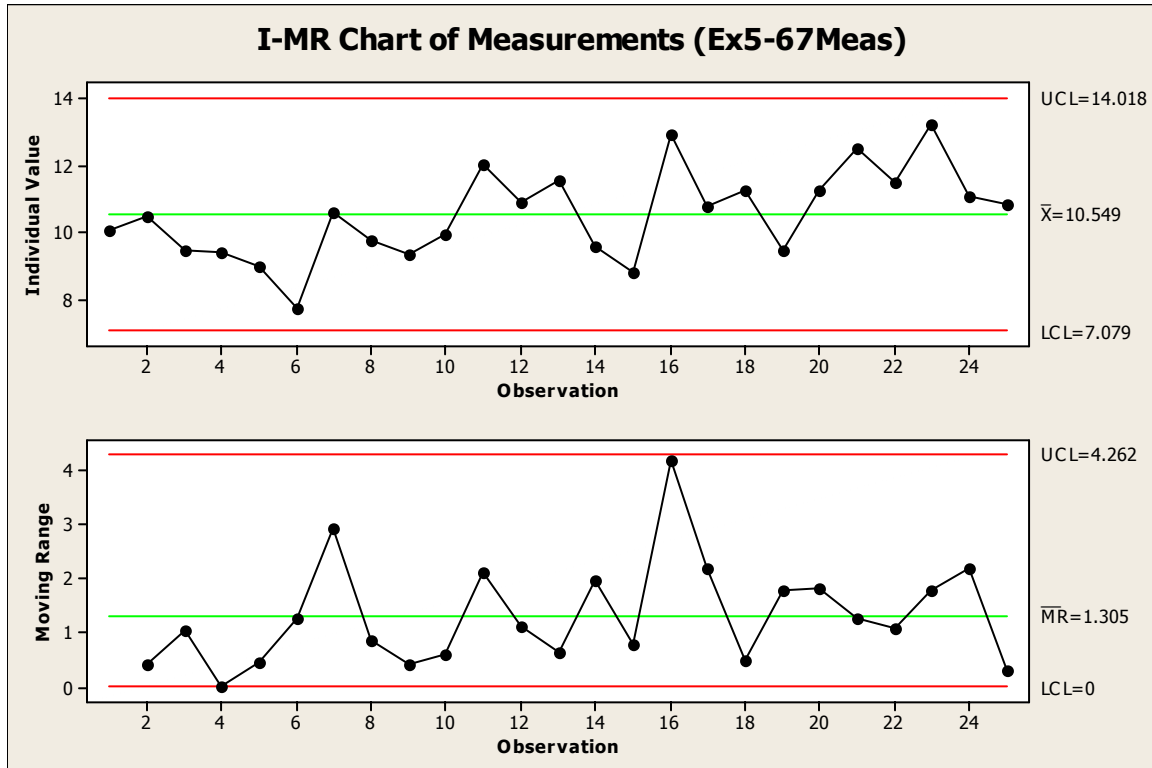
The control limits estimated by the median moving range are tighter and detect the shift in process level at an earlier sample, 33.

Chapter 5 Exercise Solutions

5-67 (5-57).

(a)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



$$\hat{\sigma}_x = \bar{R} / d_2 = 1.305 / 1.128 = 1.157$$

(b)

MTB > Stat > Basic Statistics > Descriptive Statistics

Descriptive Statistics: Ex5-67Meas				
	Total			
Variable	Count	Mean	StDev	Median
Ex5-67Meas	25	10.549	1.342	10.630

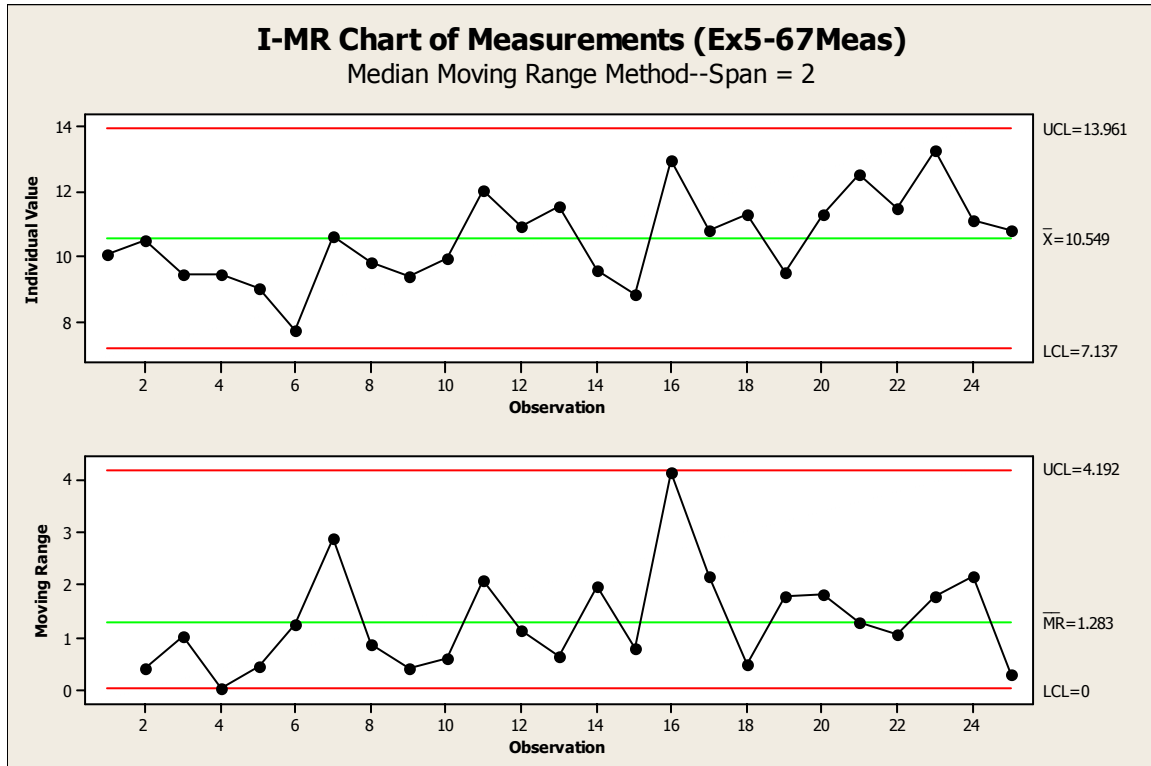
$$\hat{\sigma}_x = S / c_4 = 1.342 / 0.7979 = 1.682$$

Chapter 5 Exercise Solutions

5-67 continued

(c)

MTB > Stat > Control Charts > Variables Charts for Individuals > I-MR



$$\hat{\sigma}_x = \bar{R} / d_2 = 1.283 / 1.128 = 1.137$$

(d)

$$\text{Average MR3 Chart: } \hat{\sigma}_x = \bar{R} / d_2 = 2.049 / 1.693 = 1.210$$

$$\text{Average MR4 Chart: } \hat{\sigma}_x = \bar{R} / d_2 = 2.598 / 2.059 = 1.262$$

$$\text{Average MR19 Chart: } \hat{\sigma}_x = \bar{R} / d_2 = 5.186 / 3.689 = 1.406$$

$$\text{Average MR20 Chart: } \hat{\sigma}_x = \bar{R} / d_2 = 5.36 / 3.735 = 1.435$$

(e)

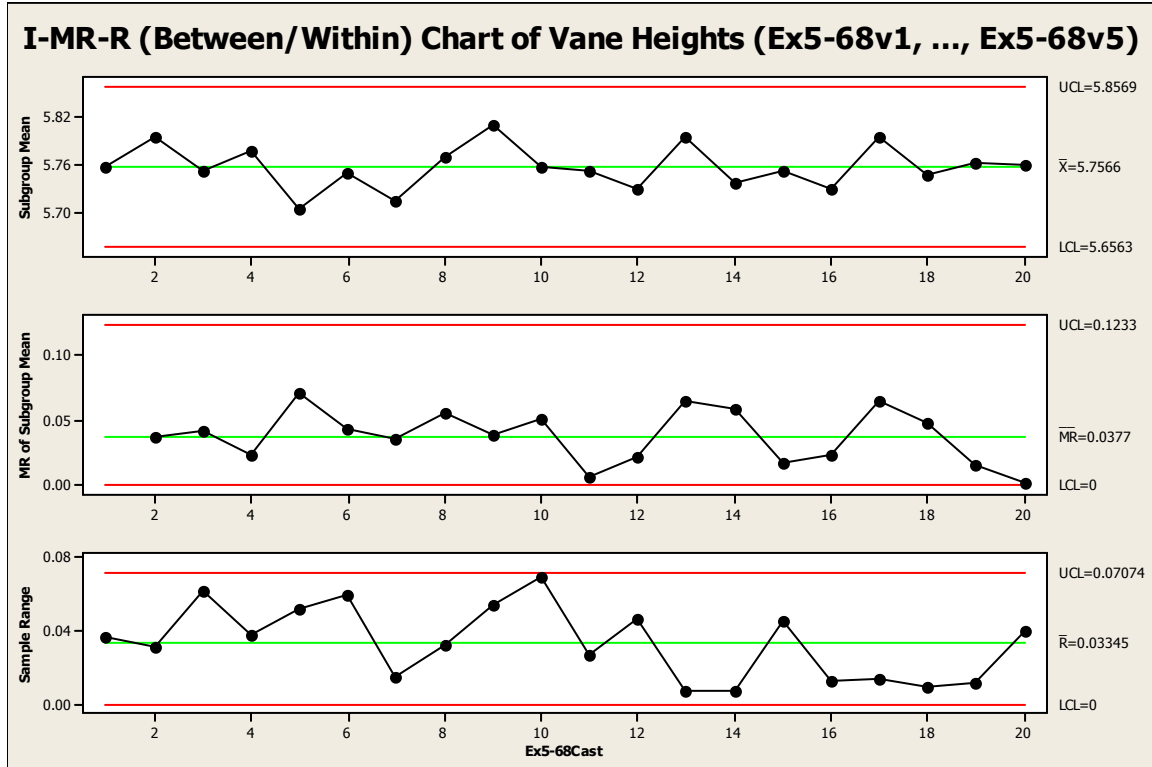
As the span of the moving range is increased, there are fewer observations to estimate the standard deviation, and the estimate becomes less reliable. For this example, σ gets larger as the span increases. This tends to be true for unstable processes.

Chapter 5 Exercise Solutions

5-68 (5-58).

MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S
(Between/Within)

Select "I-MR-R/S Options, Estimate" and choose R-bar method to estimate standard deviation



I-MR-R/S Standard Deviations of Ex5-68v1, ..., Ex5-68v5

Standard Deviations	
Between	0.0328230
Within	0.0143831
Between/Within	0.0358361

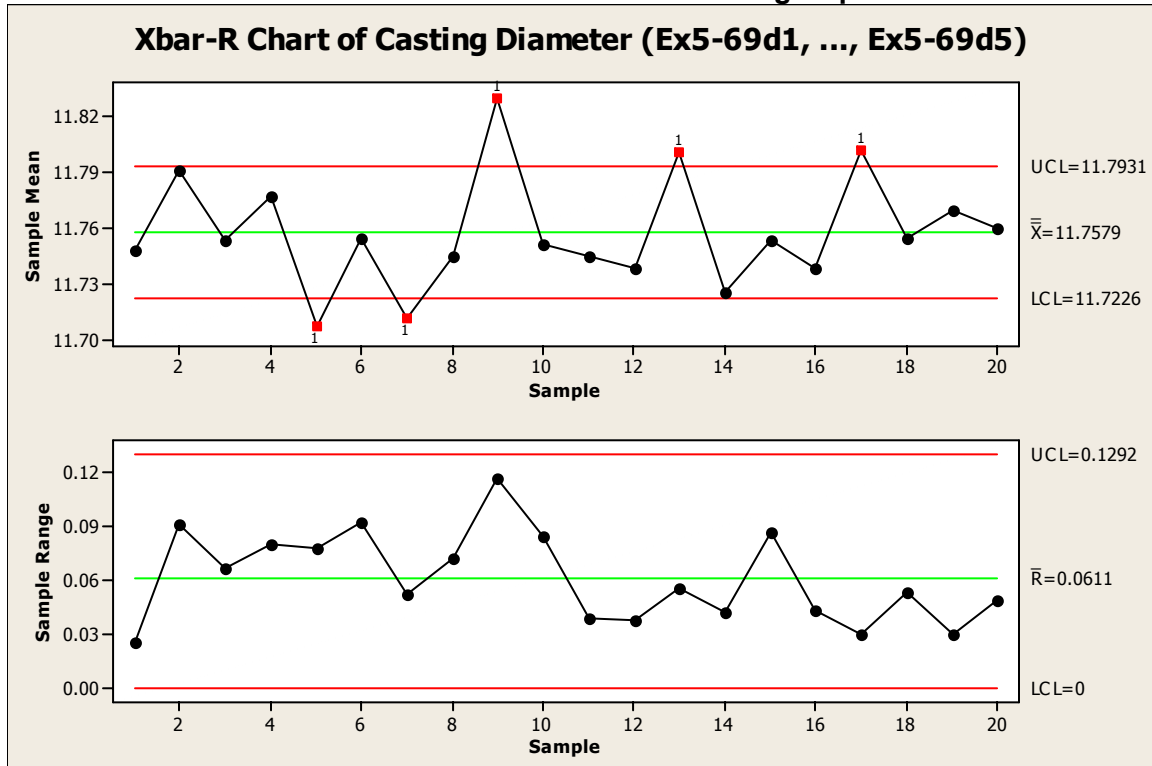
The Individuals and Moving Range charts for the subgroup means are identical. When compared to the s chart for all data, the R chart tells the same story—same data pattern and no out-of-control points. For this example, the control schemes are identical.

Chapter 5 Exercise Solutions

5-69 (5-59).

(a)

MTB > Stat > Control Charts > Variables Charts for Subgroups > Xbar-R



Xbar-R Chart of Ex5-69d1, ..., Ex5-69d5

Test Results for Xbar Chart of Ex5-69d1, ..., Ex5-69d5

TEST 1. One point more than 3.00 standard deviations from center line.

Test Failed at points: 5, 7, 9, 13, 17

TEST 5. 2 out of 3 points more than 2 standard deviations from center line (on one side of CL).

Test Failed at points: 7

(b)

Though the R chart is in control, plot points on the \bar{x} chart bounce below and above the control limits. Since these are high precision castings, we might expect that the diameter of a single casting will not change much with location. If no assignable cause can be found for these out-of-control points, we may want to consider treating the averages as an Individual value and graphing “between/within” range charts. This will lead to a understanding of the greatest source of variability, between castings or within a casting.

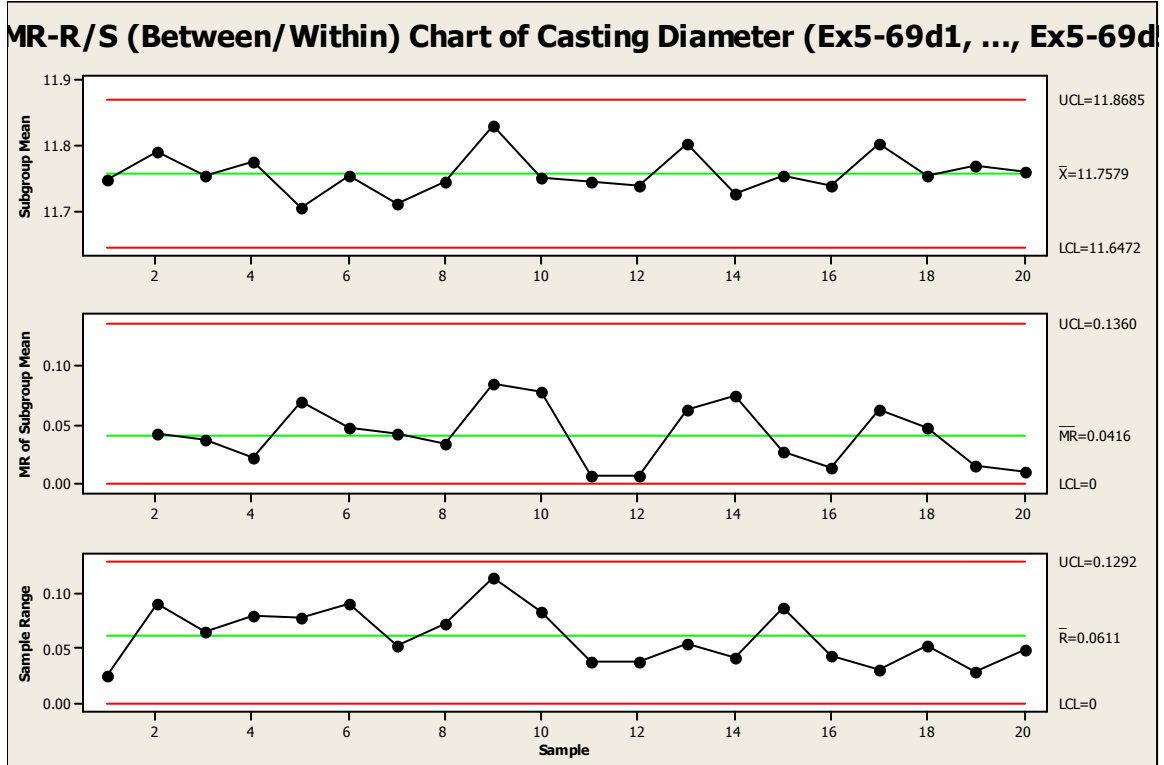
Chapter 5 Exercise Solutions

5-69 continued

(c)

MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S
(Between/Within)

Select “I-MR-R/S Options, Estimate” and choose R-bar method to estimate standard deviation



I-MR-R/S (Between/Within) Chart of Ex5-69d1, ..., Ex5-69d5

I-MR-R/S Standard Deviations of Ex5-69d1, ..., Ex5-69d5

Standard Deviations	
Between	0.0349679
Within	0.0262640
Between/Within	0.0437327

(d)

We are taking several diameter measurements on a single precision casting.

(e)

The “within” chart is the usual R chart ($n > 1$). It describes the measurement variability within a sample (variability in diameter of a single casting). Though the nature of this process leads us to believe that the diameter at any location on a single casting does not change much, we should continue to monitor “within” to look for wear, damage, etc., in the wax mold.

Chapter 5 Exercise Solutions

5-70 (5-60).

(a)

Both total process variability and the overall process average could be estimated from a single measurement on one wafer from each lot. Individuals X and Moving Range charts should be used for process monitoring.

(b)

Assuming that each wafer is processed separately, within-wafer variability could be monitored with a standard $\bar{X} - R$ control chart. The data from each wafer could also be used to monitor between-wafer variability by maintaining an individuals X and moving range chart for each of the five fixed positions. The Minitab “between/within” control charts do this in three graphs: (1) wafer mean (\bar{x}_{ww}) is an “individual value”, (2) moving range is the difference between successive wafers, and (3) sample range is the difference within a wafer (R_{ww}). Alternatively, a multivariate process control technique could be used.

(c)

Both between-wafer and total process variability could be estimated from measurements at one point on five consecutive wafers. If it is necessary to separately monitor the variation at each location, then either five $\bar{X} - R$ charts or some multivariate technique is needed. If the positions are essentially identical, then only one location, with one $\bar{X} - R$ chart, needs to be monitored.

(d)

Within-wafer variability can still be monitored with randomly selected test sites. However, no information will be obtained about the pattern of variability within a wafer.

(e)

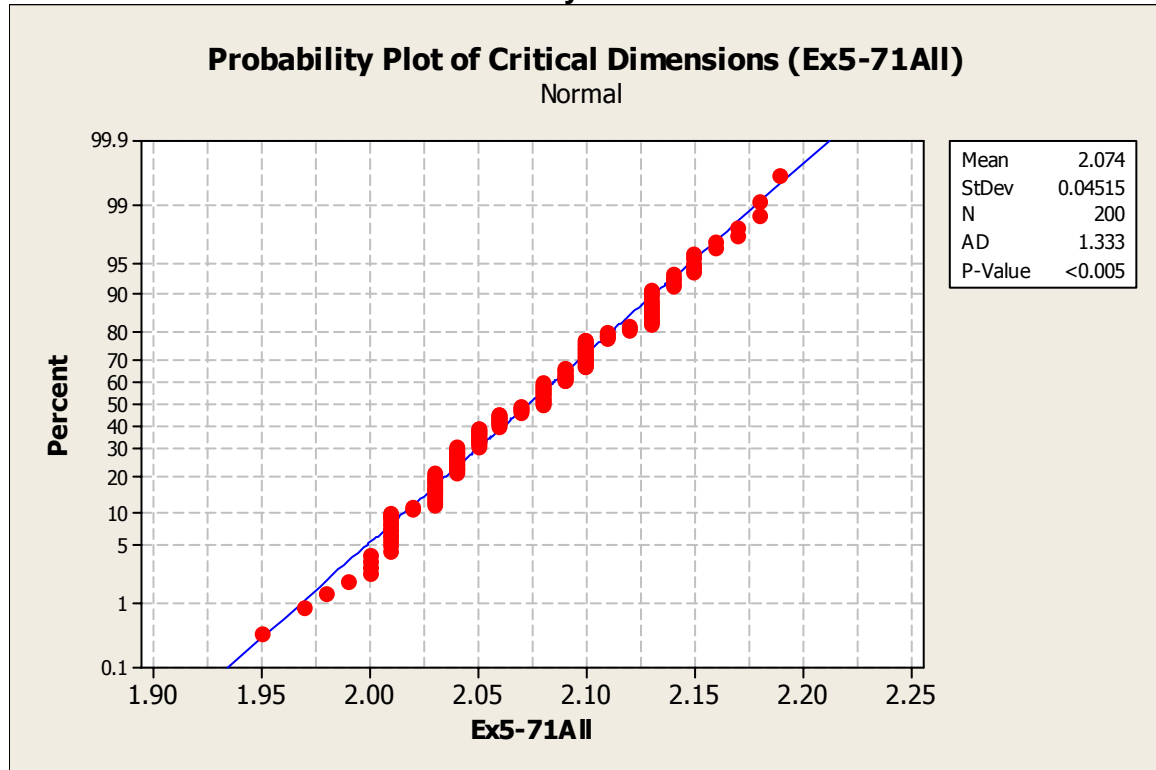
The simplest scheme would be to randomly select one wafer from each lot and treat the average of all measurements on that wafer as one observation. Then a chart for individual x and moving range would provide information on lot-to-lot variability.

Chapter 5 Exercise Solutions

5-71 (5-61).

(a)

MTB > Stat > Basic Statistics > Normality Test



Although the p-value is very small, the plot points do fall along a straight line, with many repeated values. The wafer critical dimension is approximately normally distributed.

The natural tolerance limits (± 3 sigma above and below mean) are:

$$\bar{x} = 2.074, s = 0.04515$$

$$\text{UNTL} = \bar{x} + 3s = 2.074 + 3(0.04515) = 2.209$$

$$\text{LNLT} = \bar{x} - 3s = 2.074 - 3(0.04515) = 1.939$$

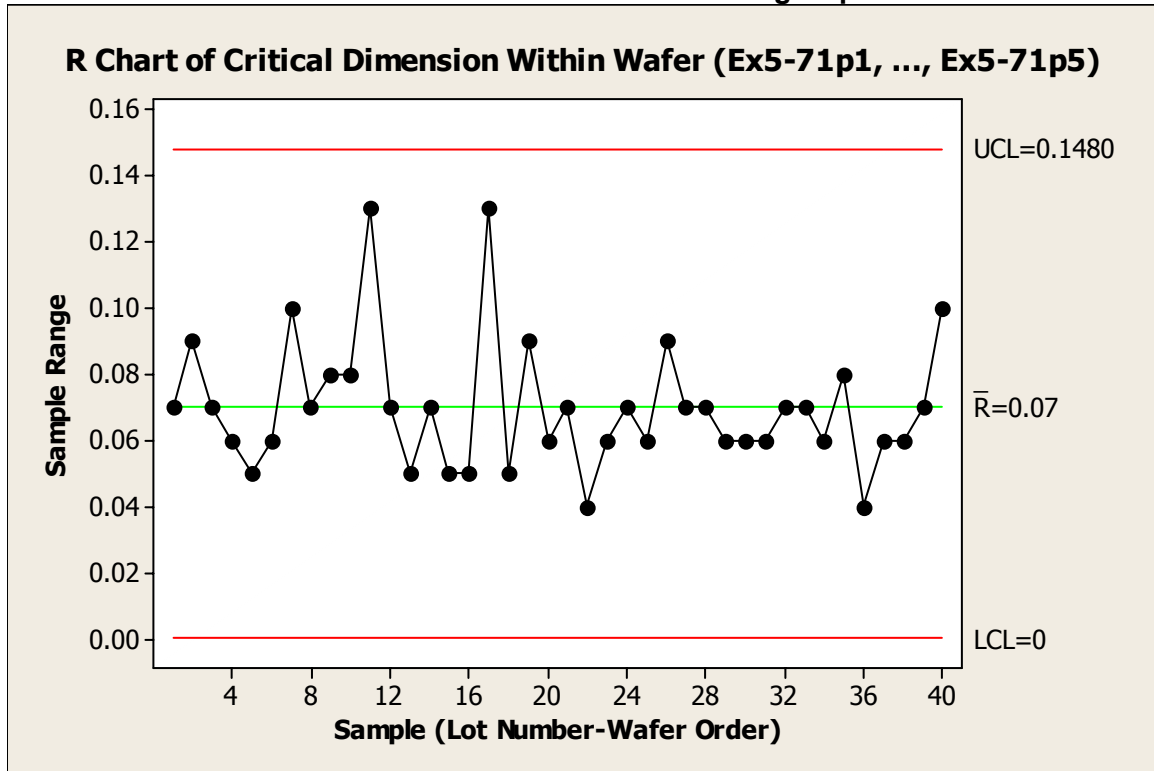
Chapter 5 Exercise Solutions

5-71 continued

(b)

To evaluate within-wafer variability, construct an R chart for each sample of 5 wafer positions (two wafers per lot number), for a total of 40 subgroups.

MTB > Stat > Control Charts > Variables Charts for Subgroups > R



The Range chart is in control, indicating that within-wafer variability is also in control.

Chapter 5 Exercise Solutions

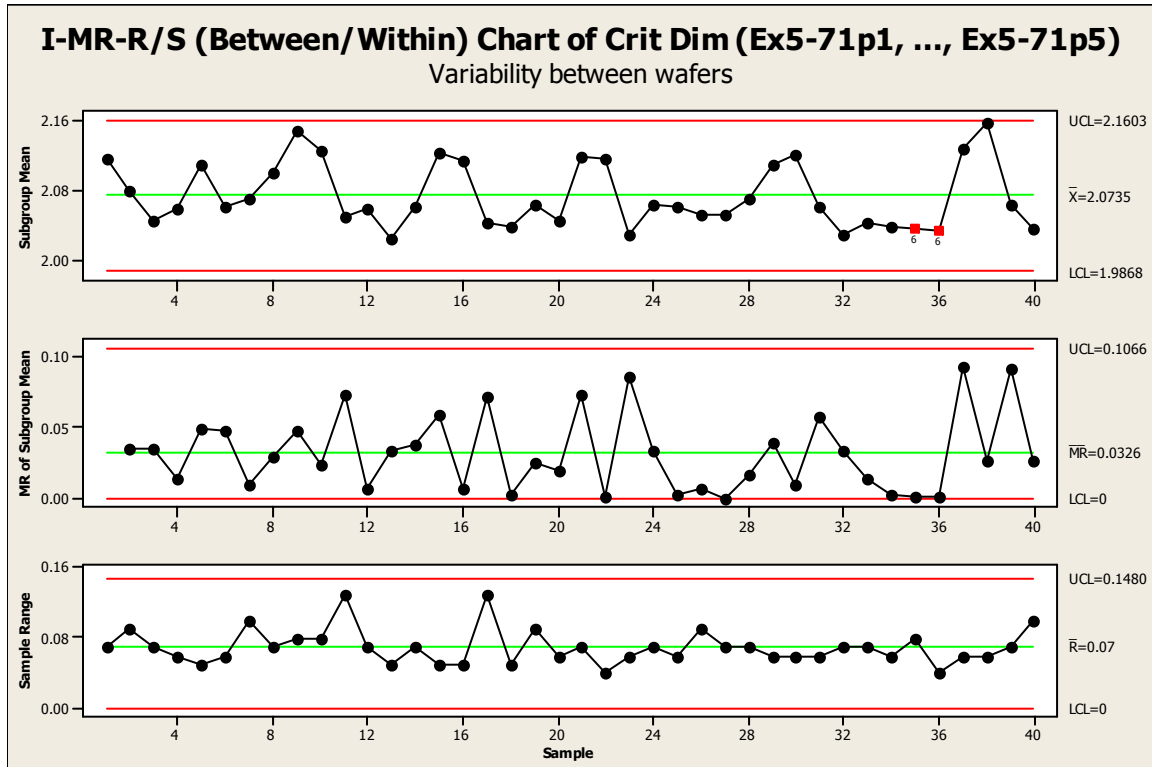
5-71 continued

(c)

To evaluate variability between wafers, set up Individuals and Moving Range charts where the \bar{x} statistic is the average wafer measurement and the moving range is calculated between two wafer averages.

MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)

Select “I-MR-R/S Options, Estimate” and choose R-bar method to estimate standard deviation



I-MR-R/S Standard Deviations of Ex5-71p1, ..., Ex5-71p5

Standard Deviations	
Between	0.0255911
Within	0.0300946
Between/Within	0.0395043

Both “between” control charts (Individuals and Moving Range) are in control, indicating that between-wafer variability is also in-control. The “within” chart (Range) is not required to evaluate variability between wafers.

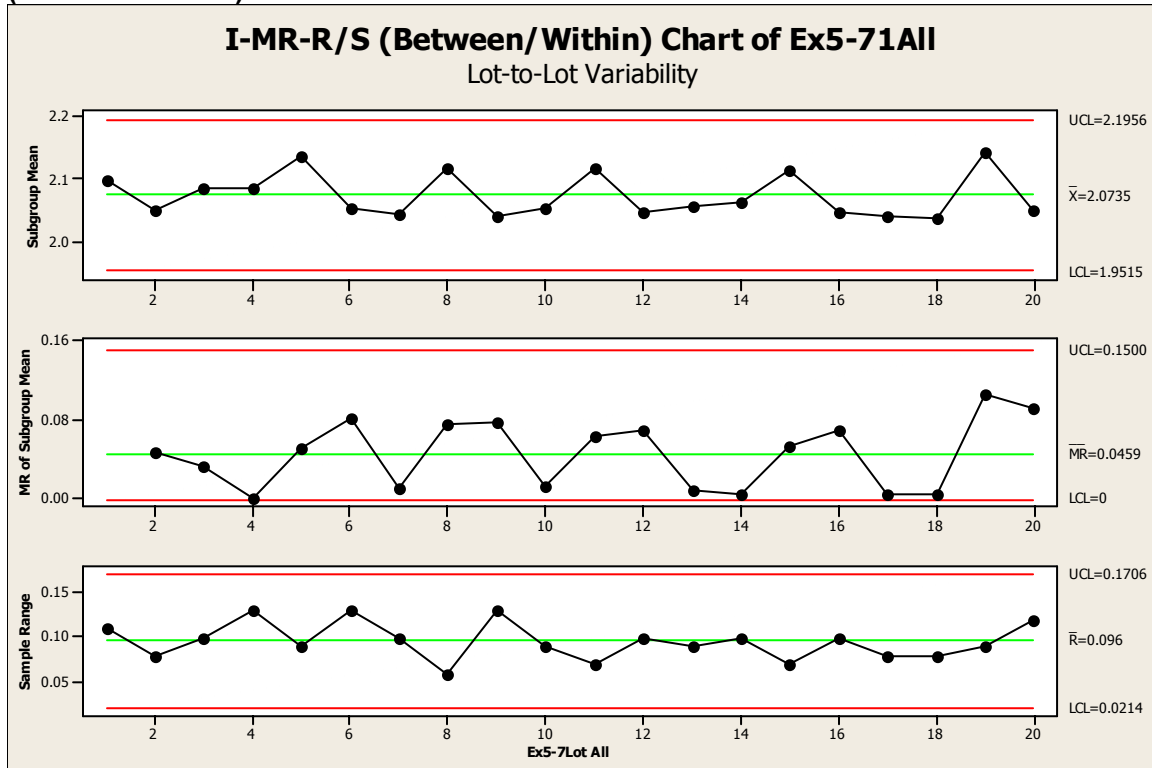
Chapter 5 Exercise Solutions

5-71 continued

(d)

To evaluate lot-to-lot variability, three charts are needed: (1) lot average, (2) moving range between lot averages, and (3) range within a lot—the Minitab “between/within” control charts.

MTB > Stat > Control Charts > Variables Charts for Subgroups > I-MR-R/S (Between/Within)



I-MR-R/S Standard Deviations of Ex5-71All

Standard Deviations	
Between	0.0394733
Within	0.0311891
Between/Within	0.0503081

All three control charts are in control, indicating that the lot-to-lot variability is also in-control.